A Few Tips on How to Write Mathematics Updated January 1997

I will be enforcing a few stylistic "rules" about the writing of graceful mathematics. As you peruse the items in the following list, please note that they are all of the "common sense" variety and do not provide any deep insights into how good mathematical writing differs from good expository writing in other areas of academic discourse. I include them here primarily to remind you that a clear mathematical presentation requires far more than what most of you are used to turning in as solutions to homework problems in algebra or calculus. As the semester progresses, I will make extensive comments on your papers pointing out refinements of and additions to these rules that should help your mathematical writing as well as any of your other expository efforts.

1. Always use complete and accurately punctuated sentences. For example: $(x^2 - 2x + 1) = 0$ solving $(x - 1)^2$ so x = 1" is a **horrible** piece of writing. Not only is it not punctuated correctly but it does not even consist of complete sentences. An improved version is: "If we now solve $x^2 - 2x + 1 = 0$, we have: $x^2 - 2x + 1 = 0$ which implies that $(x - 1)^2 = 0$ which implies that x - 1 = 0 which implies that x = 1. Hence the solution set for the equation $x^2 - 2x + 1 = 0$ is $S = \{1\}$."

Note that these sentences (although quite repetitive) make proper use of punctuation as well as being easier to follow than the original. Note also that mathematical symbols are read as words. Hence the symbol "=" is read as "equals" which is a verb. This means we can further improve the above sentence by replacing the phrase "which implies that" with its equivalent mathematical symbol " \Rightarrow ". This results in a correctly punctuated paragraph which is acceptable as a working draft. However, since it still does not read gracefully (read it out loud to see what I mean), it could be further refined in the final draft.

- 2. One should also avoid starting a sentence (or a major clause) with a mathematical symbol. For example, while "A points are on the line l." is a valid sentence, it is misleading. The intent is to use the symbol A to represent the number of points on a particular line (the fact that it is possible that some lines have only a finite number of points incident with them is explored in any non-Euclidean geometry course) However, since the indefinite article "a" is capitalized at the beginning of a sentence, there is a real chance that the reader will experience some confusion when reading the above sentence. One way to improve the above is to avoid starting the sentence with a symbol as in: "We use the symbol A to denote the number of points on the line l"
- 3. The accepted style in mathematical writing is to avoid passive voice whenever possible. Thus, sentences such as "The theorem is now proved." are rarely used. In order to avoid the use of passive voice, mathematicians assign a subject to the sentence by using the pronouns "we", or "our". Thus, an active version of the sentence above is "We have now proven the theorem." (On a more subtle stylistic point, if the proof is very long, it is better to end the proof with a sentence that reiterates what was proven. For example, if

the theorem claimed all lines had the same number of points, one could end the proof with: "This shows that every line in this finite geometry is incident with exactly 4 points.")

As in the above, whenever you use the word "we" in mathematical writing it is best to have it refer to the set consisting of the author and the readers. This is the author's way of showing some consideration to the readers in that she is saying in effect "We are working this problem together because you, the reader, are an active participant in this proof."

- 4. Because the main purpose of mathematical writing is to clearly communicate an argument, whenever you are making a mathematical point and there is a logical jump, apply the following strategy. If the reader could more easily follow your presentation if you gave them more detail and it is only necessary to write another sentence or two to provide that detail, then include those sentences. The idea in technical writing is to share knowledge. If it is not much of a burden to make it easier for the reader to understand your argument, you should do so. (Note that this rule is valid only when you have a reasonable amount of time to revise. It is not appropriate in a timed situation such as an in-class test.)
- 5. It is **NOT** necessary to include symbols on every line of your writing nor is it necessary to completely forgo their use. Mathematical and logical symbols are communication tools carefully crafted to contain a great deal of information in a small amount of space. Hence these symbols prove to be exceptionally useful to those who know their definitions. However, anything that can be communicated using symbols can also be presented without them; so make as much use of them as you like. You should find that your use of symbols will evolve naturally to a style similar to the text's as you become more familiar with their use.