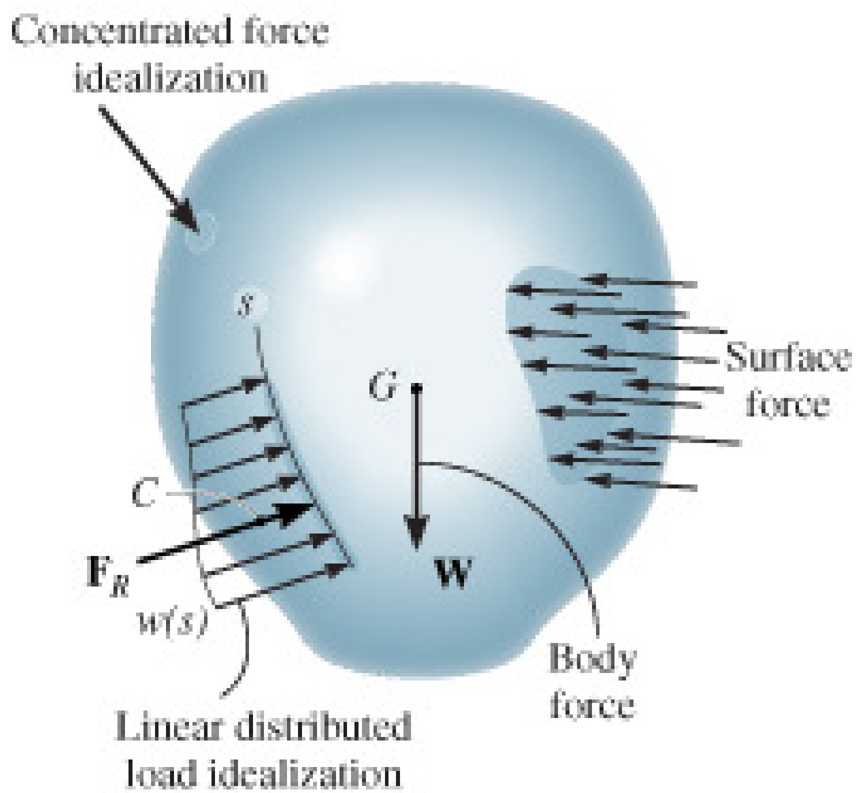
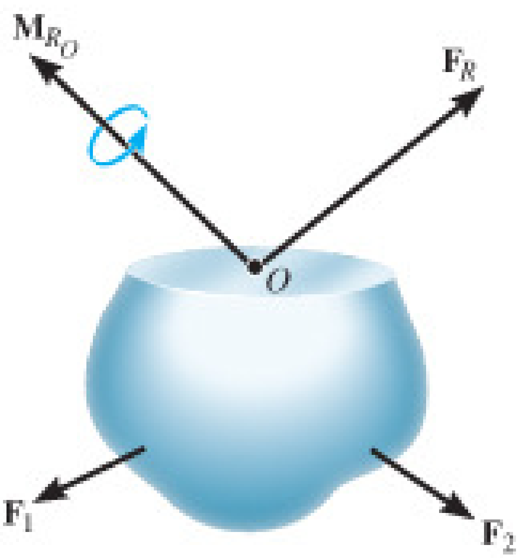
# EQUILIBRIUM OF A DEFORMABLE BODY

