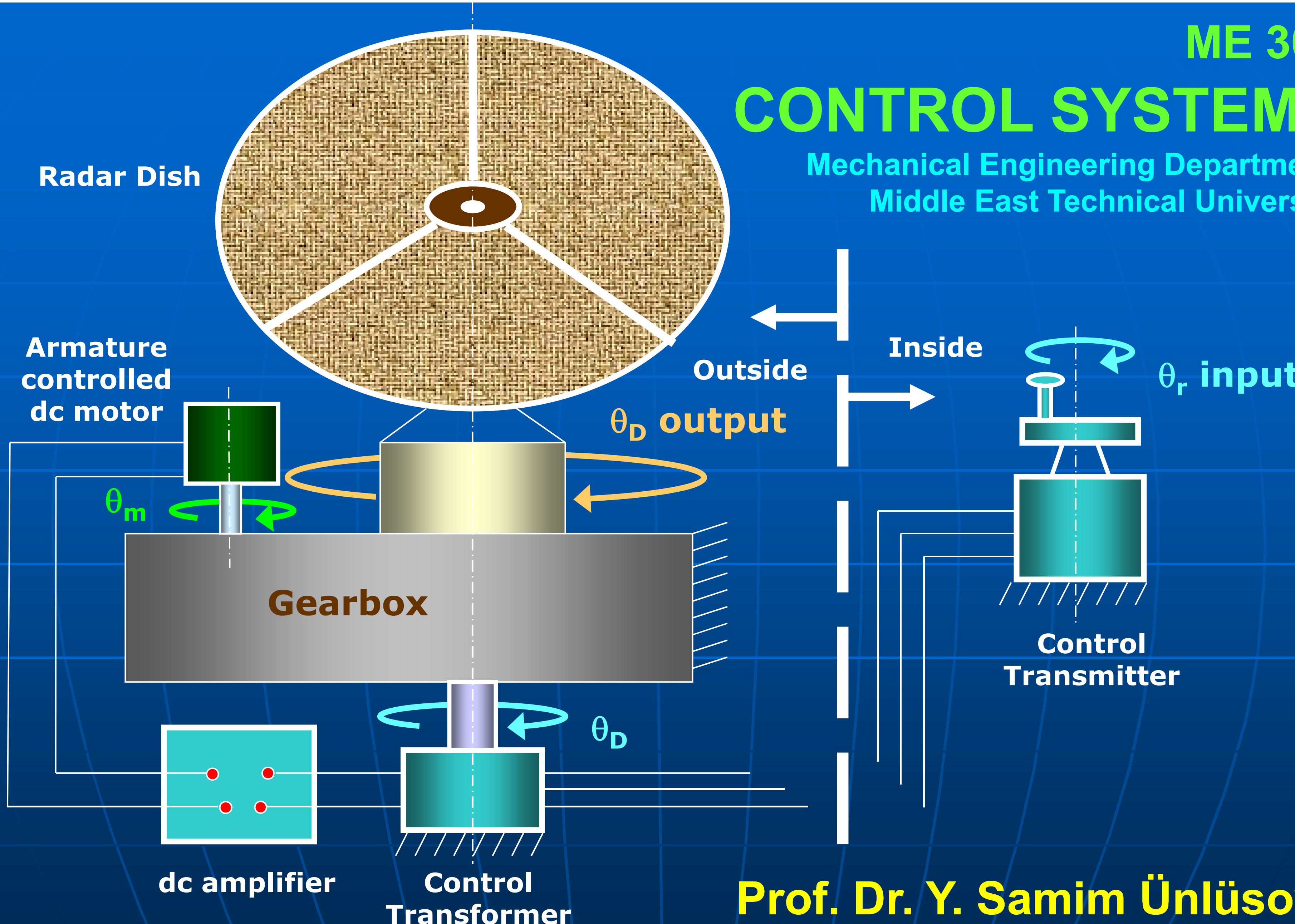
ME 304
CONTROL SYSTEMS

Mechanical Engineering Department,
Middle East Technical University



Prof. Dr. Y. Samim Ünlüsoy

CH III

COURSE OUTLINE

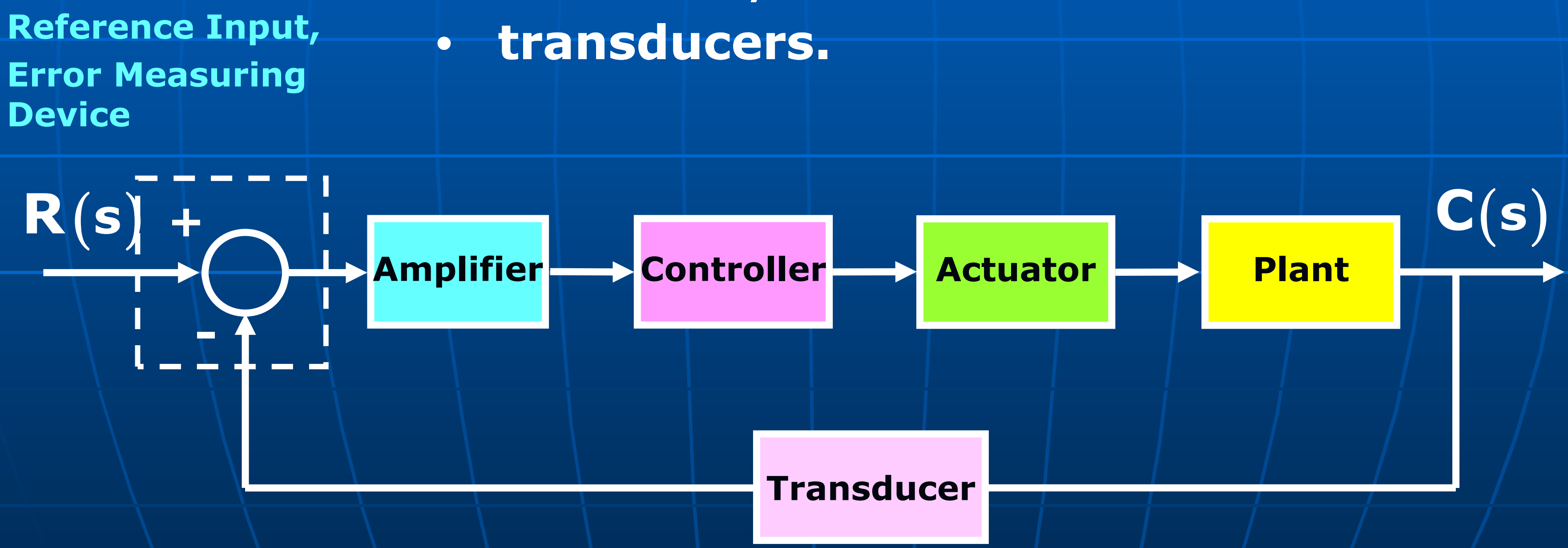
CONTROL SYSTEM COMPONENTS

OBJECTIVES

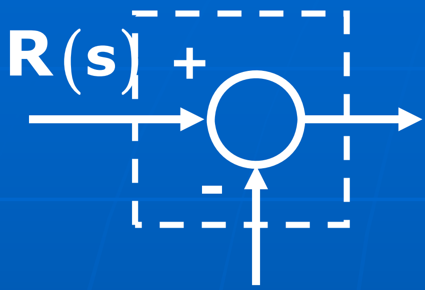
- **Getting familiar with some important electro-hydro-mechanical components commonly used in control systems and understand their functions.**
- **Obtaining the input-output relations, overall transfer functions, and block diagrams of these components.**

CONTROL SYSTEM COMPONENTS

- **Basic components of a control system**
  - **reference (command) input generators,**
  - **error measuring devices (comparators),**
  - **amplifiers,**
  - **actuators, and**
  - **transducers.**

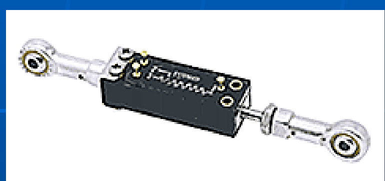


Reference Input, Error Detector

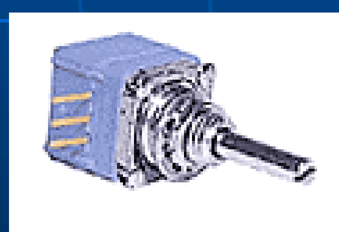


POTENTIOMETERS

Dorf&Bishop Table 2.5, p. 66



- Linear (translational) potentiometers

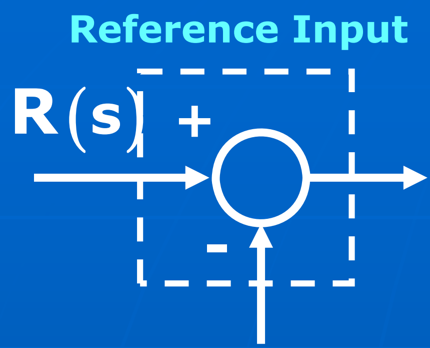


- Rotary potentiometers

- **Potentiometers** can be used
    - to set reference input, or

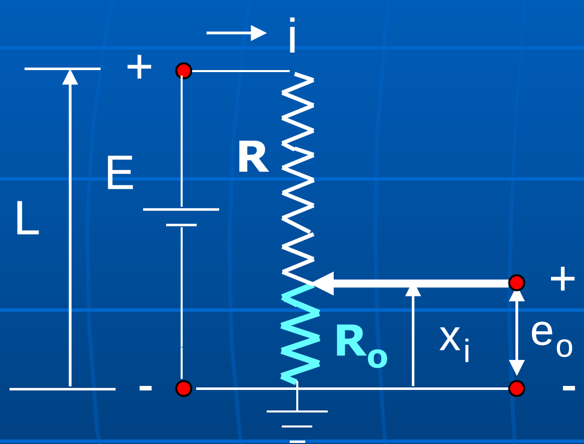
  as
    - error detectors , or
    - transducers.

- When a manually set **reference input** is to be provided in the form of a voltage signal, adjustment of a calibrated dial or slider allows the selection of any voltage from 0 to E.



LINEAR POTENTIOMETERS

- **Linear (translational, slider) potentiometer as an input device or transducer**



Translational Potentiometer

$$E = Ri$$

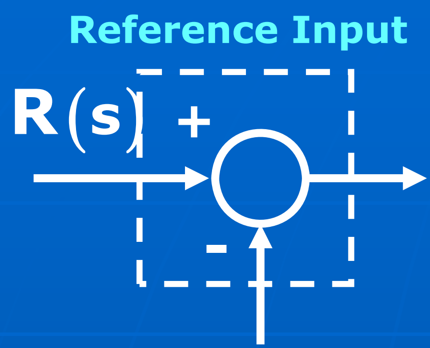
$$e_o = R_o i$$

$$e_o = \frac{R_o}{R} E$$

$$R_o = \frac{R}{L} x_i$$

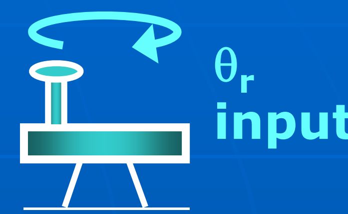
$$e_o = \left(\frac{E}{L}\right) x_i$$

$$K_p = \frac{E}{L}$$



ROTARY POTENTIOMETERS

- **Rotational potentiometer as an input device or transducer**



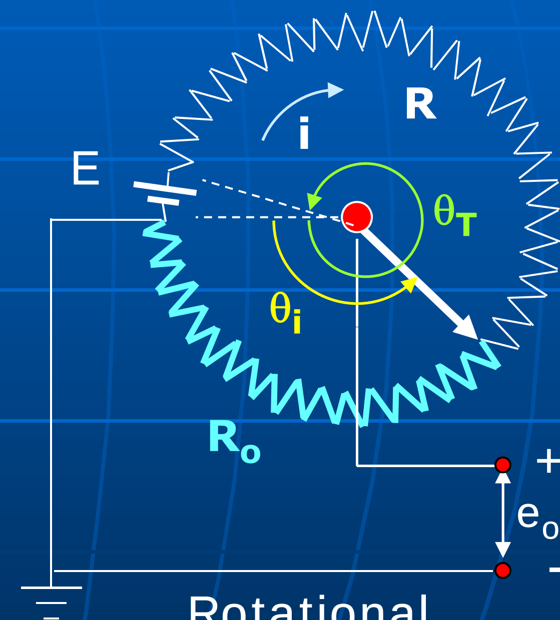
$$E = Ri$$

$$e_o = R_o i$$

$$e_o = \frac{R_o}{R} E$$

$$R_o = \frac{R}{\theta_T} \theta_i$$

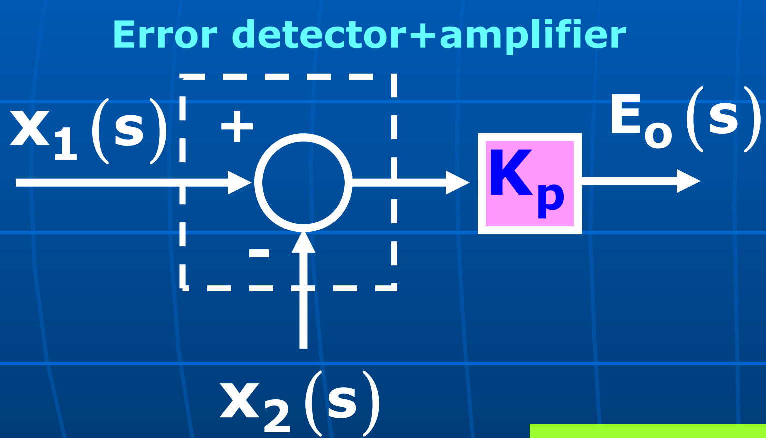
$$e_o = \left(\frac{E}{\theta_T} \right) \theta_i$$

