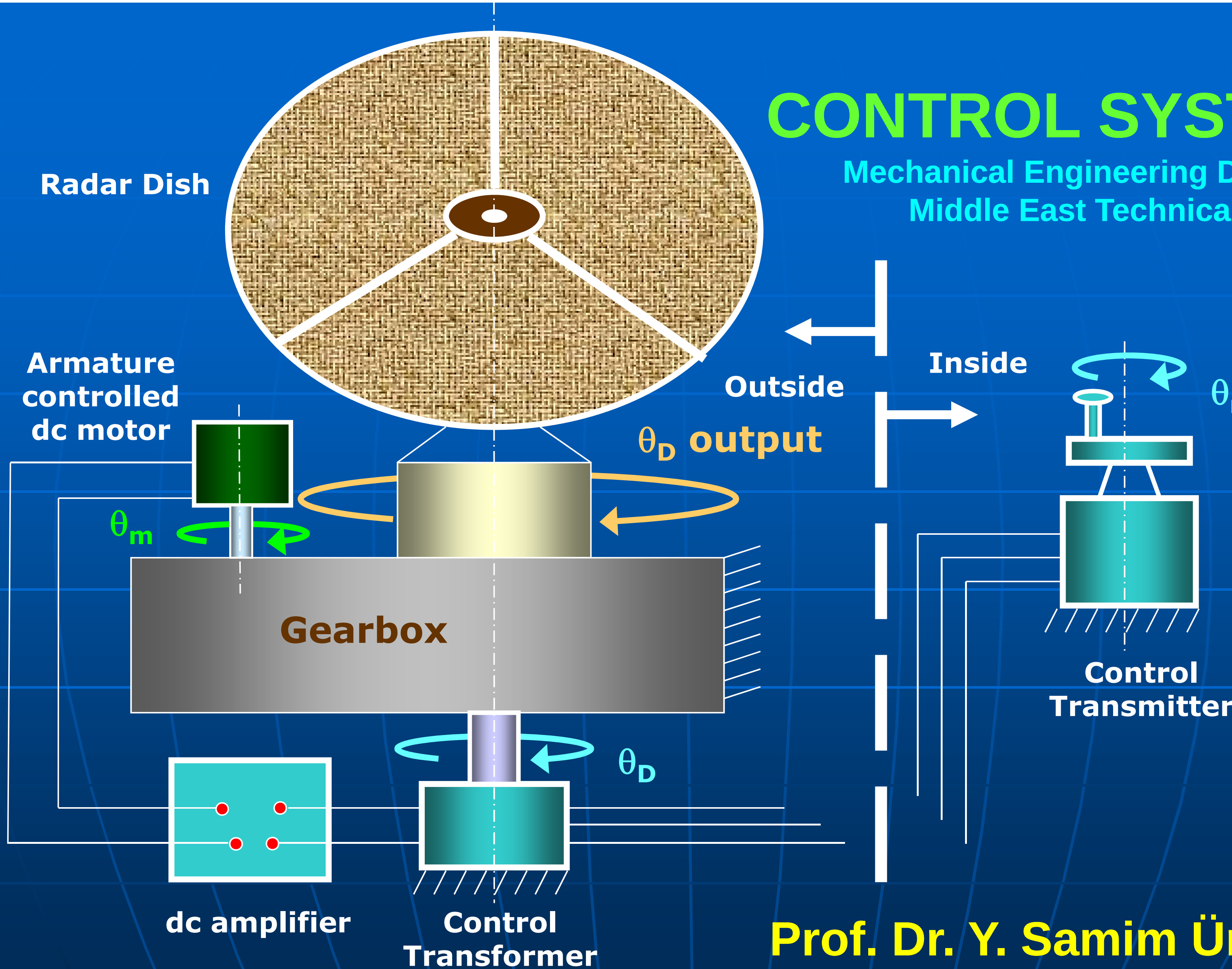
ME 304

CONTROL SYSTEMS

Mechanical Engineering Department,
Middle East Technical University



Prof. Dr. Y. Samim Ünlüsoy

CH II

COURSE OUTLINE

I. INTRODUCTION & BASIC CONCEPTS

II. MODELING DYNAMIC SYSTEMS

III. CONTROL SYSTEM COMPONENTS

IV. STABILITY

V. TRANSIENT RESPONSE

VI. STEADY STATE RESPONSE

VII. DISTURBANCE REJECTION

VIII. BASIC CONTROL ACTIONS & CONTROLLERS

IX. FREQUENCY RESPONSE ANALYSIS

X. SENSITIVITY ANALYSIS

XI. ROOT LOCUS ANALYSIS

MODELING DYNAMIC SYSTEMS

OBJECTIVES

- Deriving input-output relations of linear time invariant systems (**mechanical**, fluid, thermal, and electrical) using elemental and structural equations.
- Obtaining transfer function representation of LTI systems.
- Representing control systems with block diagrams.

We are here !

Completed

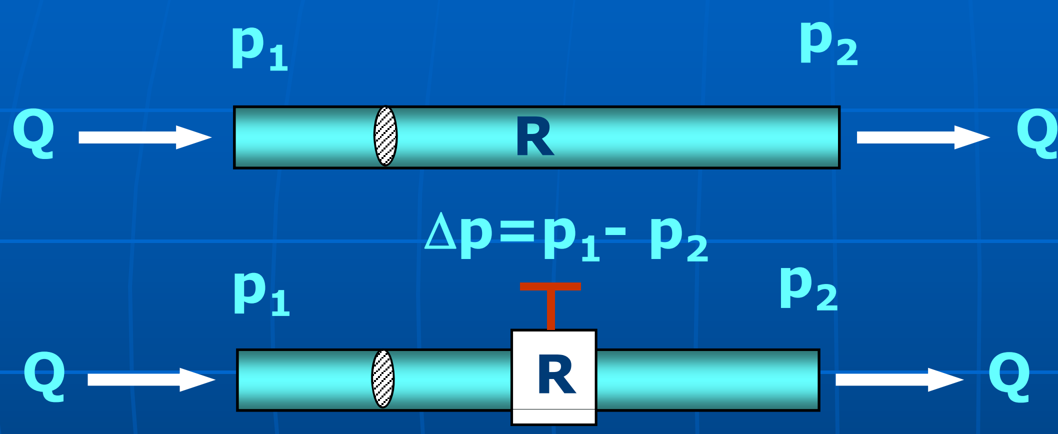
MODELING DYNAMIC SYSTEMS

REMEMBER !

- In this course, only Linear Time Invariant (**LTI**) systems will be considered. Further, they will be lumped, deterministic and continuous time.
- These systems will have **input-output** relations described by linear ordinary differential equations with constant coefficients.