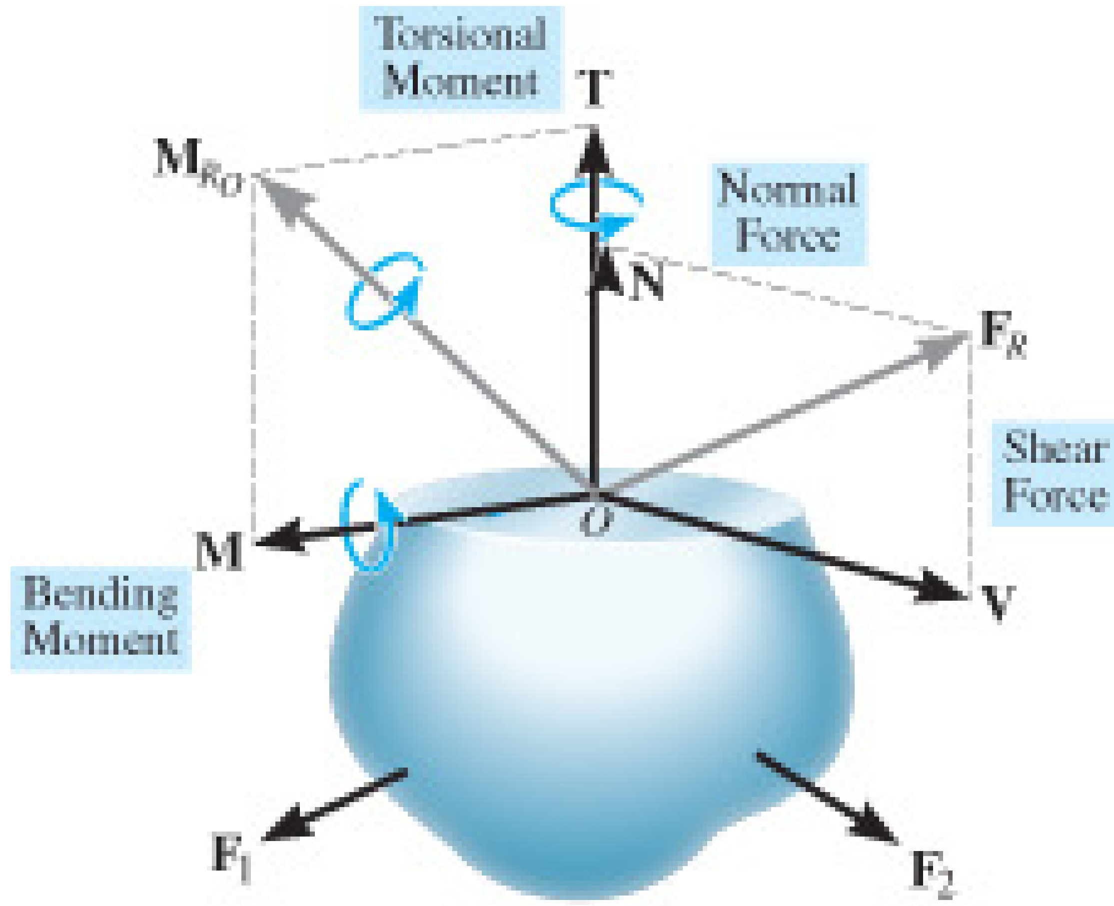
**External loads**



## Internal resultant loadings

- Define resultant force (**F**$_R$) and moment (**M**$_{Ro}$) in 3D:
  - Normal force, **N**
  - Shear force, **V**
  - Torsional moment or torque, **T**
  - Bending moment, **M**



# EQUILIBRIUM OF A DEFORMABLE BODY

## Internal resultant loadings

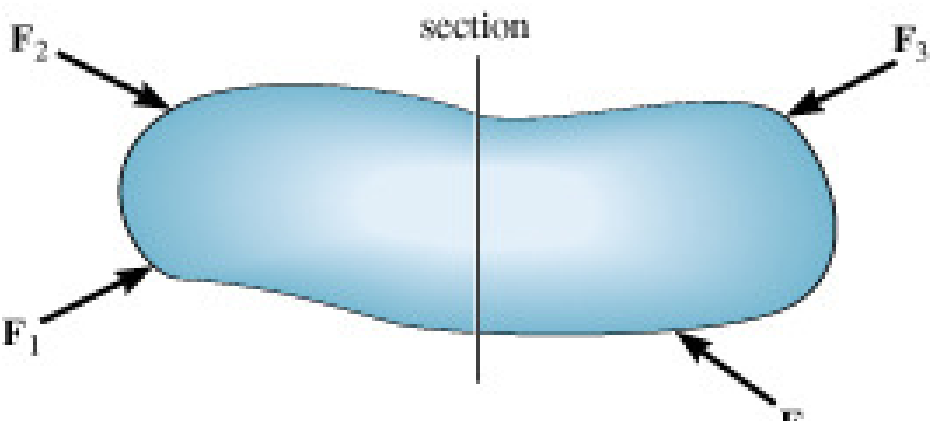
- For coplanar loadings:
  - Normal force, **N**
  - Shear force, **V**
  - Bending moment, **M**

