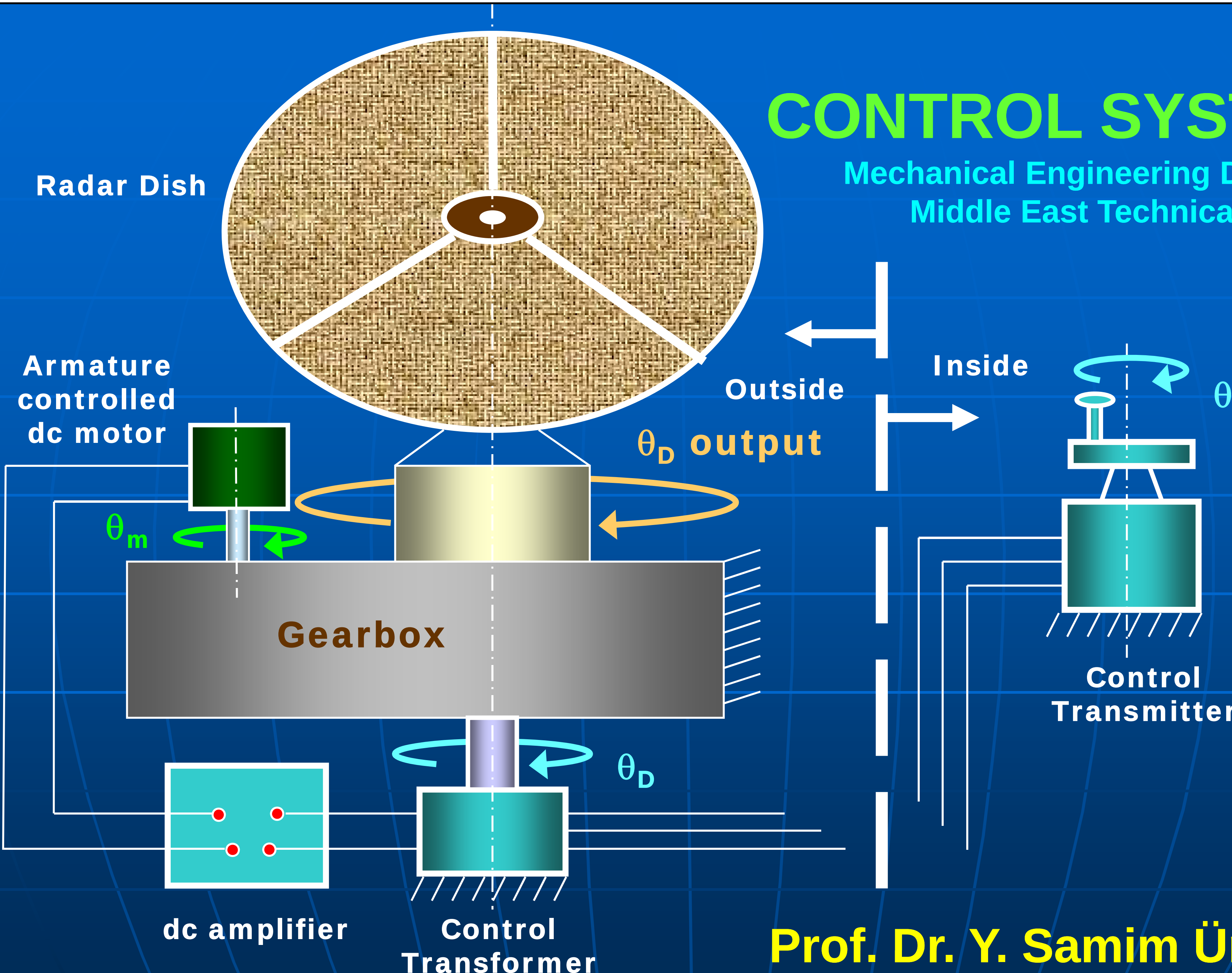
ME 304

CONTROL SYSTEMS

Mechanical Engineering Department,
Middle East Technical University



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CH X

COURSE OUTLINE

I. INTRODUCTION & BASIC CONCEPTS
II. MODELING DYNAMIC SYSTEMS
III. CONTROL SYSTEM COMPONENTS
IV. STABILITY
V. TRANSIENT RESPONSE
VI. STEADY STATE RESPONSE
VII. DISTURBANCE REJECTION
VIII. BASIC CONTROL ACTIONS & CONTROLLERS
IX. FREQUENCY RESPONSE ANALYSIS
X. SENSITIVITY ANALYSIS
XI. ROOT LOCUS ANALYSIS

SENSITIVITY - OBJECTIVES

Getting familiar with the concept of sensitivity.