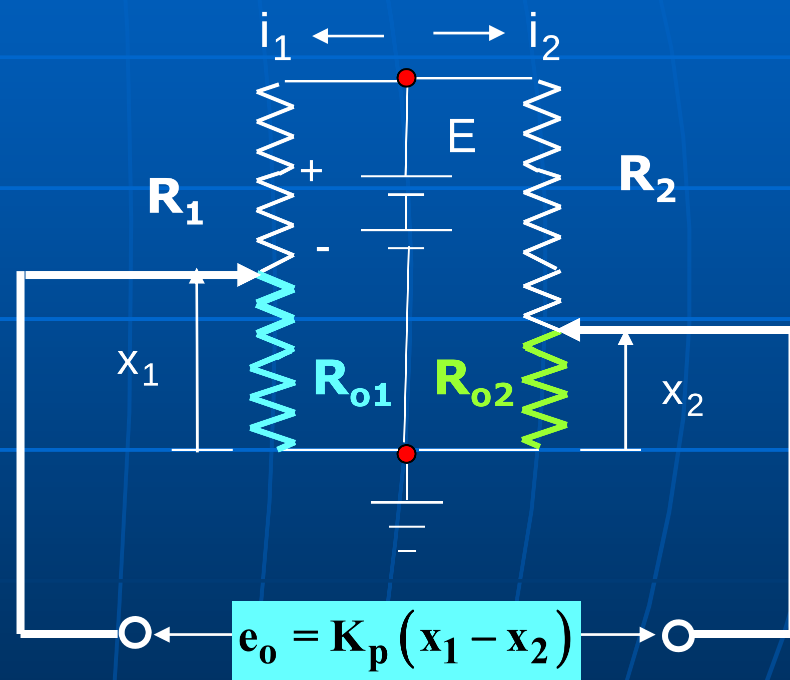


Rotational Potentiometer

POTENTIOMETERS

- **Linear (translational) potentiometers as error detector**

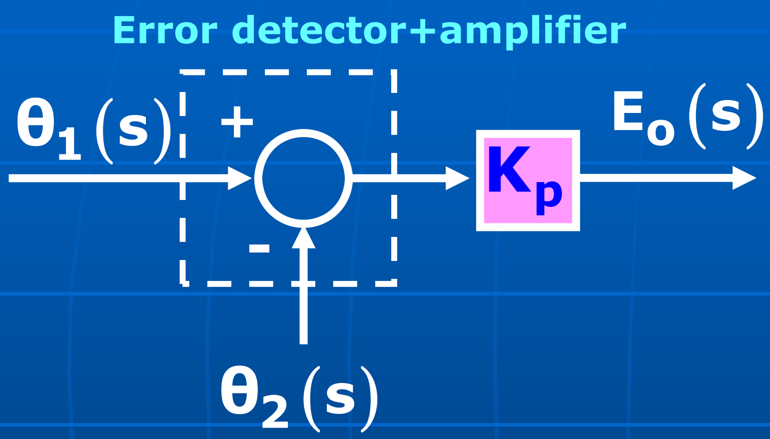


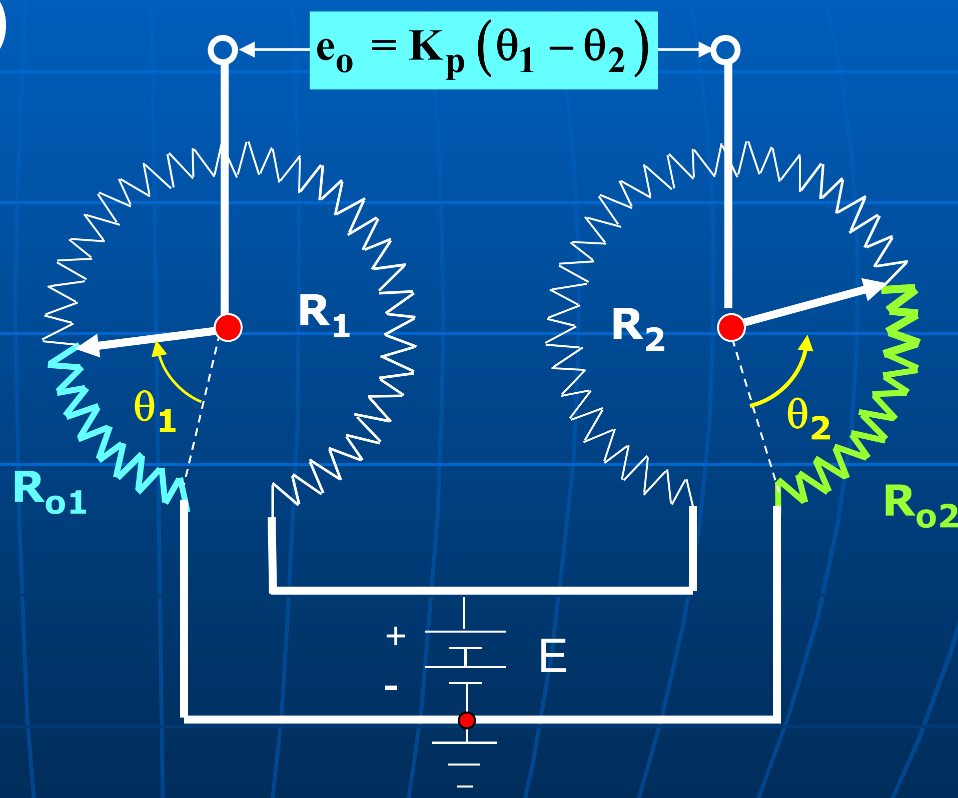


$$K_p = \frac{E}{L}$$

POTENTIOMETERS

- **Rotational potentiometers as error detector**





TACHOMETER

Nise p.547-548, Dorf&Bishop Table 2.5, p. 66

- Tachometer is essentially a special dc generator producing an output voltage proportional to its angular speed.
- They are usually provided as an integral part of electric motors.

$$\omega(t) = \dot{\theta}(t)$$

$$\Omega(s) = \dot{\theta}(s) = s\theta(s)$$

$$G_{E\Omega}(s) = \frac{E(s)}{\Omega(s)} = K_T$$

$$G_{E\theta}(s) = \frac{E(s)}{\theta(s)} = K_T s$$

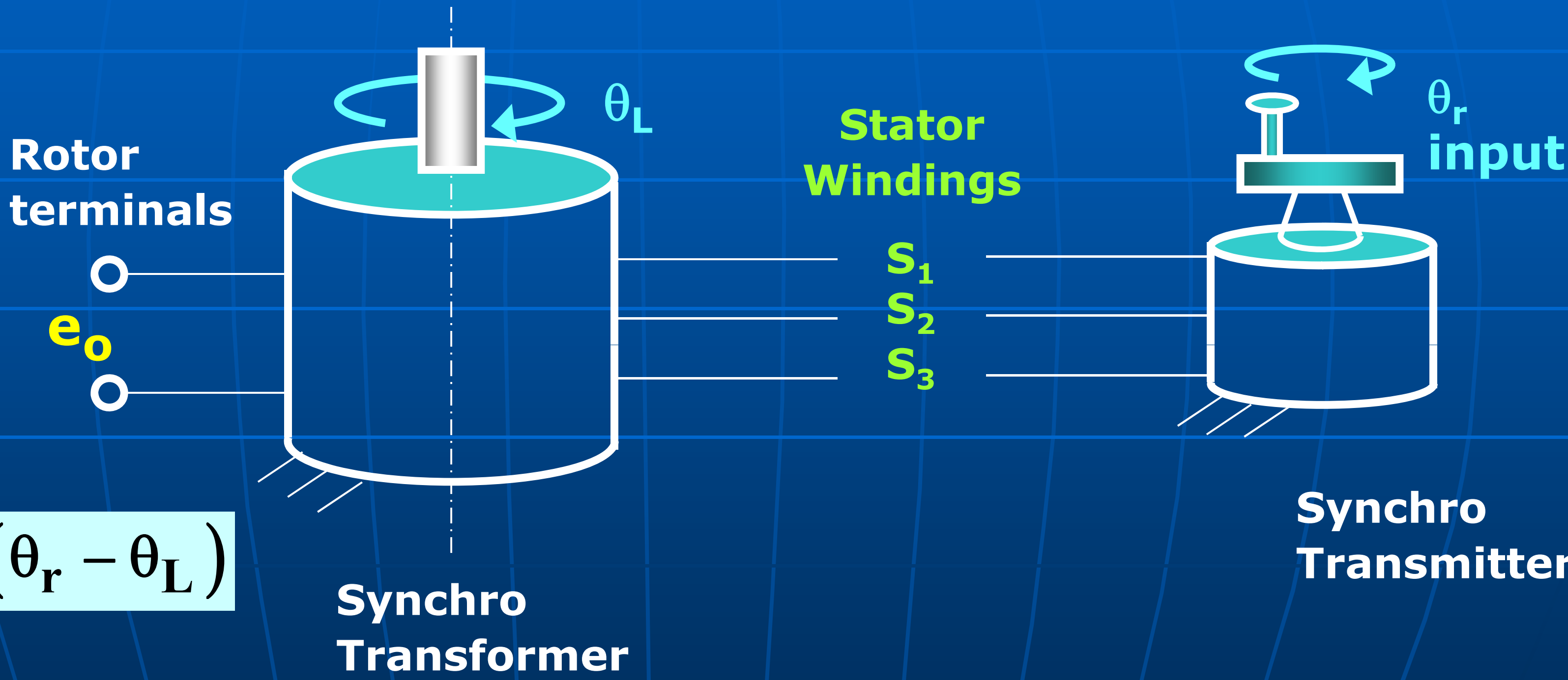
SYNCHRO

- A synchro is a device which produces a voltage as a function of the angular position of its rotor.
- There are various types of synchros, and only a single application of a synchro transmitter and a synchro control transformer will be given here.

	Inputs	Outputs
Synchro transmitter (generator)	-Angular position of rotor shaft.	-Voltages induced on stator windings S1, S2, S3.
Synchro control transformer	-Angular position of rotor shaft. -Voltages applied to stator windings	-Voltage induced between rotor terminals.

SYNCHRO

- An error detector to convert the difference between the angular positions of two shafts is obtained by connecting a transmitter and a control transformer.