FLUID SYSTEM ELEMENTS - Incompressible

- **Pipe and Valve Resistances**



- P_1, P_2 : pressure at fluid entrance and exit,
- Q : volumetric flow rate,
- R : pipe or valve resistance coefficient-constant.

$$\Delta p = R\,Q$$

For laminar flow

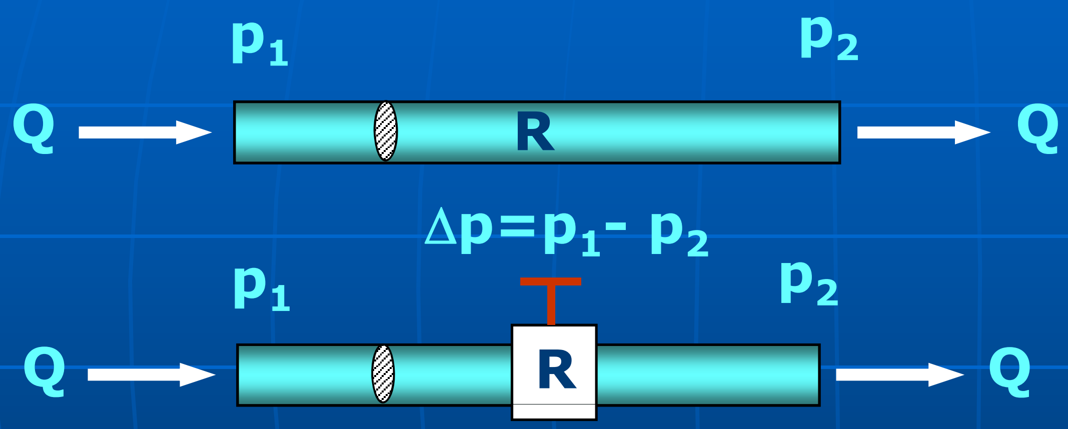
In terms of heads H_i

$$p_i = \rho g H_i$$

$$\Delta H = \left(\frac{R}{\rho g}\right) Q$$

FLUID SYSTEM ELEMENTS - Incompressible

Pipe and Valve Resistances



$$Q = K\sqrt{\Delta p}$$

For turbulent flow

$$Q = \hat{K}\sqrt{\Delta H}$$

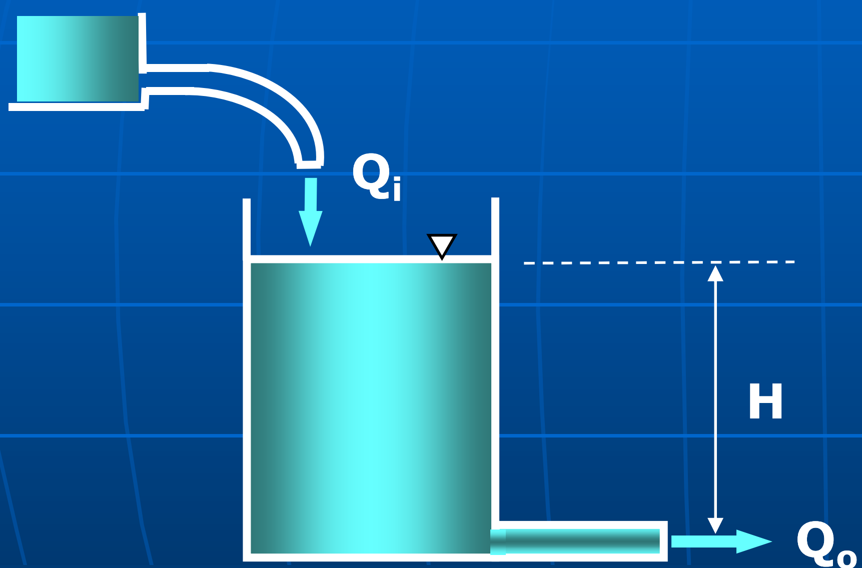
In terms of heads H_i

- For turbulent flow the **flow rate – pressure** drop relation is nonlinear.
- The resistance for turbulent flow depends on the flow rate and pressure drop.

$$R = \frac{d\left(\Delta p\right)}{dQ}$$

FLUID SYSTEM ELEMENTS - Incompressible

- **Tank Capacitance** **H : height of fluid**

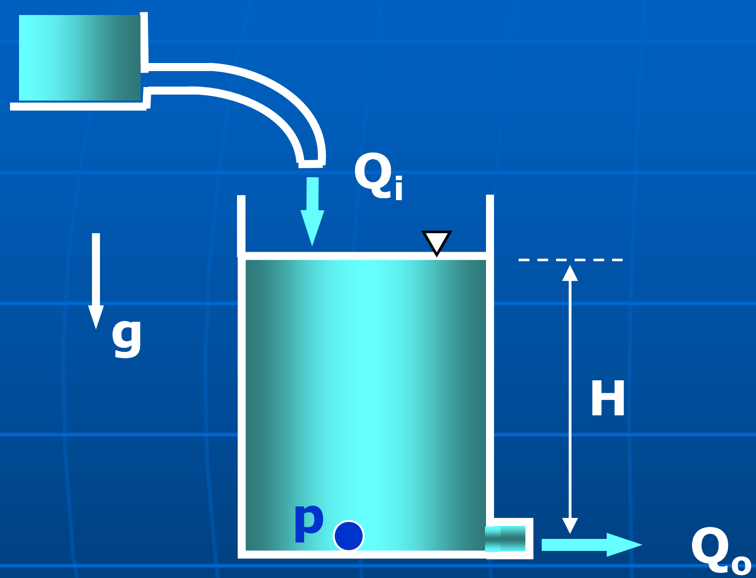


- **Tank capacitance is defined as the change of fluid volume in the tank corresponding to a change in fluid height.**

$$C_f = \frac{\Delta V}{\Delta H} = \frac{A(\Delta H)}{\Delta H} = A$$

FLUID SYSTEM ELEMENTS - Incompressible

- **Tank Capacitance**



- Q_i, Q_o : volumetric flow rate in and out of the tank,
- Q_t : net volumetric flow rate in (or out of) the tank,
- p : pressure at the bottom of the tank.

$$Q_t = C_f \frac{dH}{dt}$$

$$C_f = A$$

In terms of heads H_i

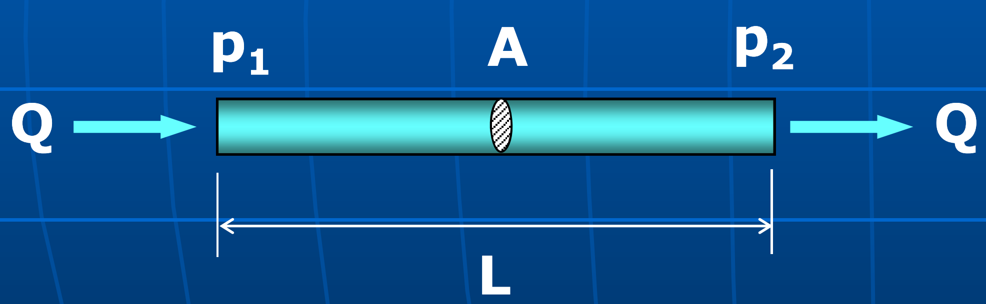
$$p = \rho g H$$

$$Q_t = C_f \frac{dp}{dt}$$

$$C_f = \frac{A}{\rho g}$$

FLUID SYSTEM ELEMENTS - Incompressible

- **Fluid Inertance**
- inertial effect of fluid flow.
- significant for long and thin pipes.