For coplanar loadings:

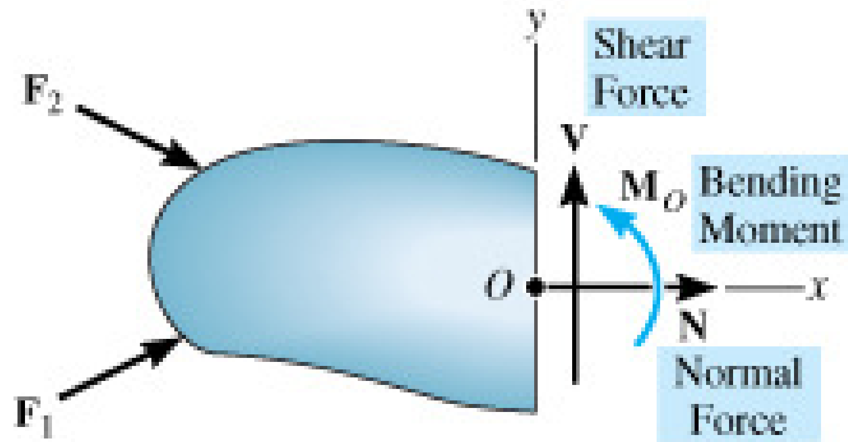
Apply $\sum F_x = 0$ to solve for **N**

Apply $\sum F_y = 0$ to solve for **V**

Apply $\sum M_O = 0$ to solve for **M**




(a)