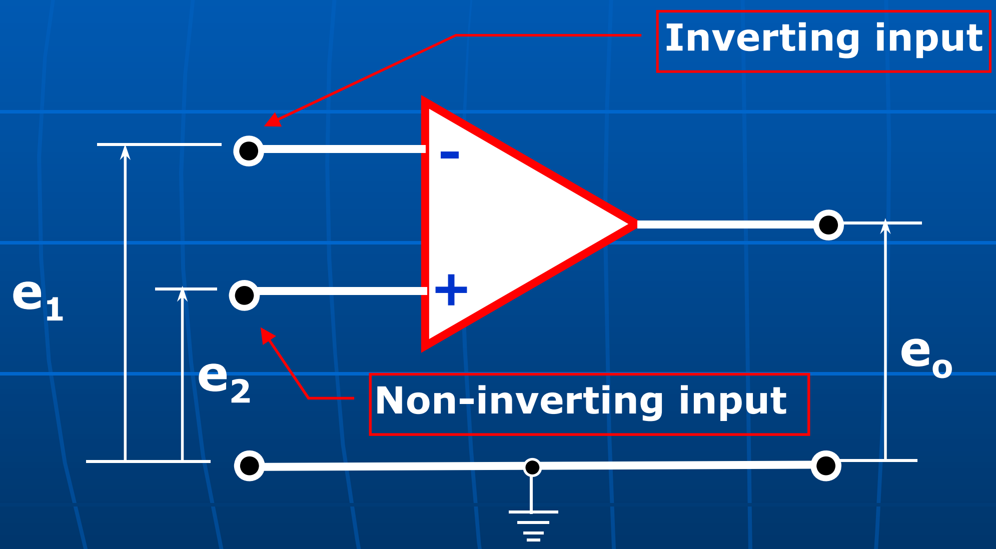


$$e_o = K_s(\theta_r - \theta_L)$$

OPERATIONAL AMPLIFIER

Nise p.64-67, Dorf&Bishop Ex. 2.3, Table 2.5, p. 64 ; Ogata Sect. 3-8

- Operational Amplifiers are commonly used to amplify signals (dc or ac) in sensor circuits.
- No current flows through an ideal operational amplifier.



$$e_o = K(e_2 - e_1)$$

- If e_1=0 : if e_2=0:

$e_o = Ke_2$ $\quad$ $e_o = -Ke_1$

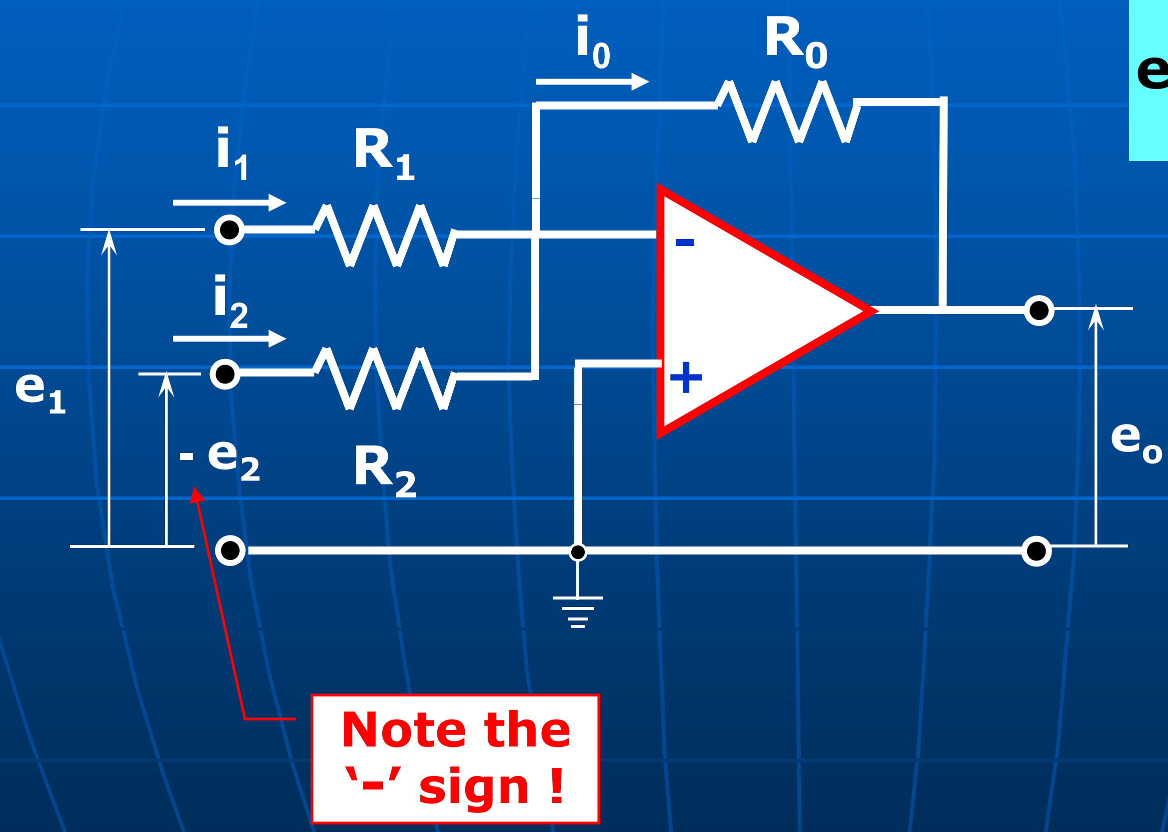
- Large gain may cause instability !

$K \cong 10^5 \text{ to } 10^6 >> 1$

for $f \leq 10\text{Hz}$

OPERATIONAL AMPLIFIER

- Summing Amplifier (Error detector)

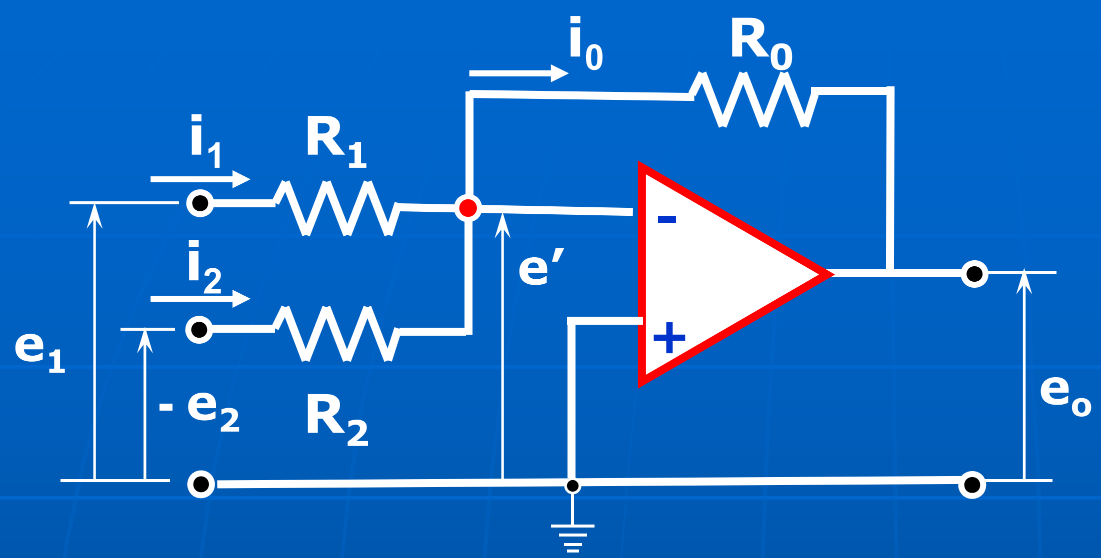


$$e_o = -\left(\frac{R_0}{R_1}e_1 - \frac{R_0}{R_2}e_2\right)$$

For

$R_o = R_1 = R_2$

$$e_o = e_2 - e_1$$



OPERATIONAL AMPLIFIER

- Summing Amplifier (Error detector)

$$e_1 - i_1 R_1 = e', \quad -e_2 - i_2 R_2 = e', \quad i_o = i_1 + i_2$$

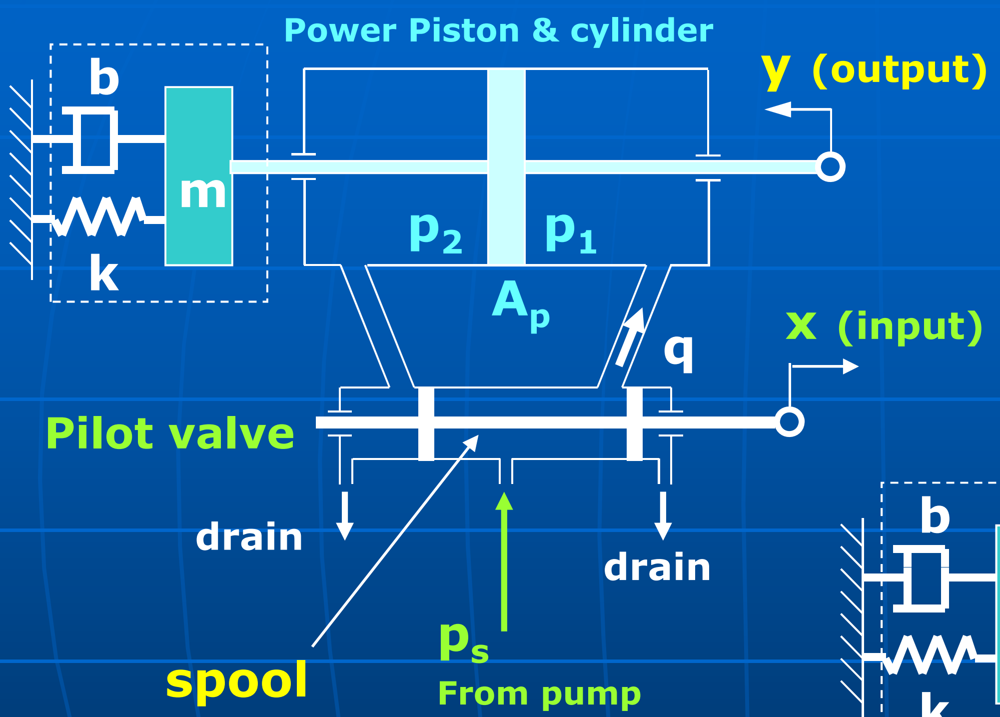
$$e_0 = e' - i_0 R_0 = e' - \left(\frac{e_1 - e'}{R_1}\right) R_0 + \left(\frac{e_2 - e'}{R_2}\right) R_0$$

$$e_0 = -Ke' \qquad K >> 1 \qquad \Rightarrow \qquad e' \cong 0$$

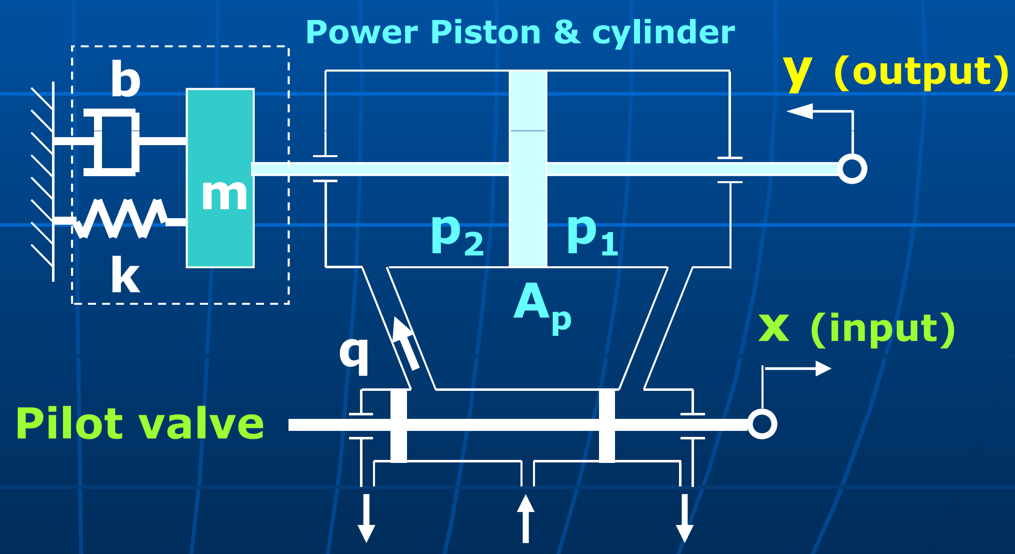
$$e_0 = -\left(\frac{R_0}{R_1} e_1 - \frac{R_0}{R_2} e_2\right)$$