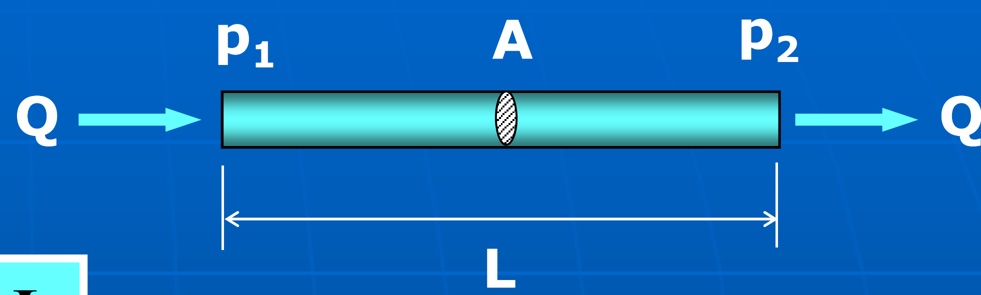


$\Delta p = p_1 - p_2$

- p_1, p_2 : pressure at fluid entrance and exit,
- Q : volumetric flow rate,
- A : pipe cross-sectional area,
- L : pipe length.

FLUID SYSTEM ELEMENTS - Incompressible

- **Fluid Inertance**
- ρ : **fluid density,**



$\Delta p = p_1 - p_2$

$$F = A(\Delta p)$$

$$m = \rho A L$$

$$F = ma$$

$$a = \dot{v} = \frac{d}{dt}\left(\frac{Q}{A}\right) = \frac{1}{A}\frac{dQ}{dt}$$

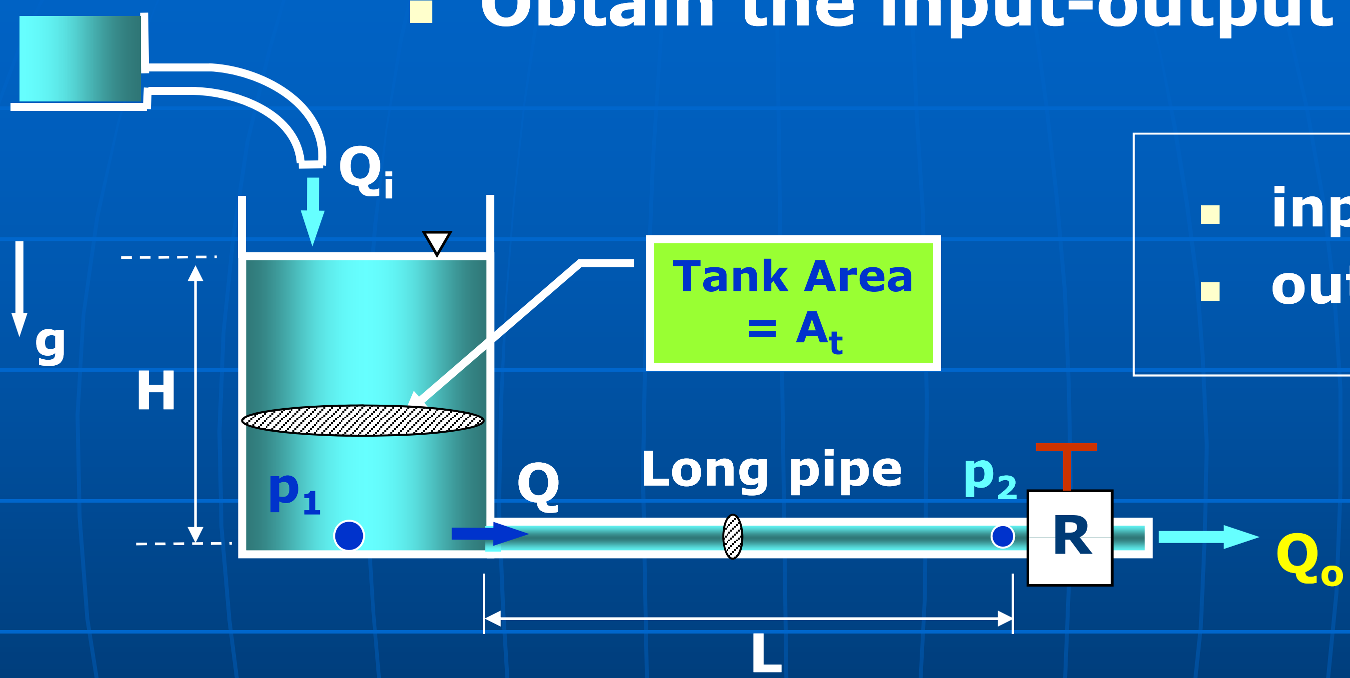
$$A(\Delta p) = (\rho A L)\frac{1}{A}\frac{dQ}{dt}$$

$$\Delta p = I\frac{dQ}{dt}$$

$$I = \frac{\rho L}{A}$$

FLUID SYSTEM ELEMENTS – EXAMPLE 1a

- Obtain the input-output relation.

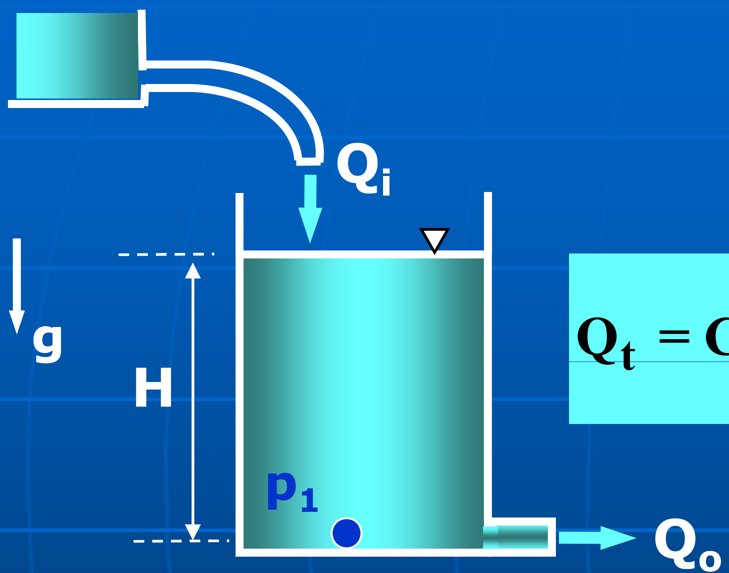


- input : Q_i
- output : Q_o

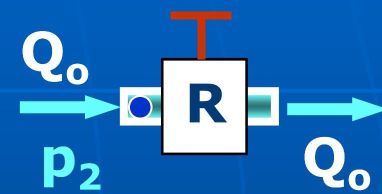
- ρ : fluid density,
- A_p : pipe cross-sectional area,

FLUID SYSTEM ELEMENTS – EXAMPLE 1b

- Identify the elements.
- Write the elemental equations

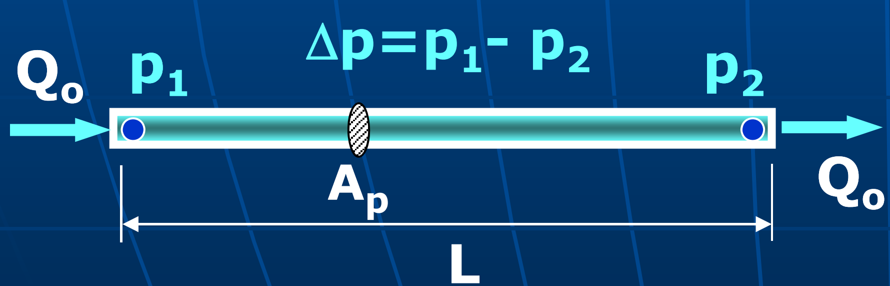


$$Q_t = C_f \frac{dH}{dt} = \left(\frac{A_t}{\rho g}\right)\frac{dp_1}{dt}$$



$$(\Delta p)_{valve} = RQ_o$$

- Long pipe will have resistance and inertance !