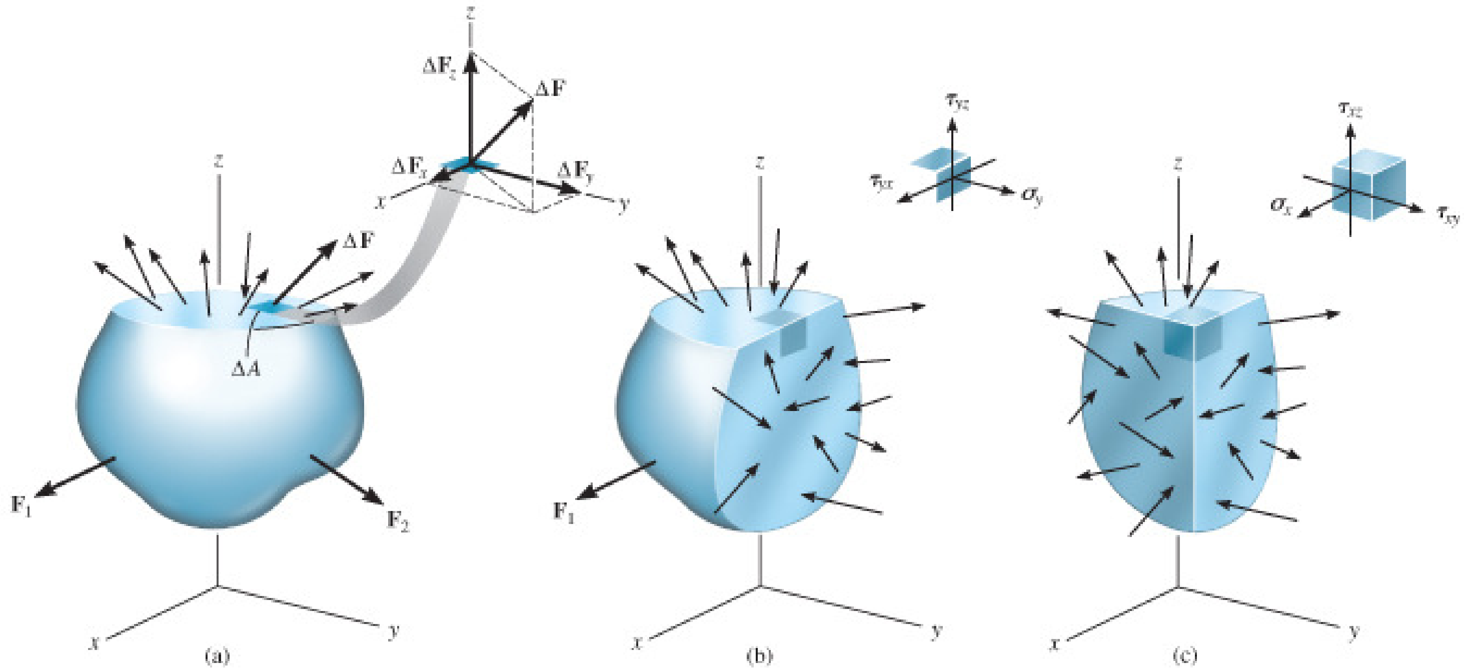



(b)

# STRESS

**Concept of stress**

- Consider $\Delta A$ in figure below
- Small finite force, $\Delta F$ acts on $\Delta A$
- As $\Delta A \rightarrow 0$, $\Delta F \rightarrow 0$
- But stress ($\Delta F / \Delta A$) → finite limit (∞)



# STRESS

## Normal stress

- *Intensity* of force, or force per unit area, acting *normal* to $\Delta A$
- Symbol used for *normal stress,* is $\sigma$ (sigma)

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

Tensile stress: normal force “pulls” or “stretches” the area element $\Delta A$

Compressive stress: normal force “pushes” or “compresses” area element $\Delta A$

# STRESS

## Shear stress

- *Intensity* of force, or force per unit area, acting *tangent* to $\Delta A$
- Symbol used for normal stress is $\tau$ (tau)

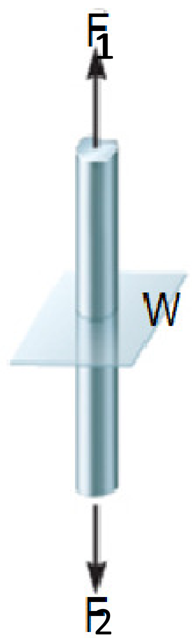
$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

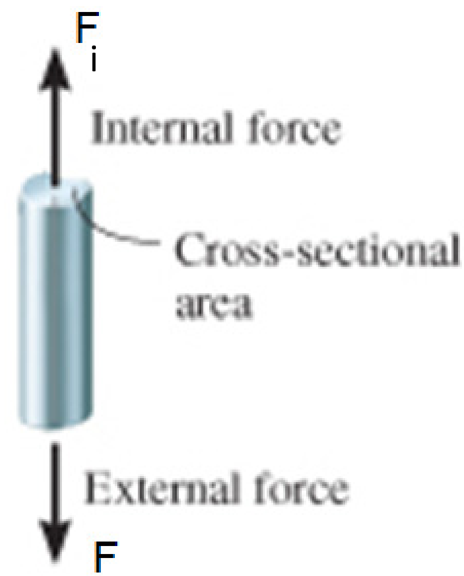
AVERAGE NORMAL STRESS IN AXIALLY LOADED BAR

**Examples of axially loaded bar**

- Usually long and slender structural members
- Truss members, hangers, bolts
- Prismatic means all the cross sections are the same




If F>>W
then neglect
effect of W



## AVERAGE NORMAL STRESS IN AXIALLY LOADED BAR

### Assumptions

1. Uniform deformation: Bar remains straight before and after load is applied, and cross section remains flat or plane during deformation
2. In order for uniform deformation, force **F** be applied along centroidal axis of cross section