HYDRAULIC SERVOMOTOR

Dorf&Bishop Example 2.6, Table 2.5, p. 65; Ogata Section 4-4



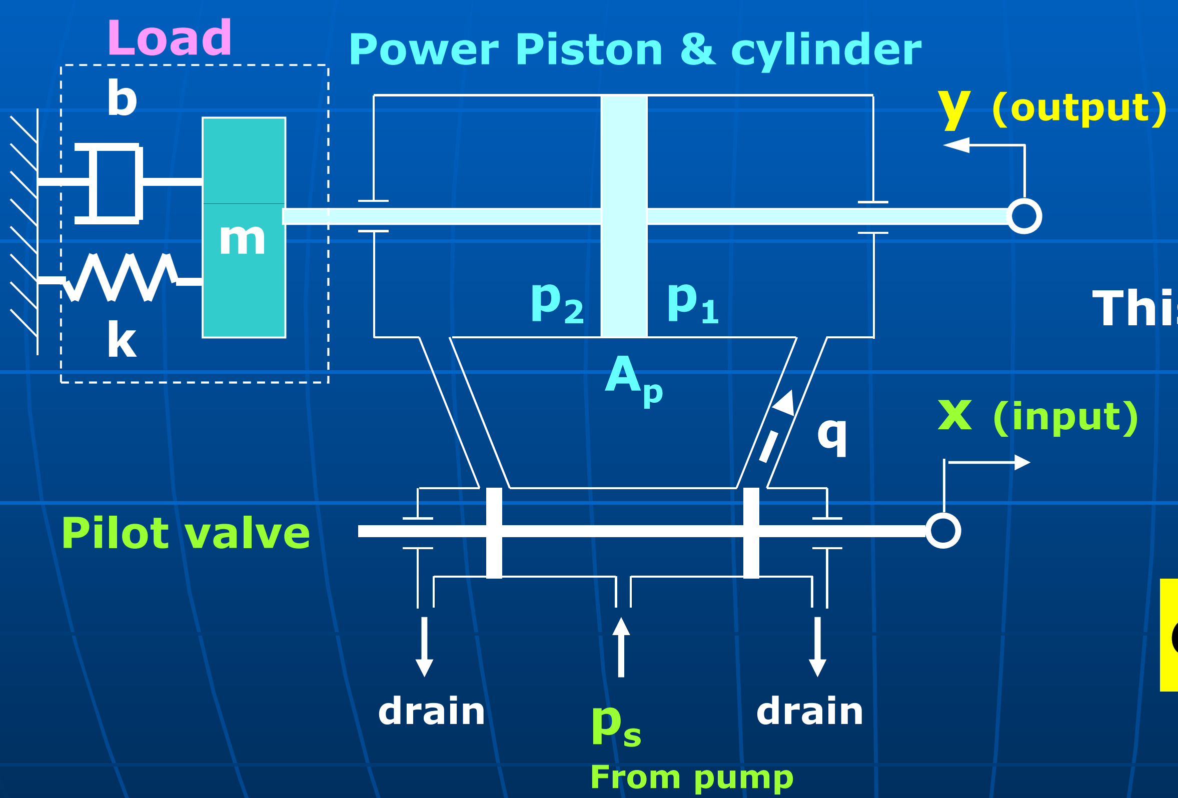
Depending on the pilot valve spool position x, fluid will be directed to the left or right of the power piston.



HYDRAULIC SERVOMOTOR

Dorf&Bishop Example 2.6, Table 2.5, p. 65; Ogata Section 4-4

- **Loads involving large forces can be controlled by applying only small forces.**



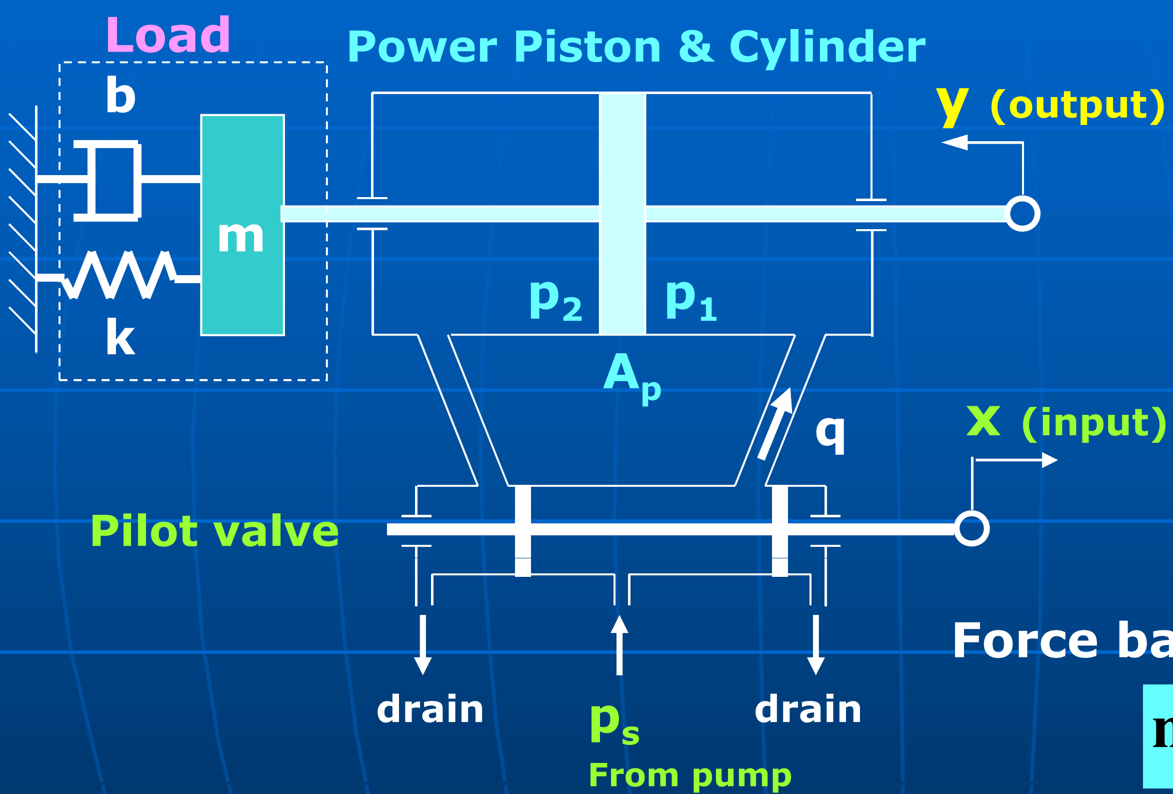
$$\Delta p = p_1 - p_2$$

$$q = f(x, \Delta p)$$

This nonlinear function is linearized around $q_o=0$, $x_o=0$, and $\Delta p_o=0$ to give :

$$q = K_1 x - K_2 \Delta p$$

HYDRAULIC SERVOMOTOR



$$q = K_1 x - K_2 \Delta p$$

$$x = \frac{1}{K_1}\left(q + K_2 \Delta p\right)$$

Power piston:

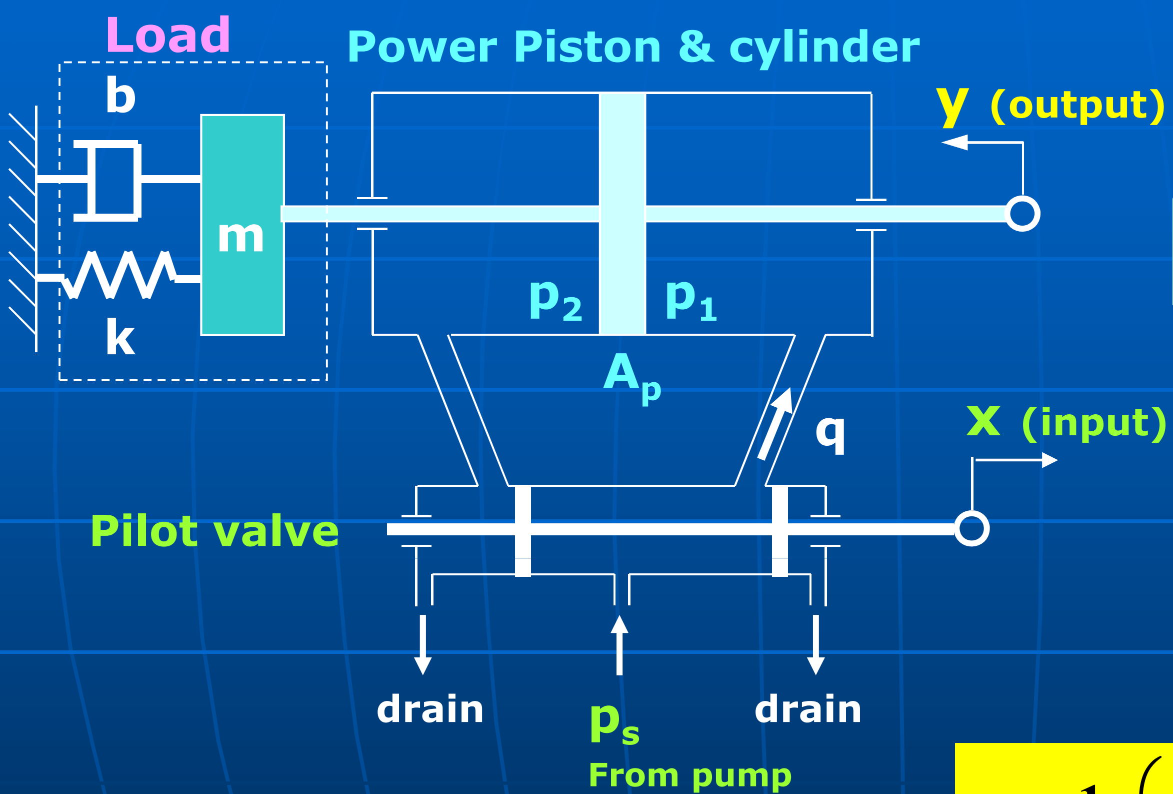
$$q = A_p \dot{y}$$

Force balance on load mass:

$$m\ddot{y} + b\dot{y} + ky = A_p \Delta p$$

$$\Delta p = \frac{1}{A_p}\left(m\ddot{y} + b\dot{y} + ky\right)$$

HYDRAULIC SERVOMOTOR



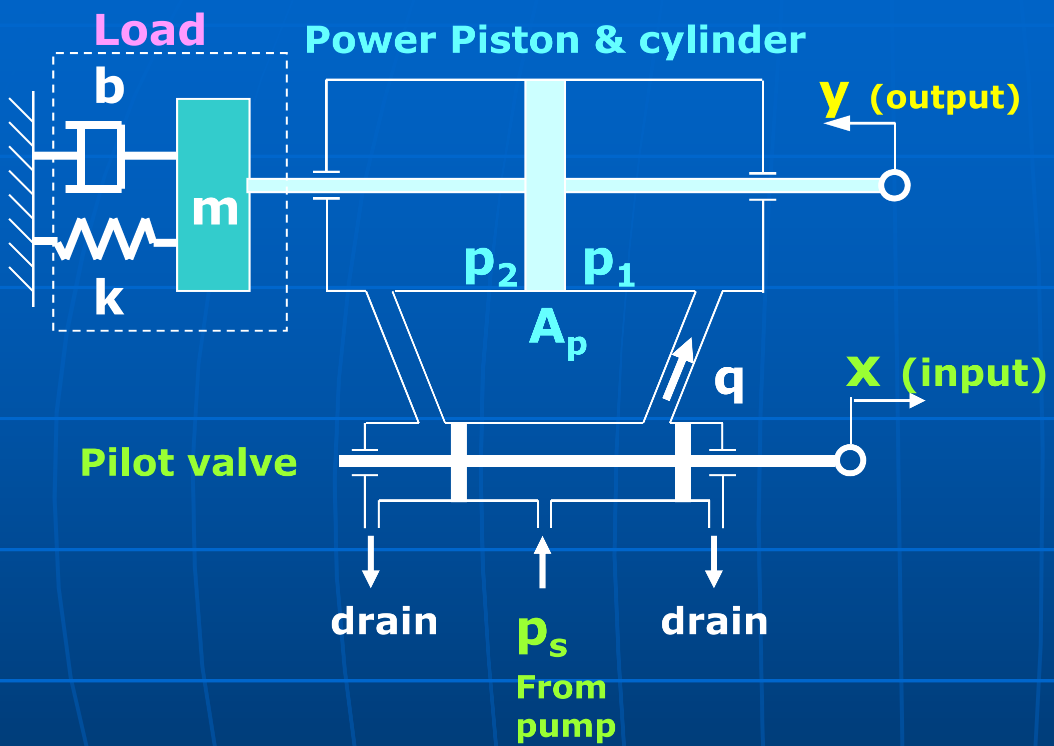
$$\Delta p = \frac{1}{A_p}\left(m\ddot{y} + b\dot{y} + ky\right)$$

$$q = A_p \dot{y}$$

$$x = \frac{1}{K_1}\left(q + K_2 \Delta p\right)$$

$$x = \frac{1}{K_1}\left(A_p \dot{y} + \frac{K_2}{A_p}\left(m\ddot{y} + b\dot{y} + ky\right)\right)$$