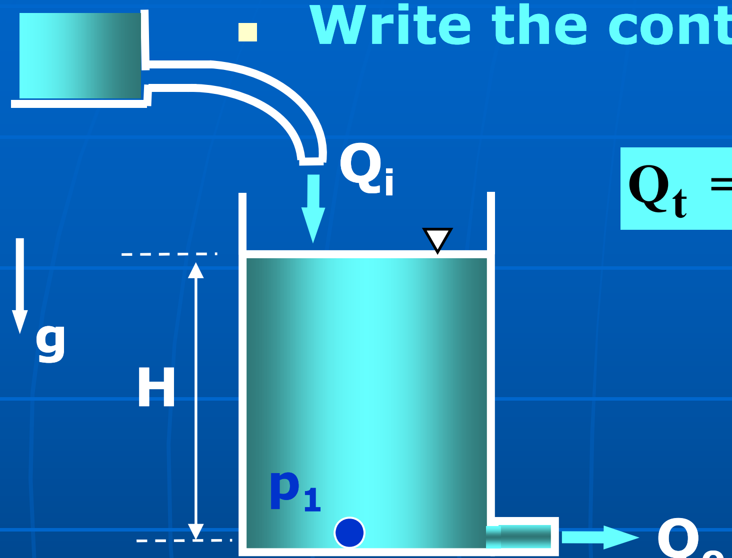


$$(\Delta p)'_{pipe} = R_p Q_o$$

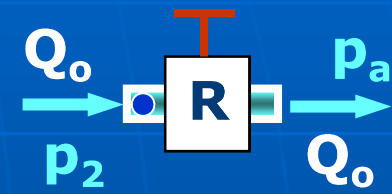
$$(\Delta p)''_{pipe} = I\frac{dQ}{dt} = \left(\frac{\rho L}{A_p}\right)\frac{dQ_o}{dt}$$

FLUID SYSTEM ELEMENTS – EXAMPLE 1c

- Write the continuity and compatibility equations.

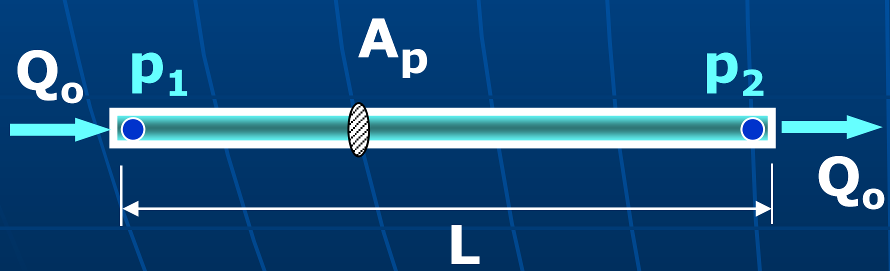


$$Q_t = Q_i - Q_o$$



- Which are continuity or compatibility equations ?

$$(\Delta p)_{valve} = p_2 - p_a$$
$$= p_2 - 0$$
$$= p_2$$



$$\Delta p = (\Delta p)'_{pipe} + (\Delta p)''_{pipe} = p_1 - p_2$$

FLUID SYSTEM ELEMENTS – EXAMPLE 1d

- Combine elemental and structural equations.

Tank

$$Q_t = \left(\frac{A_t}{\rho g}\right)\frac{dp_1}{dt}$$

+

$$Q_t = Q_i - Q_o$$

→

$$Q_i - Q_o = \left(\frac{A_t}{\rho g}\right)\frac{dp_1}{dt}$$

Pipe

$$(\Delta p)'_{pipe} = R_p Q_o$$

$$(\Delta p)''_{pipe} = \left(\frac{\rho L}{A_p}\right)\frac{dQ_o}{dt}$$

+

$$\Delta p = (\Delta p)'_{pipe} + (\Delta p)''_{pipe} = p_1 - p_2$$

→

$$p_1 - p_2 = \left(\frac{\rho L}{A_p}\right)\frac{dQ_o}{dt} + R_p Q_o$$

Valve

$$(\Delta p)_{valve} = RQ_o$$

+

$$(\Delta p)_{valve} = p_2$$

→

$$p_2 = RQ_o$$

FLUID SYSTEM ELEMENTS – EXAMPLE 1e

Tank

$$Q_i - Q_o = \left(\frac{A_t}{\rho g}\right)\frac{dp_1}{dt}$$

Pipe

$$p_1 - p_2 = \left(\frac{\rho L}{A_p}\right)\frac{dQ_o}{dt} + R_p Q_o$$

Valve

$$p_2 = RQ_o$$

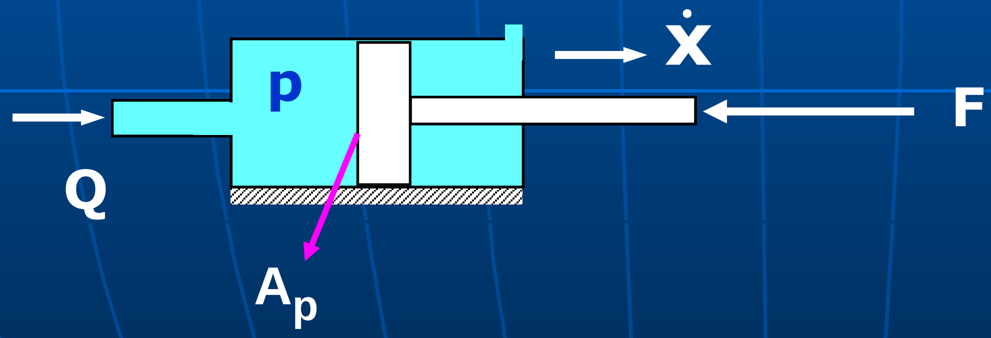
- Eliminate p_1 and p_2 from these 3 equations to obtain the relation between the input Q_i and output Q_o.

$$\left(\frac{A_t}{A_p}\right)\left(\frac{L}{g}\right)\frac{d^2Q_o}{dt^2} + \left(R + R_p\right)\left(\frac{A_t}{\rho g}\right)\frac{dQ_o}{dt} + Q_o = Q_i$$

STRUCTURAL EQUATIONS – Example

- **Piston+cylinder is a very common component in hydromechanical systems.**
- **Two structural equations can be written for this component.**

Piston+cylinder



$$Q = A_p \frac{dx}{dt}$$

$$F = A_p p$$

MODELING DYNAMIC SYSTEMS

OBJECTIVES

- **Deriving input-output relations of linear time invariant systems (mechanical, fluid, thermal, and electrical) using elemental and structural equations.**
- **Obtaining transfer function representation of LTI systems.**
- **Representing control systems with block diagrams.**

We are here !