## 1.4 AVERAGE NORMAL STRESS IN AXIALLY LOADED BAR

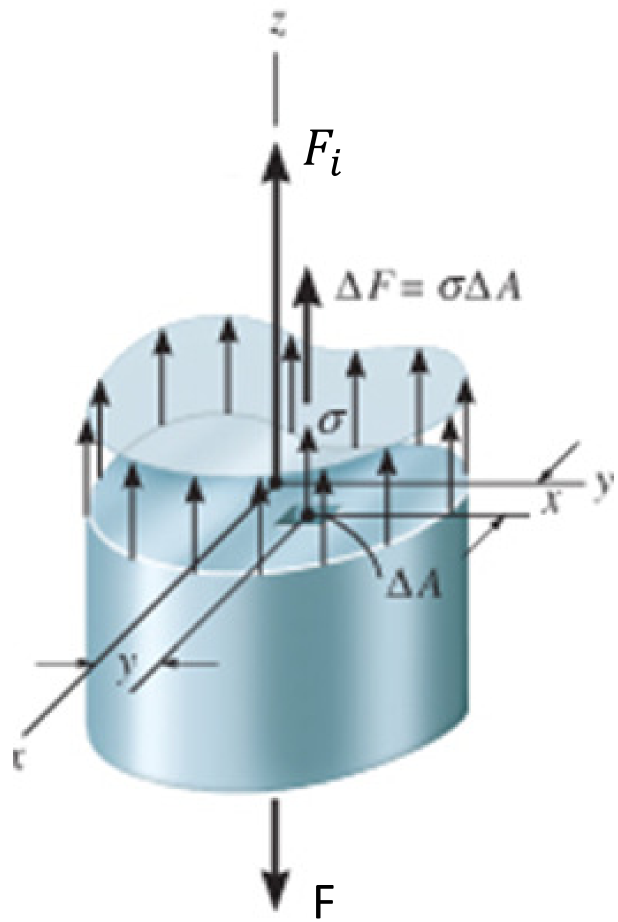
### Average normal stress distribution

$+\uparrow \quad F_{Rz} = \sum F_{xz} \qquad \int dF = \int_A \sigma \, dA$

$$F_i = \sigma A$$

$$\sigma = \frac{F_i}{A}$$

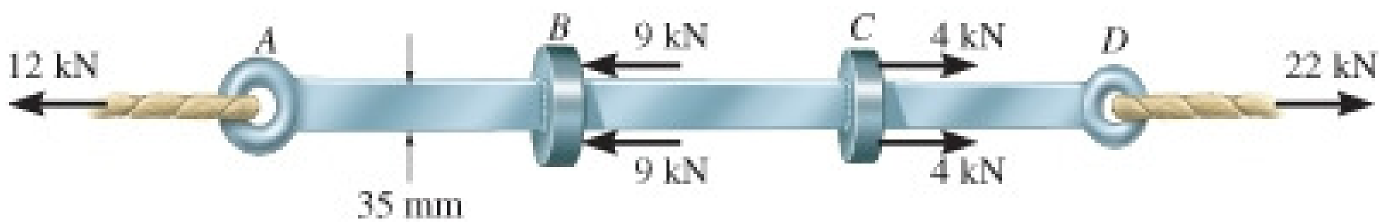
$\sigma$ = average normal stress at any point on cross sectional area
$F_i$ = internal resultant normal force
$A$ = x-sectional area of the bar