HYDRAULIC SERVOMOTOR



$$x = \frac{1}{K_1}\left(A_p\dot{y} + \frac{K_2}{A_p}\left(m\ddot{y} + b\dot{y} + ky\right)\right)$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\frac{K_1}{K_2}A_p}{ms^2 + \left(b + \frac{A_p^2}{K_2}\right)s + k}$$

If the load is not included, i.e., m=b=k=0 :

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\frac{K_1}{A_p}}{s} = \frac{K}{s}$$

Integrating Amplifier

- **Draw the block diagram.**

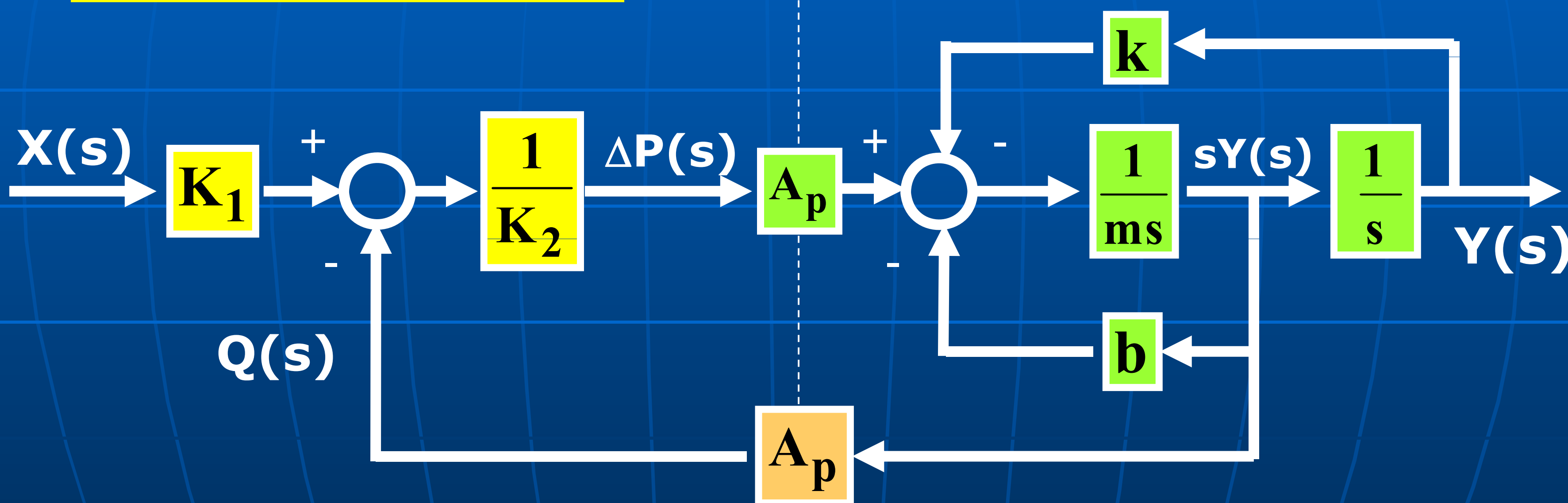
HYDRAULIC SERVOMOTOR

$$x = \frac{1}{K_1}\left(q + K_2 \Delta p\right)$$

$$\Delta p = \frac{1}{A_p}\left(m\ddot{y} + b\dot{y} + ky\right)$$

$$\frac{1}{K_2}\left[K_1 X(s) - Q(s)\right] = \Delta P(s)$$

$$A_p \Delta P(s) - bsY(s) - kY(s) = ms^2 Y(s)$$

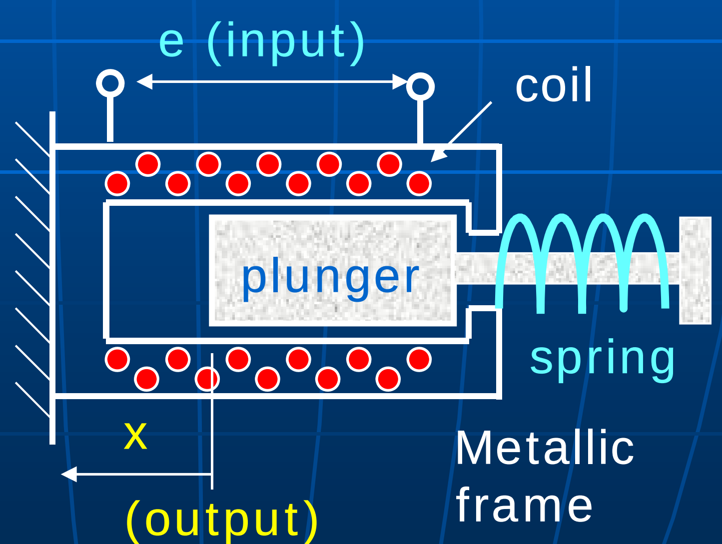


$$q = A_p \dot{y}$$

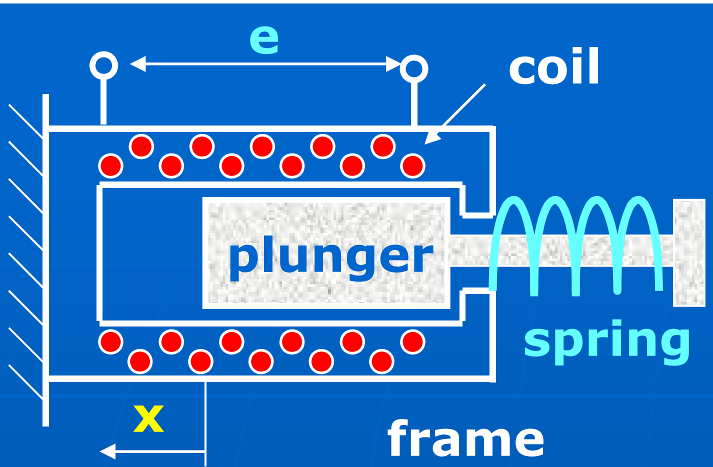
$$Q(s) = A_p sY(s)$$

LINEAR ACTUATOR (SOLENOID)

- A linear actuator consists of a coil of wire mounted in a metallic frame with a metallic plunger within the coil.
- An applied voltage (mostly dc) causes a current to flow in the coil, and a magnetic field which tends to pull the plunger is created.
- The stroke and the force exerted by the plunger at a given voltage are the basic specifications for a solenoid.
- Electro-Mechanical component

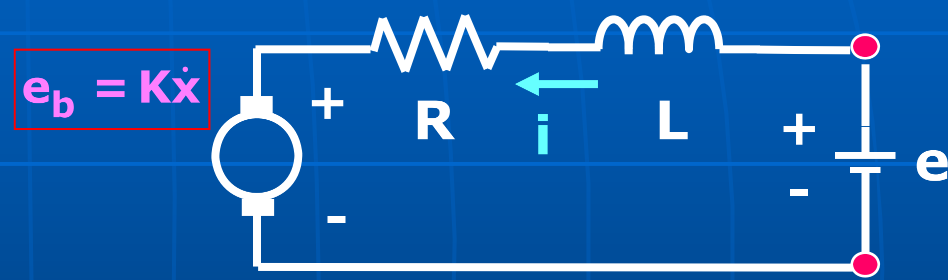


LINEAR ACTUATOR (SOLENOID)



K : Electromagnetic coupling constant

e_b : Back voltage



$$f_n = m\ddot{x}$$

$$m\ddot{x} + b\dot{x} + kx = Ki$$

$$\left(ms^2 + bs + k\right)X(s) = KI(s)$$

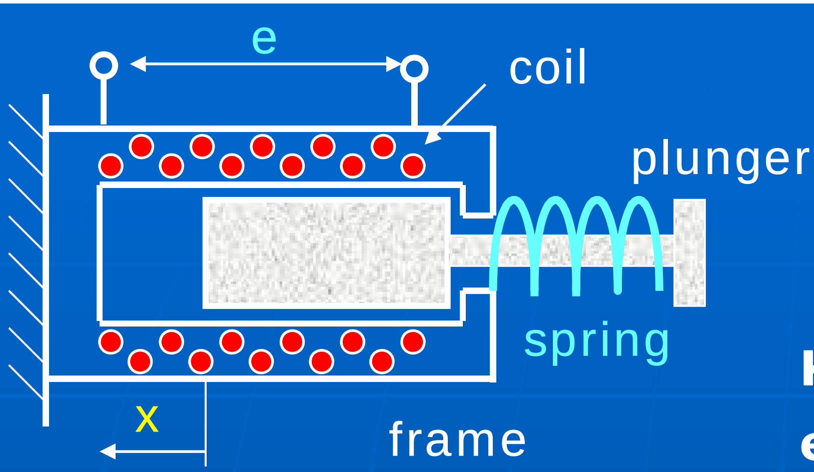
$$e = Ri + L\frac{di}{dt} + K\dot{x}$$

$e_b = K\dot{x}$

$$E(s) = \left(R + Ls\right)I(s) + KsX(s)$$

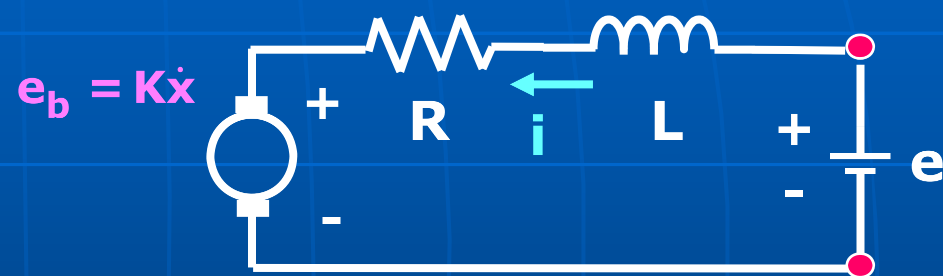
Mechanical ⟷ Electrical

LINEAR ACTUATOR (SOLENOID)



K : Electromagnetic coupling constant

e_b : Back voltage



$$\left(ms^2 + bs + k\right)X(s) = KI(s)$$

$$E(s) = \left(R + Ls\right)I(s) + KsX(s)$$

$$G(s) = \frac{X(s)}{E(s)} = \frac{K}{mLs^3 + \left(mR + bL\right)s^2 + \left(bR + kL + K^2\right)s + kR}$$

LINEAR ACTUATOR (SOLENOID)