Applying sensitivity analysis to basic components of control systems.

Investigation of sensitivity of control systems to parameter variations.

SENSITIVITY - Definition

Nise Section 7.7, Dorf&Bishop Section 4.2

- There are many uncertainties encountered in the design and analysis of control systems.
- In modeling and parameter selection imperfections and inaccuracies always exist.
- Parameters may also vary in time due to age, wear, etc.

SENSITIVITY - Definition

- One of the basic design criteria for control systems is to minimize the sensitivity of the response to modelling inaccuracies and parameter variations.
- In control system terminology, a system that is insensitive to external disturbances and parameter variations is called a robust system.

SENSITIVITY - Definition

- The sensitivity function of a system is defined as the ratio of the percentage variation of the system transfer function, M, to the percentage variation of a system parameter, k.

$$S_k^M = \frac{\dfrac{dM}{M}}{\dfrac{dk}{k}} = \frac{\text{percentage change in M (due to a change in k)}}{\text{percentage change in k}}$$

SENSITIVITY

Open Loop Control System

- Consider the open loop control system.

$$C(s) = G(s)R(s)$$

- Sensitivity of C(s) w.r.t. G(s) $S_G^C = \frac{dC/C}{dG/G}$

SENSITIVITY

Open Loop Control System

$$C(s) = G(s)R(s) \quad \longrightarrow \quad dC(s) = dG(s)R(s)$$

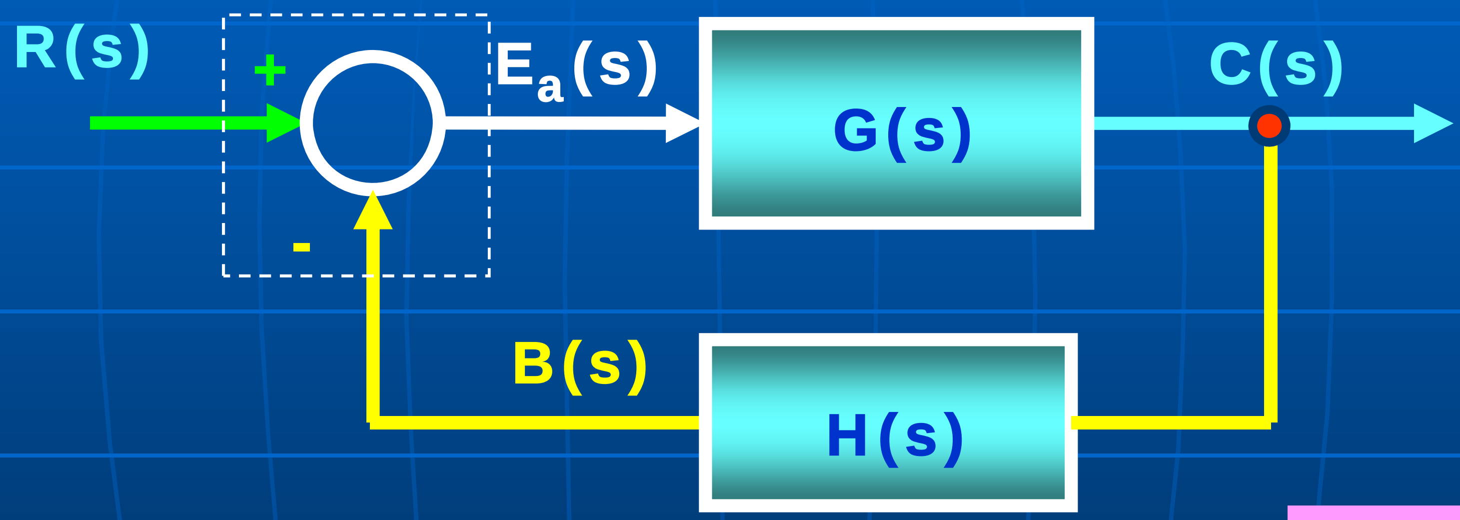
$$S_G^C = \frac{G}{C}\frac{dC}{dG} = \frac{G}{GR}\frac{(dG)R}{dG} = 1$$

- **Therefore, any change or uncertainity in the transfer function of the open loop system is directly reflected in the output of the system.**

SENSITIVITY

Closed Loop System

- Consider the general (canonical) form of the closed loop control system.