# EXAMPLE

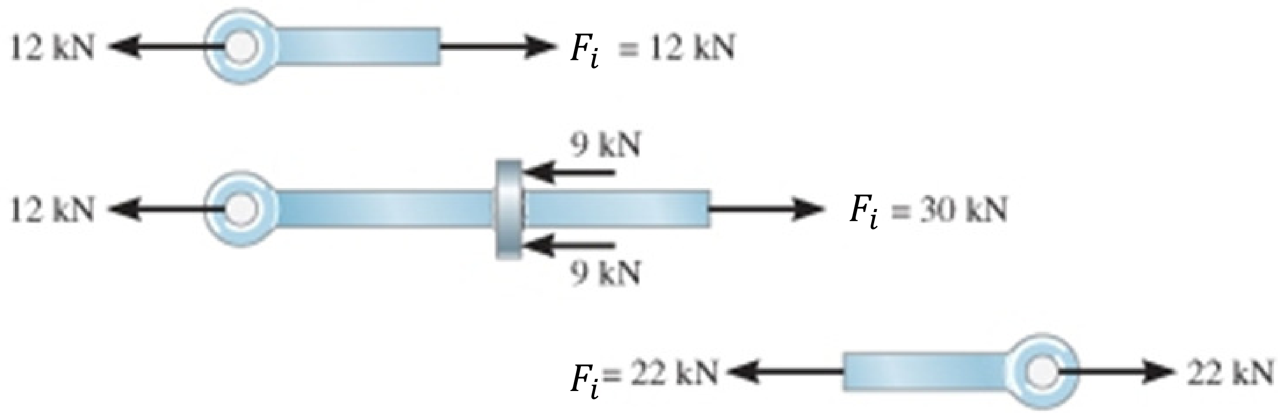
Bar width = 35 mm, thickness = 10 mm

Determine max. average normal stress in bar when subjected to loading shown.



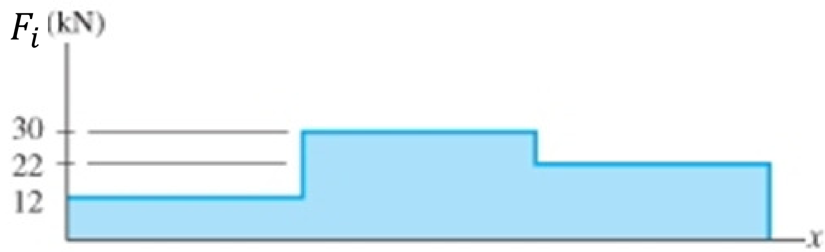
# EXAMPLE (SOLN)

**Internal loading**




**Normal force diagram**

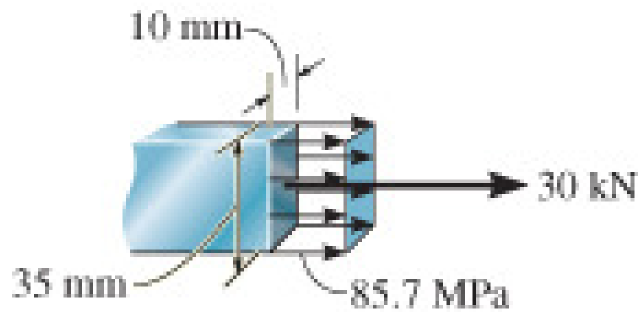
By inspection, largest loading area is *BC*, where $F_i$ = 30 kN



# EXAMPLE (SOLN)

**Average normal stress**

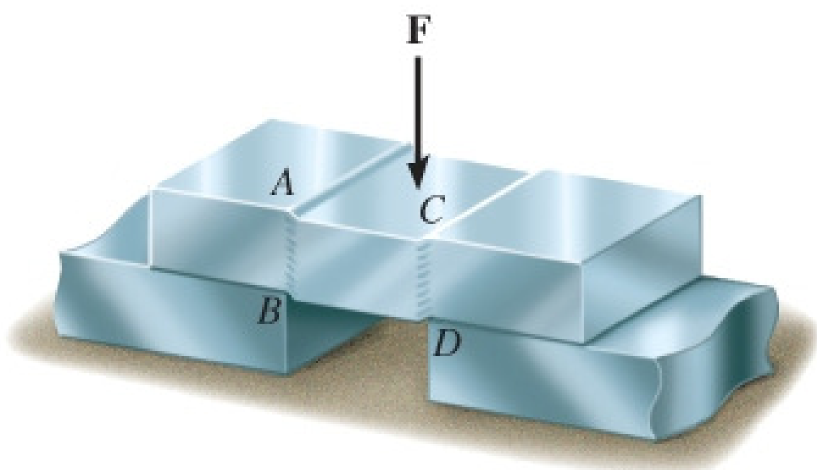
$$\sigma_{BC} = \frac{F_i}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = \mathbf{85.7\ MPa}$$