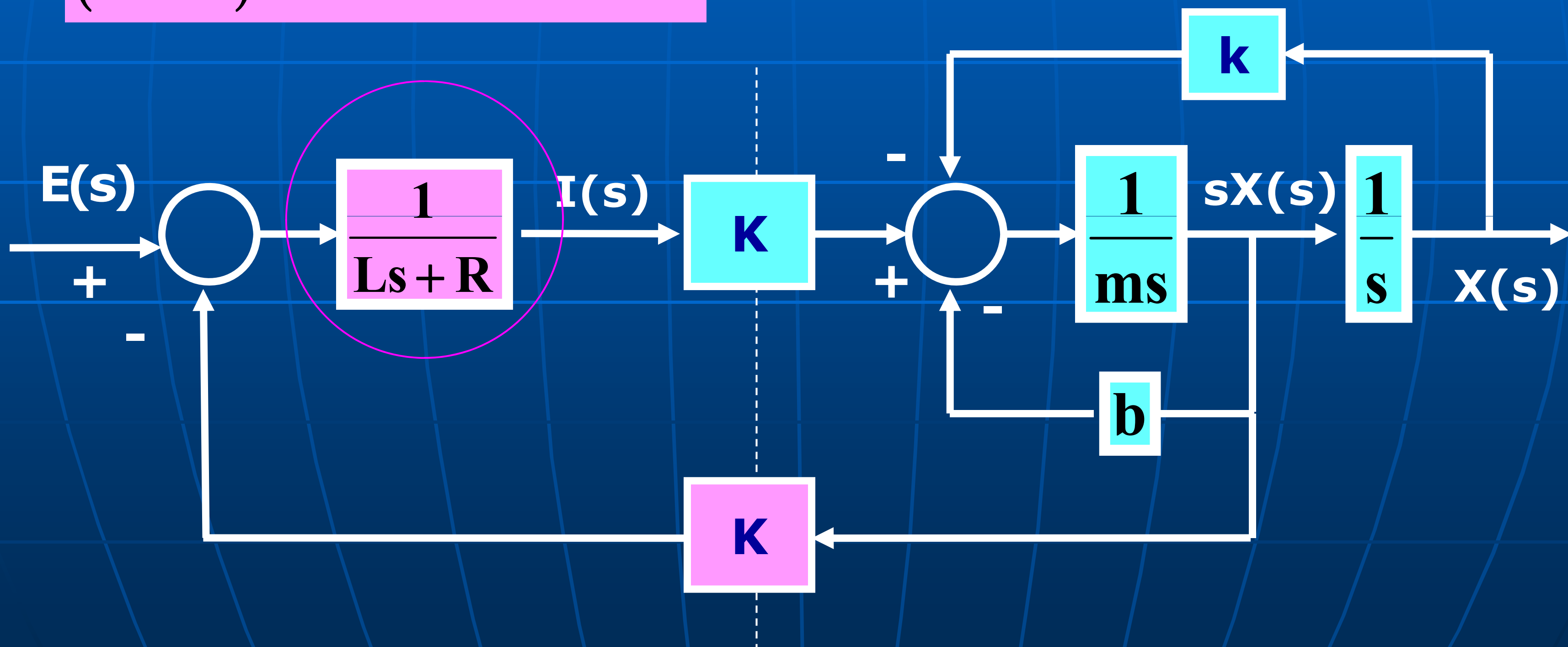
Draw the block diagram

$$E(s) = (R + Ls)I(s) + KsX(s)$$

$$\frac{1}{(R+Ls)}[E(s) - KsX(s)] = I(s)$$

$$(ms^2 + bs + k)X(s) = KI(s)$$

$$X(s) = \frac{1}{ms^2}[KI(s) - bsX(s) - kX(s)]$$



LINEAR ACTUATOR (SOLENOID)

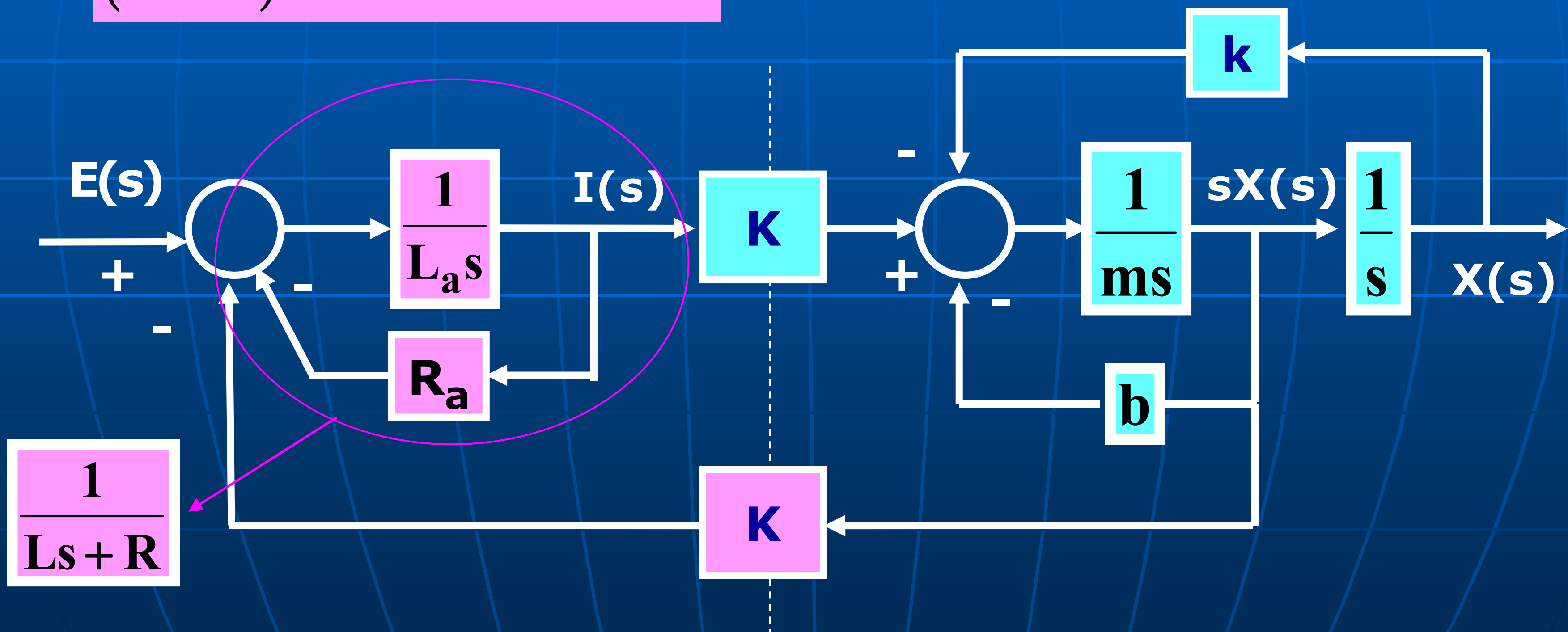
Draw the block diagram

$$E(s) = (R + Ls)I(s) + KsX(s)$$

$$\frac{1}{(R+Ls)}\left[E(s) - KsX(s)\right] = I(s)$$

$$\left(ms^2 + bs + k\right)X(s) = KI(s)$$

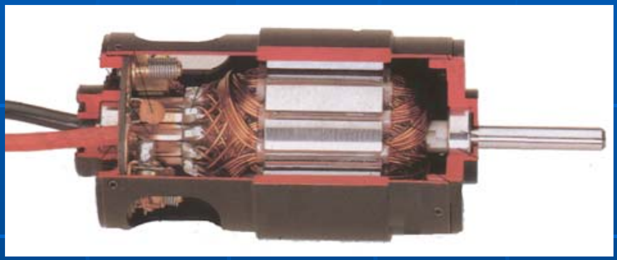
$$X(s) = \frac{1}{ms^2}\left[KI(s) - bsX(s) - kX(s)\right]$$



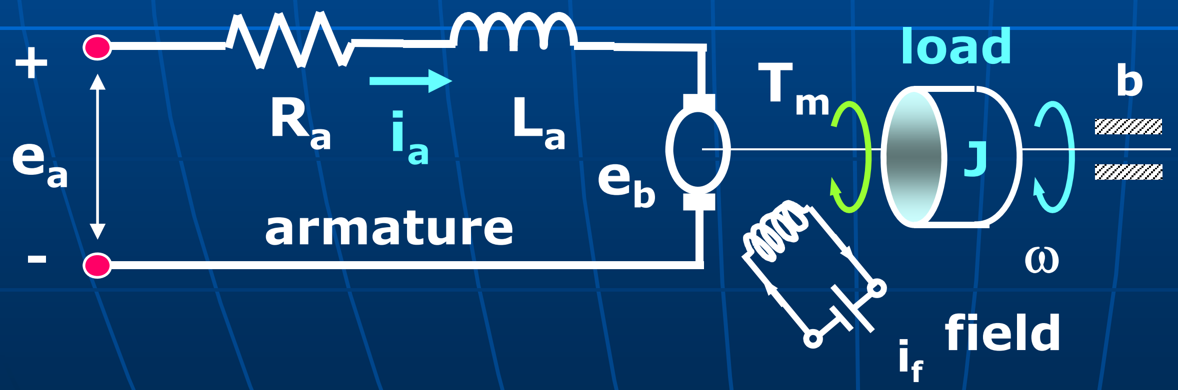
ROTARY ACTUATOR - DC SERVOMOTOR

Nise Sect. 2.8, Dorf&Bishop Ex. 2.5, Ogata Ex. A-3-23

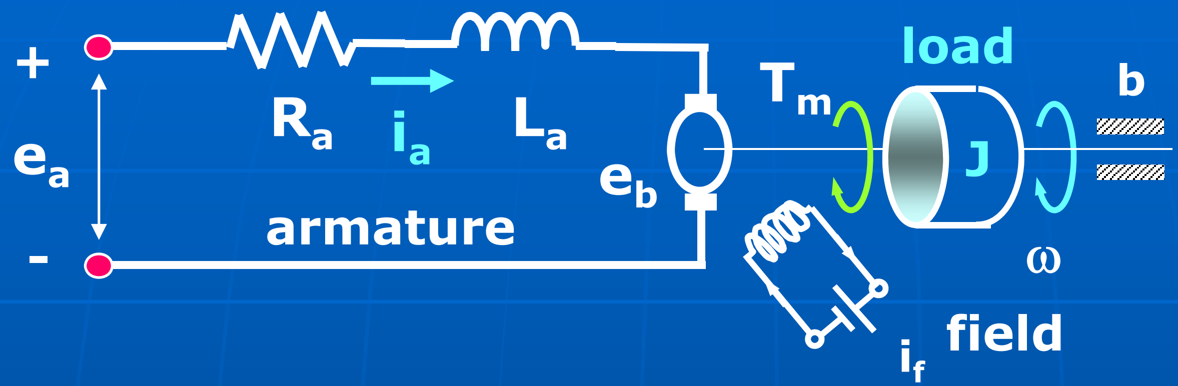
- DC motors are more commonly used in control systems, as AC motors are more difficult to control and their characteristics are highly nonlinear.



- The most commonly used control configuration with dc motors is the separately excited field windings. The control is through the applied armature voltage, keeping field current constant.



DC SERVOMOTOR



- For fixed field current, the torque produced by the motor is proportional to armature current.

$$T_m = K_t i_a$$

  K_t : motor torque constant

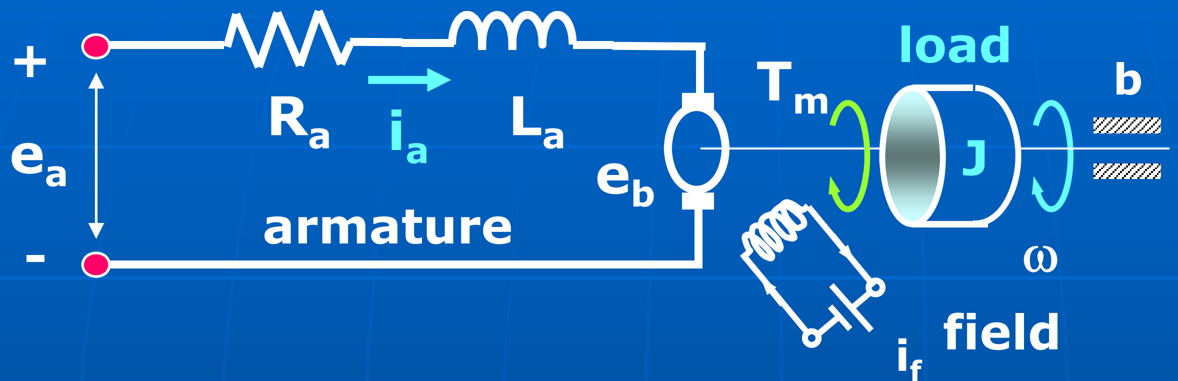
- When the armature rotates, a back emf or voltage is produced in the armature.

$$e_b = K_b \omega$$

  K_b : back emf constant

- __Note :__ In a consistent set of units, the value of K_t is equal to the value of K_b !