Completed

THERMAL SYSTEM ELEMENTS

- **Thermal systems are those which involve heat transfer between elements. Energy is stored and transferred as heat.**
- **The two variables associated with thermal system elements are thus**
  - **temperature**
  - **heat flow**

THERMAL SYSTEM ELEMENTS

- **Thermal system elements are modelled as lumped thermal capacitance and lumped thermal resistance.**
- **There is no thermal inertance element.**

THERMAL SYSTEM ELEMENTS

- **Thermal Capacitance :** relates an object's temperature to the amount of heat energy stored by this object.

$$E = C_T T \qquad C_T = mc_p$$

E : heat energy,

C_T : thermal capacitance,

m : mass,

c_p : specific heat.

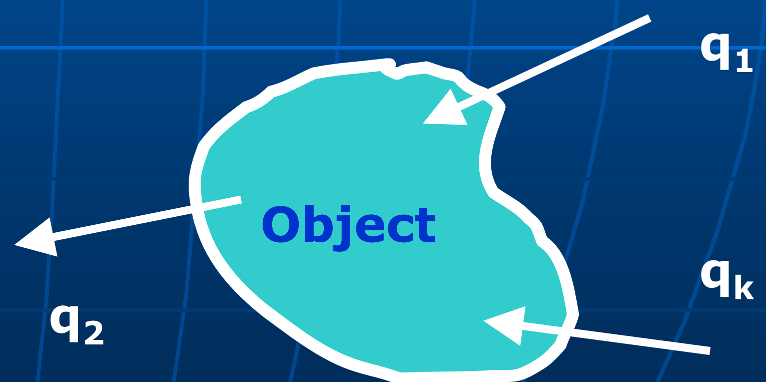
THERMAL SYSTEM ELEMENTS

- **To find the heat flow rate, one differentiates the energy equation. The rate of change of heat energy will be equal to the net heat flow in or out of the object.**

$$q_n = \frac{dE}{dt} = \frac{d}{dt}\left(C_T T\right) = C_T \frac{dT}{dt}$$

q_n : net heat flow rate.

$$q_n = \sum_{1}^{k} q_i$$



THERMAL SYSTEM ELEMENTS

- **Thermal Resistance**

The relation between the heat flow rate as a function of the temperature difference is given by :

$$\Delta T = R_T q_h$$

Why is it called resistance ?

ΔV (Δp) → ΔT

R → R_T

i (Q) → q_h

R_T : thermal resistance,

q_h : heat flow rate,

ΔT : temperature difference.

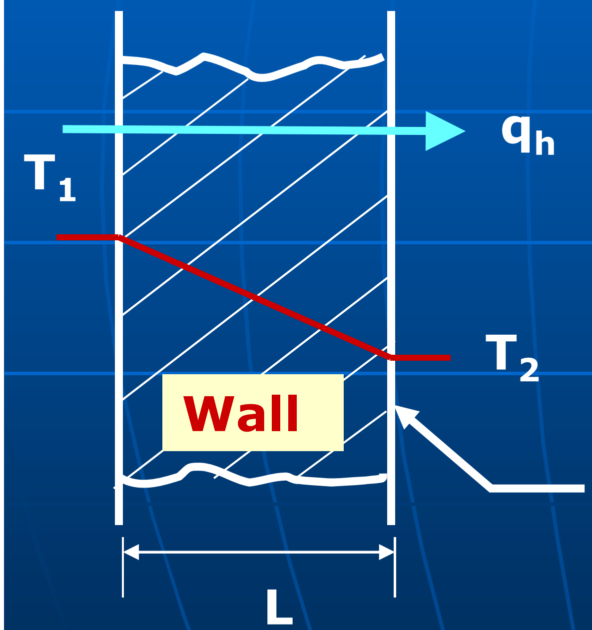
THERMAL SYSTEM ELEMENTS

- **Heat can be transferred by :**
  - **Conduction (diffusion through a substance),**
  - **Convection (fluid transport),**
  - **Radiation.**
- **In this course only the first two will be considered. Why ?**

$$q_h = \beta\left(T_1^4 - T_2^4\right)$$

THERMAL SYSTEM ELEMENTS

$$\Delta T = R_T q_h$$



A : Heat flow area

- **Thermal Resistance**

If the heat transfer is by conduction :

$$q_h = \left(\frac{kA}{L}\right)\left(T_1 - T_2\right)$$

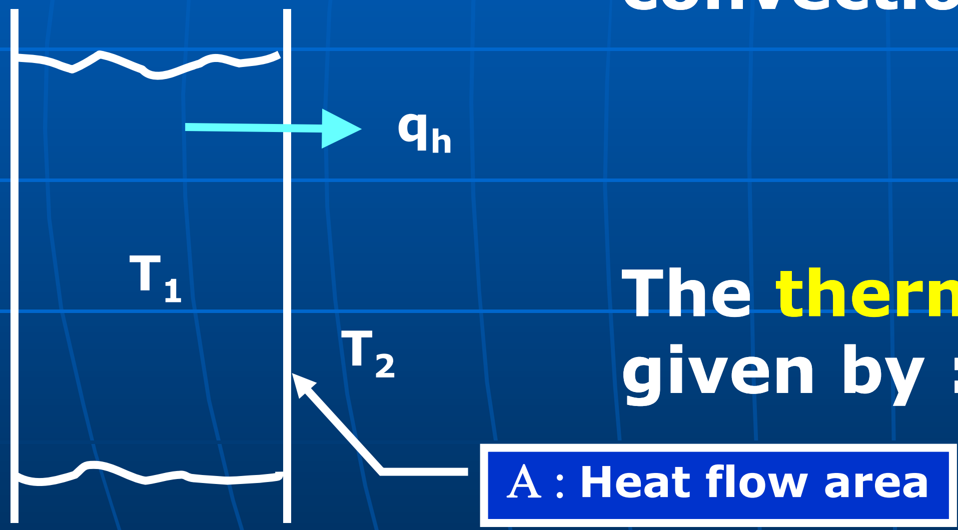
Thus the thermal resistance is given by :

$$R_T = \frac{L}{kA}$$

k : thermal conductivity.

THERMAL SYSTEM ELEMENTS

$$\Delta T = R_T q_h$$



- **Thermal Resistance**

If the heat transfer is by convection :

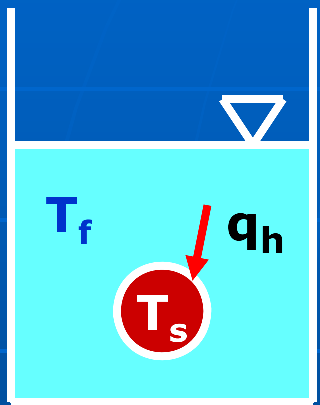
$$q_h = hA\left(T_1 - T_2\right)$$

The thermal resistance is then given by :

$$R_T = \frac{1}{hA}$$

h : film coefficient.