(d)

# AVERAGE SHEAR STRESS

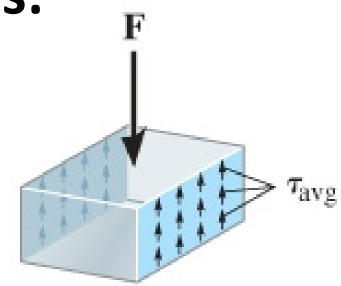
- Shear stress is the stress component that act in the plane of the sectioned area.
- Consider a force **F** acting to the bar
- For rigid supports, and **F** is large enough, bar will deform and fail along the planes identified by *AB* and *CD*
- Free-body diagram indicates that shear force, $V = F/2$ be applied at both sections to ensure equilibrium



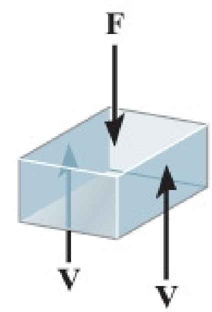
# AVERAGE SHEAR STRESS

**Average shear stress over each section is:**

$$\tau_{avg} = \frac{V}{A}$$




$\tau_{avg}$ = average shear stress at section, assumed to be same at each pt on the section




$V$ = internal resultant shear force at section determined from equations of equilibrium

$A$ = area of section

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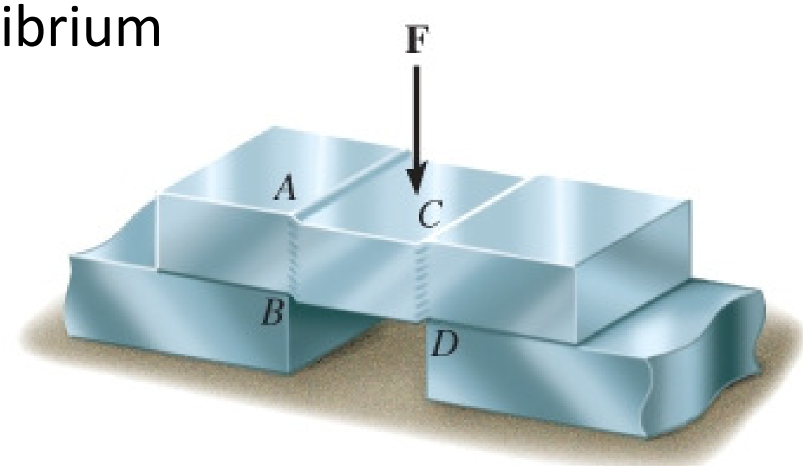

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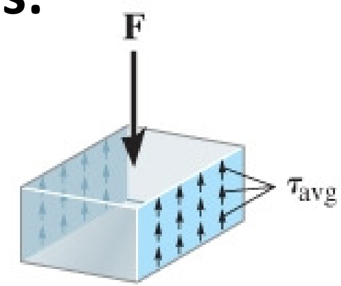


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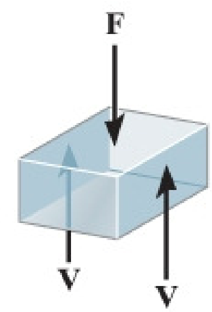