DC SERVOMOTOR



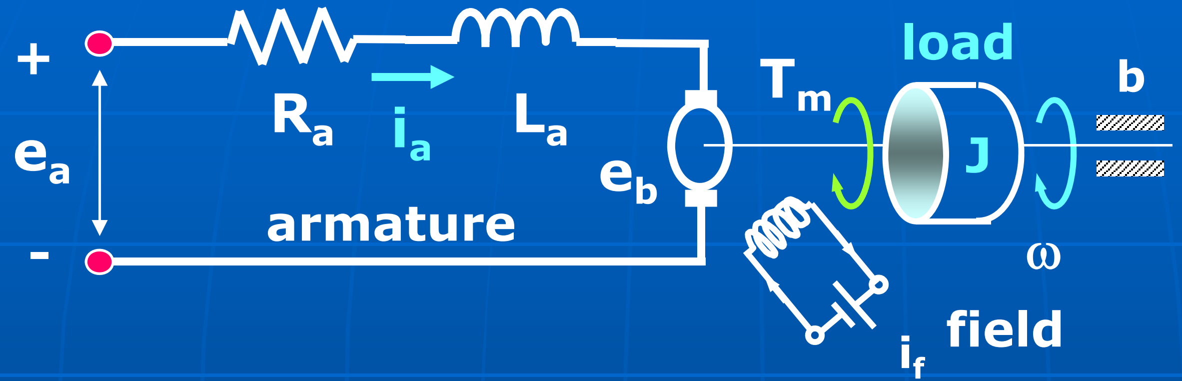
- The speed of the motor is controlled by the armature voltage which is supplied by an amplifier. For the armature circuit :

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

- Torque balance on the equivalent mass moment of inertia of the motor and the load gives :

$$J\dot{\omega} + b\omega = T_m = K_t i_a$$

DC SERVOMOTOR



$$e_b = K_b \omega$$

$$E_b(s) = K_b s\theta(s)$$

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

$$\left(L_a s + R_a\right) I_a(s) + E_b(s) = E_a(s)$$

$$J\dot{\omega} + b\omega = T_m = K_t i_a$$

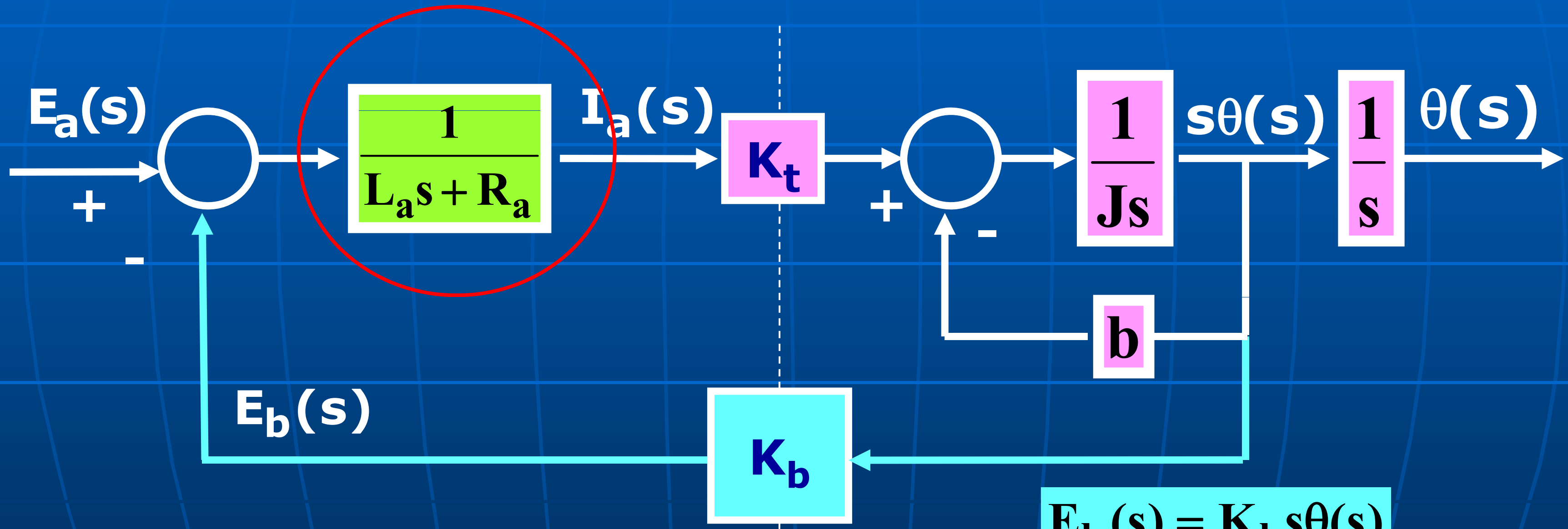
$$\left(Js^2 + bs\right)\theta(s) = T_m(s) = K_t I_a(s)$$

DC SERVOMOTOR

Draw the block diagram

$$\left(L_a s + R_a\right) I_a(s) = E_a(s) - E_b(s)$$

$$Js^2\theta(s) = K_t I_a(s) - bs\theta(s)$$

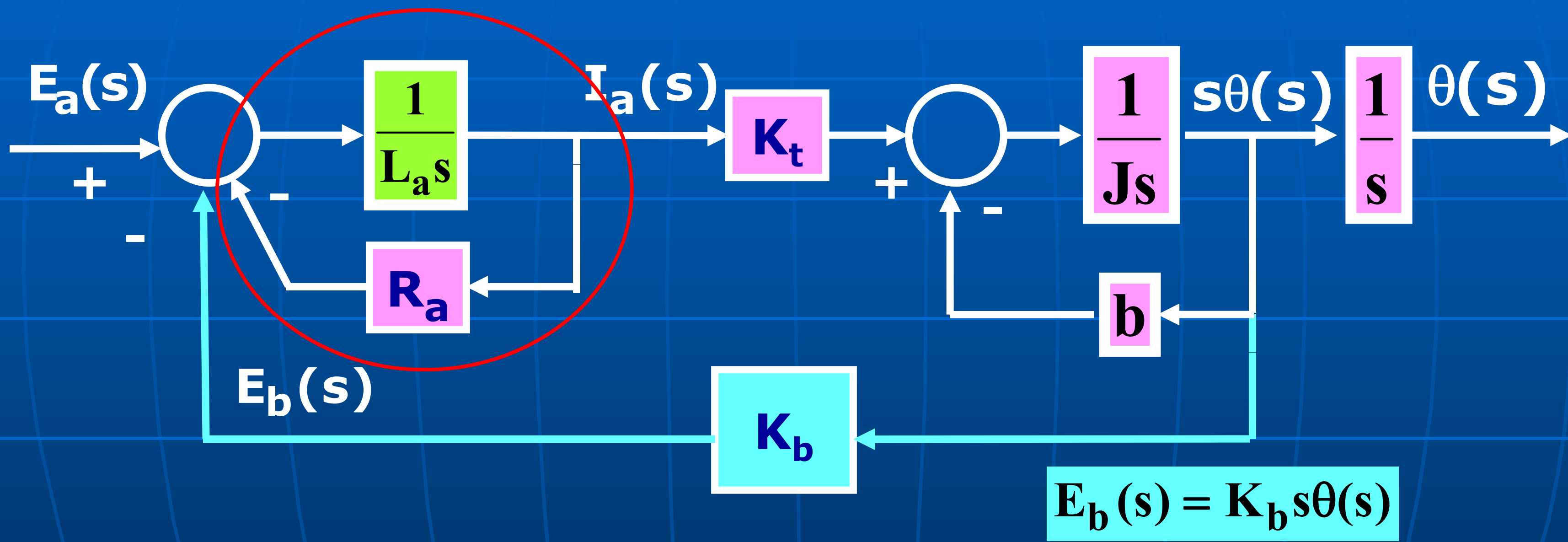


$$E_b(s) = K_b s\theta(s)$$

Electrical ⟷ **Mechanical**

DC SERVOMOTOR

Modify the block diagram



It is observed that even though the system is basically open loop, there is a built-in feedback.

DC SERVOMOTOR

The overall transfer function

$$G(s) = \frac{\theta(s)}{E_a(s)} = \frac{K_t}{s\left[JL_a s^2 + \left(bL_a + JR_a\right)s + R_a b + K_t K_b\right]}$$

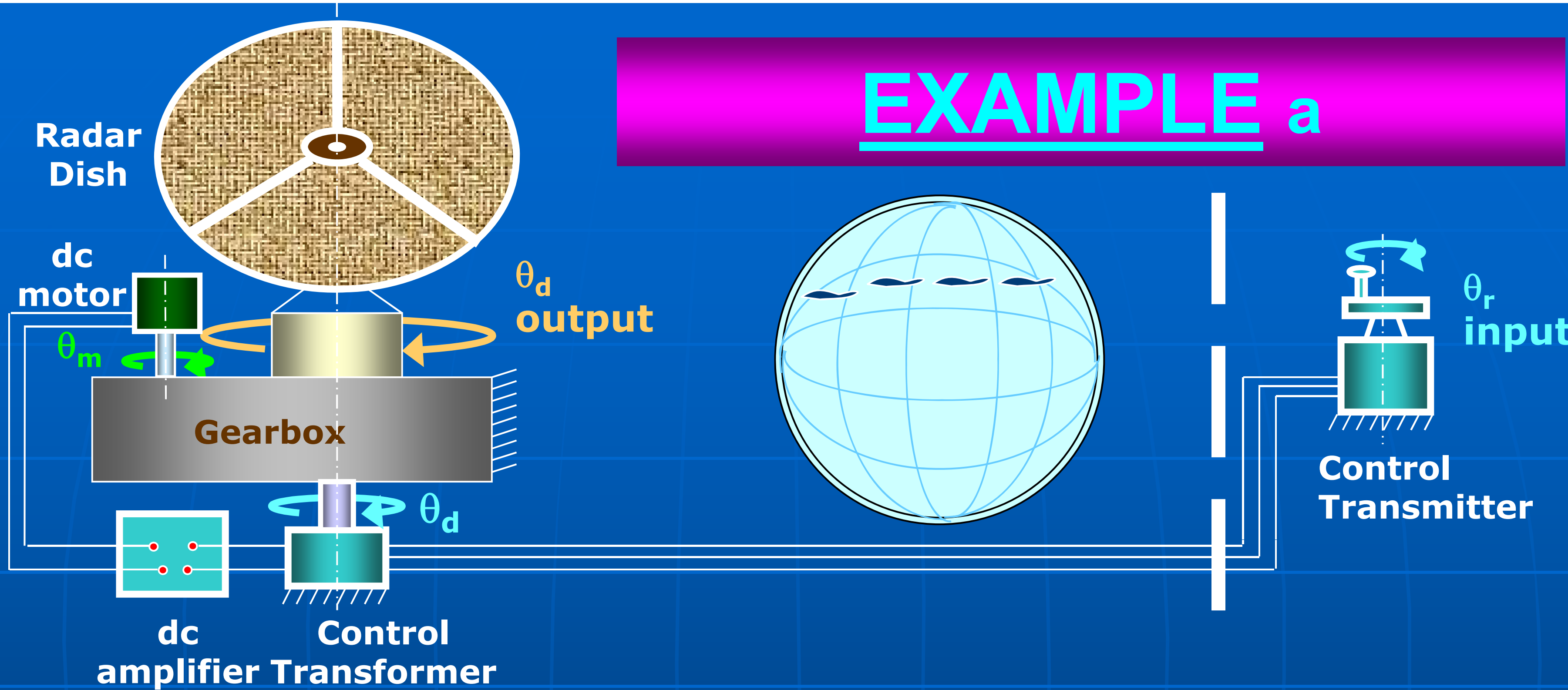
L_a is usually small and can be neglected. Thus

$$G(s) = \frac{\theta(s)}{E_a(s)} = \frac{\dfrac{K_t}{\left(K_t K_b + R_a b\right)}}{s\left(\dfrac{JR_a}{K_t K_b + R_a b}s + 1\right)} = \frac{K_m}{s\left(T_m s + 1\right)}$$

K_m : Motor gain constant,

T_m : Motor time constant.