THERMAL SYSTEM ELEMENTS – Example 2



- **Dynamics of quenching**

  A copper sphere is immersed in a hot fluid (infinite mass). Write the relation between the temperature of the sphere and that of the fluid.

m : mass of copper sphere,
c_p : specific heat of copper,
h : film coefficient.

Elemental equations :

$$q_n = C_T \frac{dT_s}{dt}$$

$$C_T = mc_p$$

$$R_T q_h = T_f - T_s$$

$$R_T = \frac{1}{hA}$$

Continuity equation :

$$q_n = q_h$$

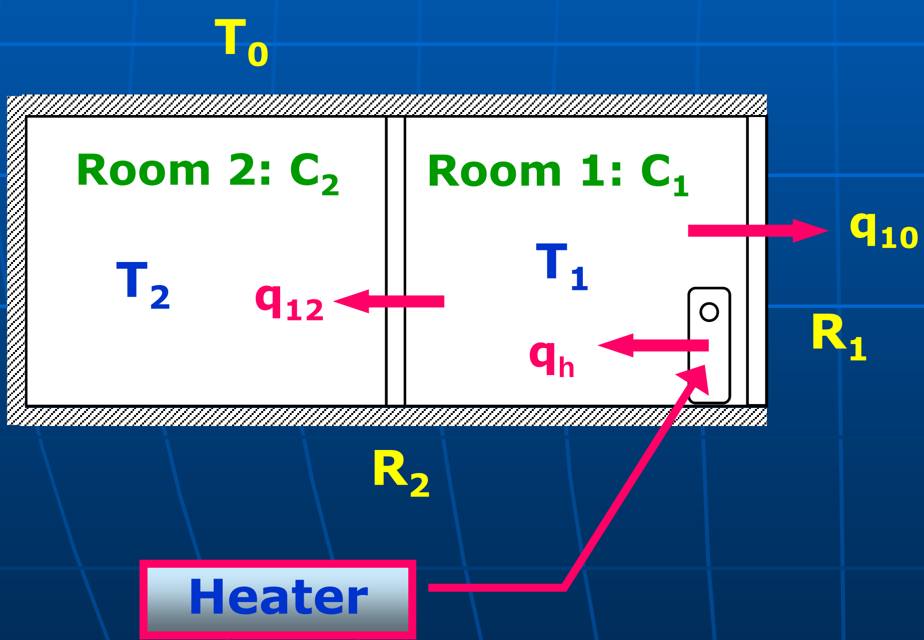
Eliminate q_n and q_h.

$$mc_p \frac{dT_s}{dt} = hA\left(T_f - T_s\right)$$

$$mc_p \frac{dT_s}{dt} + hAT_s = hAT_f$$

THERMAL SYSTEM ELEMENTS – Example 3a

- **Temperature dynamics of two rooms**
  - **Upper, lower, and left sides are perfectly insulated, i.e., there is no heat transfer through them.**



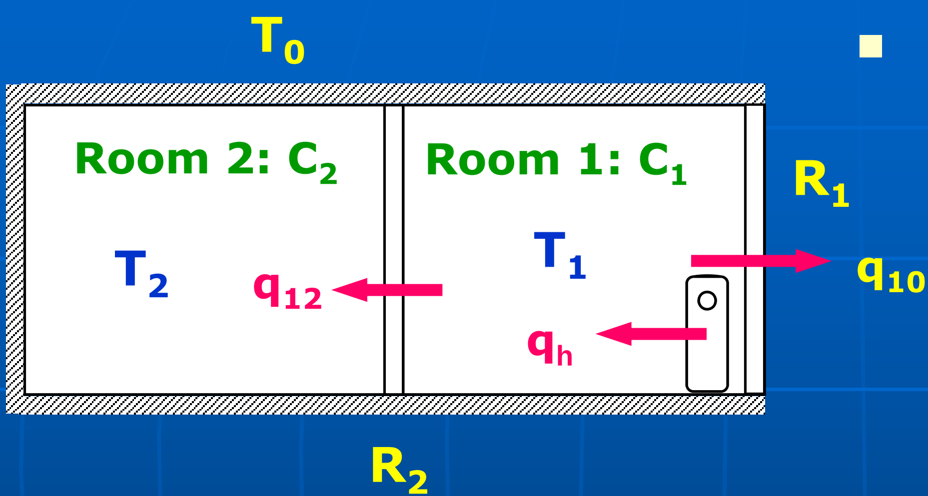
Input(s) :

1. **Heat supply rate of heater, q_h**
2. **Outside temperature, T_o**

Output :

1. **2nd room temperature, T_2**

THERMAL SYSTEM ELEMENTS – Example 3b



- **Thermal Capacitances :**

  - Room 1

$$q_{n1} = C_1 \frac{dT_1}{dt}$$

  - Room 2

$$q_{n2} = C_2 \frac{dT_2}{dt}$$

- **Thermal Resistances :**

  - Intermediate Wall

$$R_2 q_{12} = T_1 - T_2$$

- - Right wall

$$R_1 q_{10} = T_1 - T_0$$

THERMAL SYSTEM ELEMENTS – Example 3c

- **Structural equations :**

Room 1 $q_{n1} = q_h - q_{10} - q_{12}$

Room 2 $q_{n2} = q_{12}$

Insert the structural equations into the elemental equations.

$$q_h - q_{10} - q_{12} = C_1 \frac{dT_1}{dt}$$

$$q_{12} = C_2 \frac{dT_2}{dt}$$

THERMAL SYSTEM ELEMENTS – Example 3d

- **Eliminate q_{ij} and T_1 from the equations.**

$$q_{12} = C_2 \frac{dT_2}{dt}$$

$$R_2 q_{12} = T_1 - T_2$$

$$T_1 = R_2 C_2 \frac{dT_2}{dt} + T_2$$

$$\frac{dT_1}{dt} = R_2 C_2 \frac{d^2 T_2}{dt^2} + \frac{dT_2}{dt}$$

$$R_1 q_{10} = T_1 - T_0$$

$$q_{10} = \frac{C_2 R_2}{R_1} \frac{dT_2}{dt} + \frac{1}{R_1} T_2 - \frac{1}{R_1} T_0$$

THERMAL SYSTEM ELEMENTS – Example 3e

$$q_{10}=\frac{C_2R_2}{R_1}\frac{dT_2}{dt}+\frac{1}{R_1}T_2-\frac{1}{R_1}T_0$$

$$q_{12}=C_2\frac{dT_2}{dt}$$

$$q_h-q_{10}-q_{12}=C_1\frac{dT_1}{dt}$$

$$\frac{dT_1}{dt}=R_2C_2\frac{d^2T_2}{dt^2}+\frac{dT_2}{dt}$$

$$\left(C_1C_2R_1R_2\right)\frac{d^2T_2}{dt^2}+\left(C_1R_1+C_2R_1+C_2R_2\right)\frac{dT_2}{dt}+T_2=R_1q_h+T_0$$