- Sensitivity of C(s) w.r.t. G(s)
- Sensitivity of C(s) w.r.t. H(s)

$$S_G^C = \frac{dC/C}{dG/G}$$

$$S_H^C = \frac{dC/C}{dH/H}$$

SENSITIVITY

Feedforward and feedback elements

- **Express the output in terms of the input.**

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \;\Rightarrow\; C(s) = \frac{G(s)}{1+G(s)H(s)}R(s)$$

$$dC = \frac{dG(1+GH) - G(GdH + HdG)}{(1+GH)^2}R = \frac{dG - G^2 dH}{(1+GH)^2}R$$

$$\frac{dC}{C} = \left[\frac{dG - G^2 dH}{(1+GH)^2}R\right]\left(\frac{1+GH}{G}\right)\frac{1}{R} = \frac{dG - G^2 dH}{G(1+GH)}$$

$$\frac{dC}{C} = \frac{1}{1+GH}\frac{dG}{G} - \frac{GH}{1+GH}\frac{dH}{H}$$

SENSITIVITY
Feedforward and feedback elements

$$\frac{dC}{C} = \frac{1}{1+GH}\frac{dG}{G} - \frac{GH}{1+GH}\frac{dH}{H}$$

- **Let dH=0 :**

$$S_G^C = \frac{dC/C}{dG/G} = \frac{1}{1+GH}$$

- **Let dG=0 :**

$$S_H^C = \frac{dC/C}{dH/H} = -\frac{GH}{1+GH}$$

SENSITIVITY

Feedforward and feedback elements

- If large controller gains are chosen so that :

$$\left|G(s)H(s)\right| \to \infty$$

- Then

$$S_G^C = \frac{dC/C}{dG/G} = \frac{1}{1+GH}$$

$$\lim_{GH\to\infty} S_G^C = 0$$

which is very good !

$$S_H^C = \frac{dC/C}{dH/H} = -\frac{GH}{1+GH}$$

$$\lim_{GH\to\infty} S_H^C = -1$$

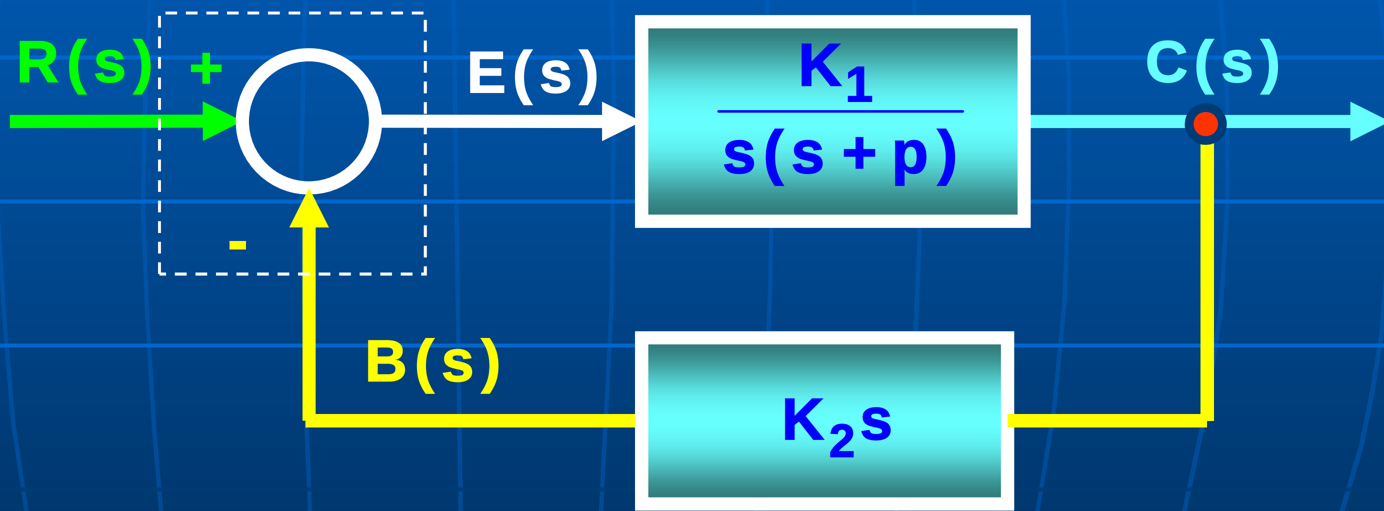
which is not so good !