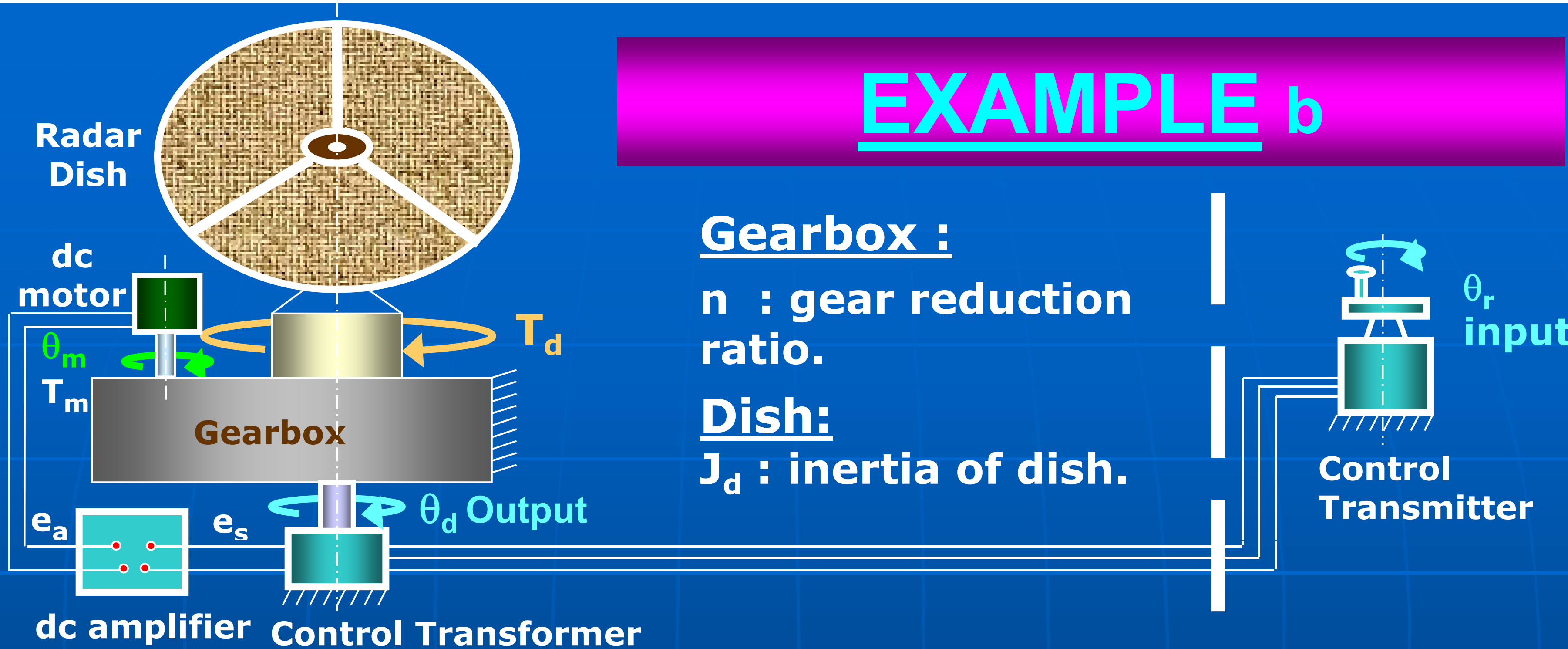
EXAMPLE a



A servomechanism designed for a radar operator to direct a remotely positioned radar dish is illustrated in the figure.

The operator keeps the images of the aircraft in the vicinity within the radar screen by rotating the hand wheel and thus rotating the dish.

EXAMPLE b



Gearbox :

n : gear reduction ratio.

Dish:

J_d : inertia of dish.

Armature controlled dc electric motor :

i_a : armature current, R : armature resistance

L_a : armature inductance,

K_t : motor torque constant, K_b: back emf constant.

Synchro pair :

K_s : error detector gain.

Dc amplifier :

K_a : amplifier gain.

EXAMPLE c

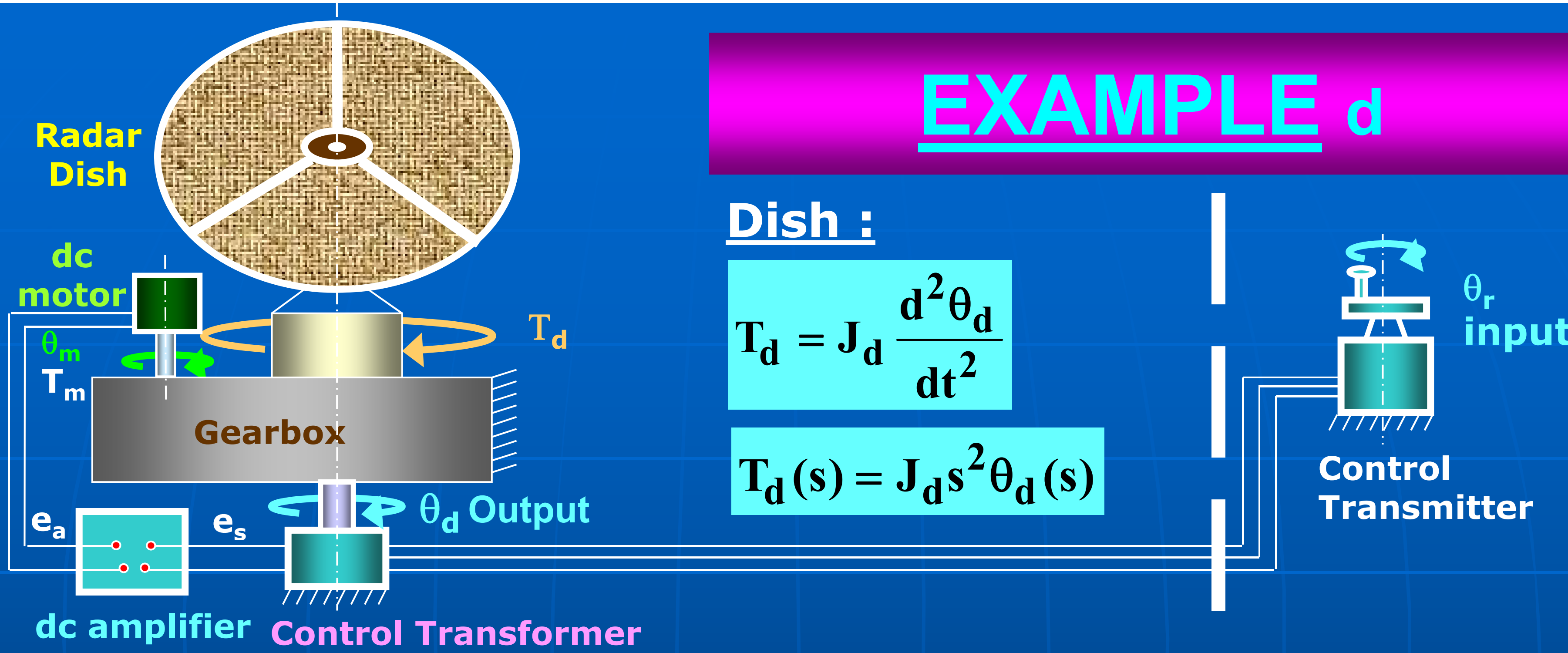
Synchro Pair : $e_s = K_s(\theta_r - \theta_d)$ $\quad E_s(s) = K_s[\theta_r(s) - \theta_d(s)]$

Amplifier : $e_a = K_a e_s$ $\quad E_a(s) = K_a E_s(s)$

Gearbox : $T_d = nT_m$ $\quad T_d(s) = nT_m(s)$

$\theta_m = n\theta_d$ $\quad \theta_m(s) = n\theta_d(s)$

EXAMPLE d



Dish :

$$T_d = J_d \frac{d^2\theta_d}{dt^2}$$

$$T_d(s) = J_d s^2 \theta_d(s)$$

Dc motor :

$$L_a \frac{di_a}{dt} + R_a i_a + K_b \omega_m = e_a$$

$$T_m = K_t i_a$$

$$\left(L_a s + R_a\right) I_a(s) + K_b s \theta_m(s) = E_a(s)$$

$$T_m(s) = K_t I_a(s)$$

EXAMPLE e

Synchro Pair : $E_s(s) = K_s[\theta_r(s) - \theta_d(s)]$

Amplifier : $E_a(s) = K_a E_s(s)$

Gearbox : $T_d(s) = nT_m(s)$ $\theta_m(s) = n\theta_d(s)$

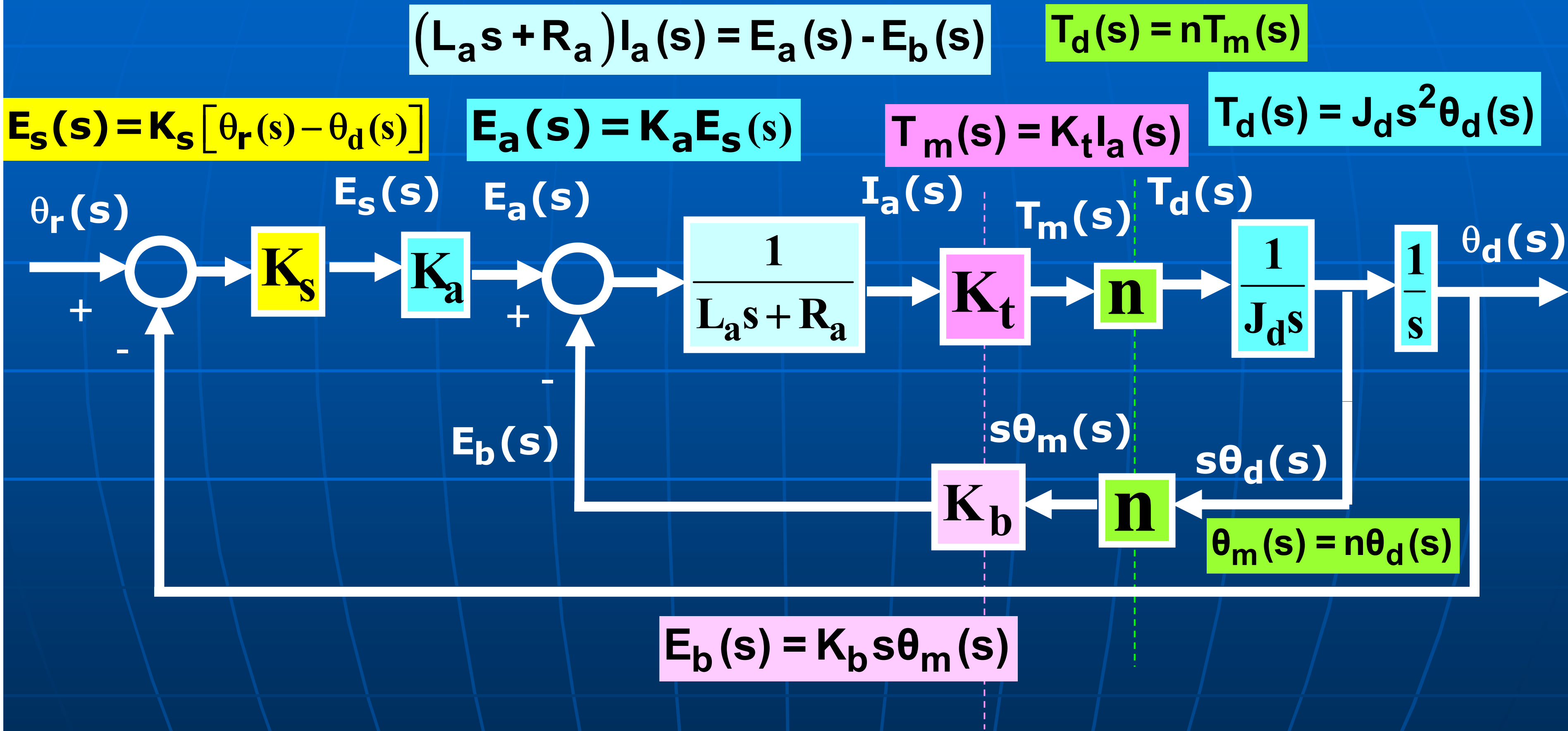
Dc motor : $(L_a s + R_a) I_a(s) + K_b s\theta_m(s) = E_a(s)$

$T_m(s) = K_t I_a(s)$

Dish : $T_d(s) = J_d s^2 \theta_d(s)$

Block Diagram

EXAMPLE f



TYPICAL SENSORS

Sensor	Input	Output
Potentiometer	Position , x	Voltage, E
	Angular position, θ	Voltage, E
Encoder	Angular position , θ	Voltage, E
LVDT	Position, x	Voltage, E
Tachometer	Angular velocity, ω	Voltage, E
Resolver	Angular velocity , ω	Voltage, E
Thermocouple	Temperature, T	Voltage, E
Pressure Transducer	Pressure, p	Voltage, E

TYPICAL SENSORS

Sensor	Input	Output
LVDT	Position, x	Voltage, E

- The letters LVDT stand for **Linear Variable Differential Transformer**, a common type of electromechanical transducer that can convert the rectilinear motion of an object into a corresponding electrical signal.
- **LVDT linear position sensors** are available to measure displacements as small as a few millionths of a centimeter up to several centimeters, but some are also capable of measuring displacements up to ±0.5 metres.