THERMAL SYSTEM ELEMENTS – Example 3f

$$\left(C_1C_2R_1R_2\right)\frac{d^2T_2}{dt^2}+\left(C_1R_1+C_2R_1+C_2R_2\right)\frac{dT_2}{dt}+T_2=R_1q_h+T_0$$

- Note that this equation can be written in the form :

$$\left(C_1C_2R_1R_2\right)\frac{d^2\Delta T}{dt^2}+\left(C_1R_1+C_2R_1+C_2R_2\right)\frac{d\Delta T}{dt}+\Delta T=R_1q_h$$

- where $\Delta T=T_2-T_0$

MODELING DYNAMIC SYSTEMS

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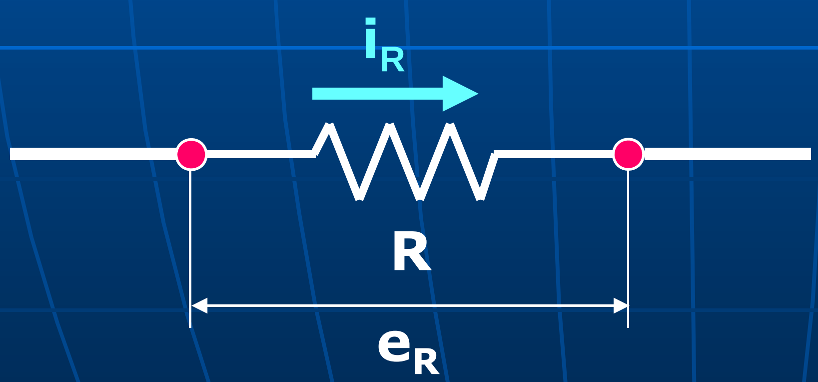
ELECTRICAL SYSTEM ELEMENTS

- **Electrical Resistance**

An electrical resistance, R, tries to prevent the flow of electrical current, i, and converts electrical energy to heat.

Thus, it is a dissipative element, equivalent to a damper in mechanical systems.



$$e_R = Ri_R$$

ELECTRICAL SYSTEM ELEMENTS

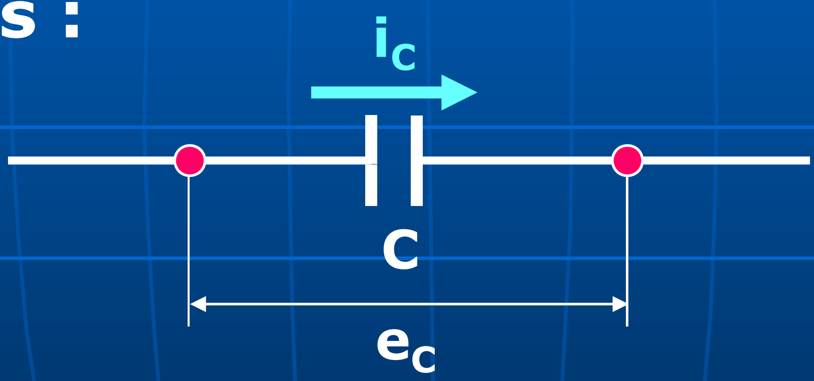
- **Electrical Capacitance**

Capacitance is the property that allows charge, q, to be stored.

$$e_C = \frac{1}{C} q_C$$

The relation between charge and current is :

$$i_C = \frac{dq_C}{dt}$$



$$i_C = C \frac{de_C}{dt}$$

An electrical capacitance is equivalent of mass in mechanical systems.

ELECTRICAL SYSTEM ELEMENTS

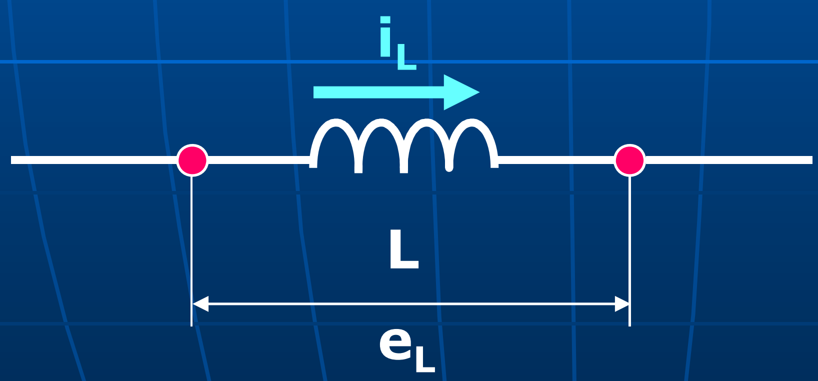
- ## Electrical Inductance

Inductance, L, is defined as the coefficient of the relation between magnetic flux and current.

$$L = \frac{\phi}{i_L}$$

The relation between flux and voltage is :

$$e_L = \frac{d\phi}{dt}$$



Thus :

$$e_L = L\frac{di_L}{dt}$$

Active Elements

All the elements considered so far have all been **passive elements**, i.e., they can store energy and release the stored energy into the system. Since they do not have an external power supply, however, they can only deliver the energy stored previously.

Active elements, on the other hand, possess their own external source of power. They can deliver external energy into the system.

Active Elements

A couple of familiar examples to active elements are the current and voltage sources used in electrical systems. They can maintain a set current or voltage, irrespective of the system behavior.

Current Source Voltage Source

Active Elements

A cam might be considered as the mechanical equivalent to the voltage source.

Similarly, a positive displacement pump could provide a (almost) constant flow rate irrespective of the system pressure. Thus it may be treated as equivalent to a current source.

A large reservoir, say a lake, may be considered to be equivalent to a voltage source. It can provide a pressure at a certain depth which is not affected by the amount of flow rate drawn from the lake .

Active Elements

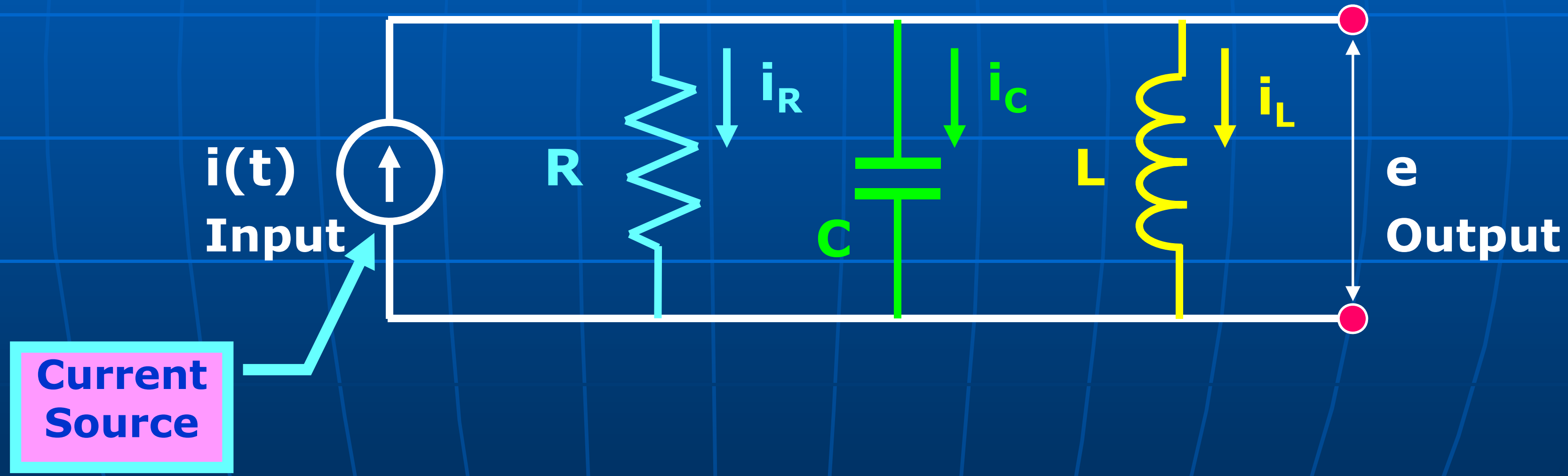
A thermal equivalent to a current source could be an electric heater which can provide a specified heat flow rate irrespective of the temperature difference with the surroundings.