SENSITIVITY
Feedforward and feedback elements

- Therefore by choosing $|G(s)H(s)| \to \infty$
- The effects of parameter variations in feedforward elements (i.e. the controller, the actuator, the plant) can be suppressed.
- The effects of the parameter variations in feedback elements (sensors) cannot be suppressed and their variations will be fully reflected in the variation of the output.
- Conclusion : Quality of a feedback control system depends on the quality of measurement.

SENSITIVITY - Parameter Variations

- Let us determine the sensitivity of the transfer function of the system represented by the block diagram shown, to variations in each of the parameters K_1, K_2, and p.



$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

SENSITIVITY - Parameter Variations

- **Sensitivity of the transfer function w.r.t. variations in the feedforward gain K_1.**

$$T(s) = \frac{\dfrac{K_1}{s(s+p)}}{1 + \dfrac{K_1 K_2 s}{s(s+p)}} = \frac{K_1}{s(s+p+K_1K_2)}$$

$$S_{K_1}^{T}(s) = \frac{K_1}{T}\frac{dT}{dK_1} = \frac{K_1}{\dfrac{K_1}{s(s+p+K_1K_2)}}\,\frac{1}{s}\,\frac{(s+p+K_1K_2) - K_1(K_2)}{(s+p+K_1K_2)^2}$$

$$S_{K_1}^{T}(s) = \frac{s+p}{s+p+K_1K_2}$$

SENSITIVITY - Parameter Variations

- **Sensitivity of the transfer function w.r.t. variations in the feedback gain K_2.**

$$T(s) = \frac{\dfrac{K_1}{s(s+p)}}{1 + \dfrac{K_1 K_2 s}{s(s+p)}} = \frac{K_1}{s(s+p+K_1 K_2)}$$

$$S_{K_2}^{T}(s) = \frac{K_2}{T}\frac{dT}{dK_2} = \frac{K_2}{\dfrac{K_1}{s(s+p+K_1 K_2)}}\frac{1}{s}\frac{-K_1(K_1)}{(s+p+K_1 K_2)^2}$$

$$S_{K_2}^{T}(s) = \frac{-K_1 K_2}{s+p+K_1 K_2}$$

SENSITIVITY - Parameter Variations

- **Sensitivity of the transfer function w.r.t. variations in the open loop pole p.**

$$T(s) = \frac{\dfrac{K_1}{s(s+p)}}{1 + \dfrac{K_1 K_2 s}{s(s+p)}} = \frac{K_1}{s(s+p+K_1K_2)}$$

$$S_p^T(s) = \frac{p}{T}\frac{dT}{dp} = \frac{p}{\dfrac{K_1}{s(s+p+K_1K_2)}} \frac{1}{s} \frac{-K_1(1)}{(s+p+K_1K_2)^2}$$

$$S_p^T(s) = \frac{-p}{s+p+K_1K_2}$$

SENSITIVITY - Parameter Variations

- It is noted that the sensitivity expressions are in general functions of s.

$$S_{K_1}^T(s) = \frac{s+p}{s+p+K_1K_2}$$

$$S_{K_2}^T(s) = \frac{-K_1K_2}{s+p+K_1K_2}$$

$$S_p^T(s) = \frac{-p}{s+p+K_1K_2}$$

- The steady state sensitivities can be obtained by letting **s=0** in those expressions.

$$S_{K_1}^T\Big|_{ss} = S_{K_1}^T(0) = \frac{p}{p+K_1K_2}$$

$$S_{K_2}^T\Big|_{ss} = S_{K_2}^T(0) = \frac{-K_1K_2}{p+K_1K_2}$$

$$S_p^T\Big|_{ss} = S_p^T(0) = \frac{-p}{p+K_1K_2}$$

SENSITIVITY - Parameter Variations

- The dynamic sensitivity is obtained by letting

$$s = j\omega$$

in the sensitivity expressions, and then by plotting the sensitivity versus frequency.

- It is also possible to determine the sensitivity of the roots of the characteristic equation for small variations of a parameter in a similar manner.

SENSITIVITY - Reading

- **Study Examples 7.10-7.12, and solve Exercise 7.21 and problems 7.44-7.45 in Nise.**

  Dorf & Bishop 4.8 Design Example : Mars Rover Vehicle, pp. 212-214.