The atmosphere may be considered as the equivalent of a voltage source, as it provides a temperature variation which is not affected by the thermal systems that may be in operation.

ELECTRICAL SYSTEM ELEMENTS – Example 4a

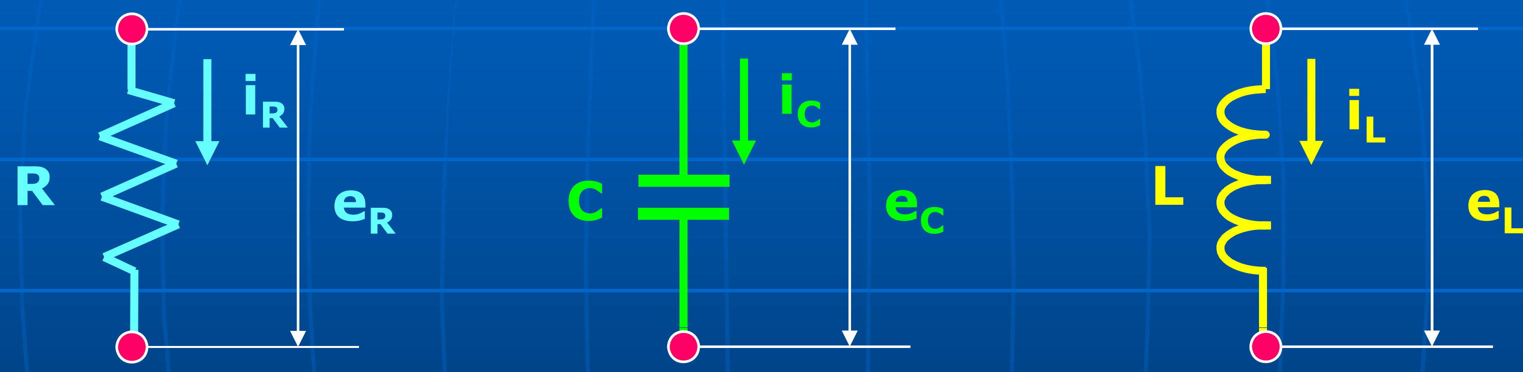
See Dorf& Bishop, Example 2.4

- **Determine the relation between the current input and the voltage output.**



ELECTRICAL SYSTEM ELEMENTS – Example 4b

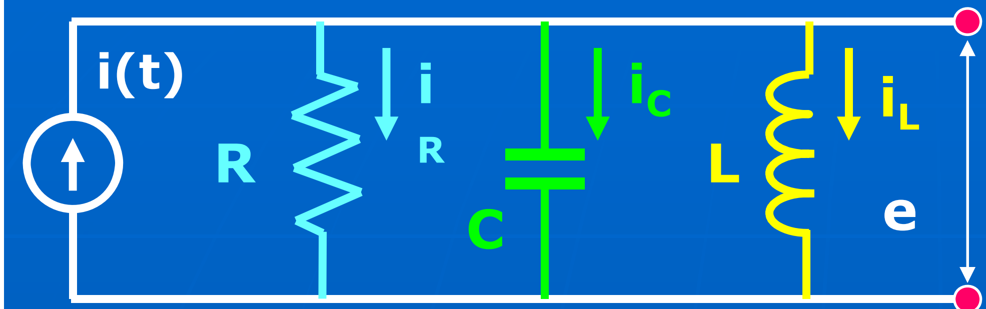
- **Write the elemental equations.**



$$e_R = R i_R$$

$$i_C = C\frac{de_C}{dt}$$

$$e_L = L\frac{di_L}{dt}$$



ELECTRICAL SYSTEM ELEMENTS – Example 4c

- Write the structural equations (Kirchoff's laws).
  - Continuity equation (node equation) :

$$i_R + i_C + i_L = i(t)$$

  - Compatibility equation (loop equations):

$$e_R = e_C = e_L = e$$

ELECTRICAL SYSTEM ELEMENTS – Example 4d

$$e_R = e_C = e_L = e$$

$$e_R = Ri_R \qquad i_C = C\frac{de_C}{dt} \qquad e_L = L\frac{di_L}{dt}$$

$$i_R + i_C + i_L = i(t)$$

- Combine elemental and structural equations to obtain the input-output relation.

$$\frac{1}{R}e + C\frac{de}{dt} + \frac{1}{L}\int e\,dt = i(t)$$

$$C\frac{d^2e}{dt^2} + \frac{1}{R}\frac{de}{dt} + \frac{1}{L}e = \frac{di(t)}{dt}$$

$$RCL\frac{d^2e}{dt^2} + L\frac{de}{dt} + Re = RL\frac{di(t)}{dt}$$

OBSERVATIONS

It seems that variables can be classified in two groups* :

Through variables :	Across variables :
Force,	Velocity,
Torque,	Angular Velocity,
Flow rate,	Pressure (Head),
Heat Flow Rate, and	Temperature, and
Current.	Voltage.

* Dorf & Bishop, Table 2.1

OBSERVATIONS

Nise Tables 2.3, 2.4, 2.5

Dorf & Bishop, Table 2.2

	Storage	Dissipative	Elastic
Mechanical			
• Translational	$F = m\frac{dV}{dt}$	$\Delta V = \frac{1}{b}F$	$\Delta V = \frac{1}{k}\frac{dF}{dt}$
• Rotational	$T = J\frac{d\omega}{dt}$	$\Delta\omega = \frac{1}{b}T$	$\Delta\omega = \frac{1}{k}\frac{dT}{dt}$
Fluid	$Q = C\frac{dH}{dt}$	$\Delta p = R\,Q$	$\Delta p = I\frac{dQ}{dt}$
Thermal	$q = C\frac{dT}{dt}$	$\Delta T = R_T q_h$	
Electrical	$i = C\frac{de}{dt}$	$e_R = R i_R$	$e = L\frac{di}{dt}$

ABBREVIATIONS

- **A/P → Autopilot**
- **R/A→ Radio Altimeter**
- **ILS → Instrument Landing System**
- **INOP → Inoperational**