$A$ = area of section

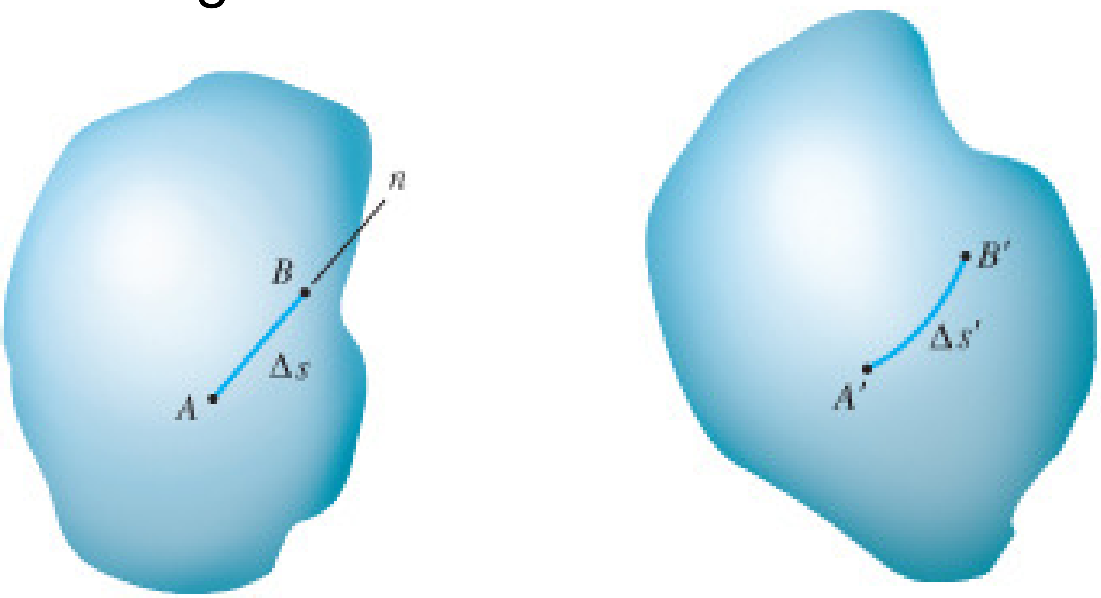
# DEFORMATION
# STRAIN

**To simplify study of deformation**

- Assume lines to be very short and located in neighborhood of a point, and
- Take into account the orientation of the line segment at the point

## Normal strain

- Defined as the elongation or contraction of a line segment per unit of length
- Consider line *AB* in figure below
- After deformation, $\Delta s$ changes to $\Delta s'$




Undeformed body (a)

Deformed body (b)

## Normal strain

- Defining *average normal strain* using $\varepsilon_{avg}$ (epsilon)

$$\varepsilon_{avg} = \frac{\Delta s - \Delta s'}{\Delta s}$$

- As $\Delta s \to 0$, $\Delta s' \to 0$

$$\varepsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s - \Delta s'}{\Delta s}$$

# Mechanical Properties of Materials

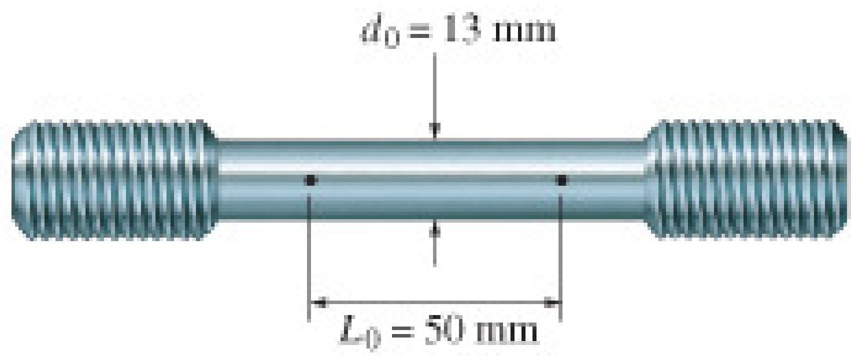
## Tension& Compression Test

- Strength of a material can only be determined by *experiment*
- One test used by engineers is the *tension or compression test*
- This test is used primarily to determine the relationship between the average normal stress and average normal strain in common engineering materials, such as metals, ceramics, polymers and composites

# TENSION & COMPRESSION TEST

**Performing the tension or compression test**

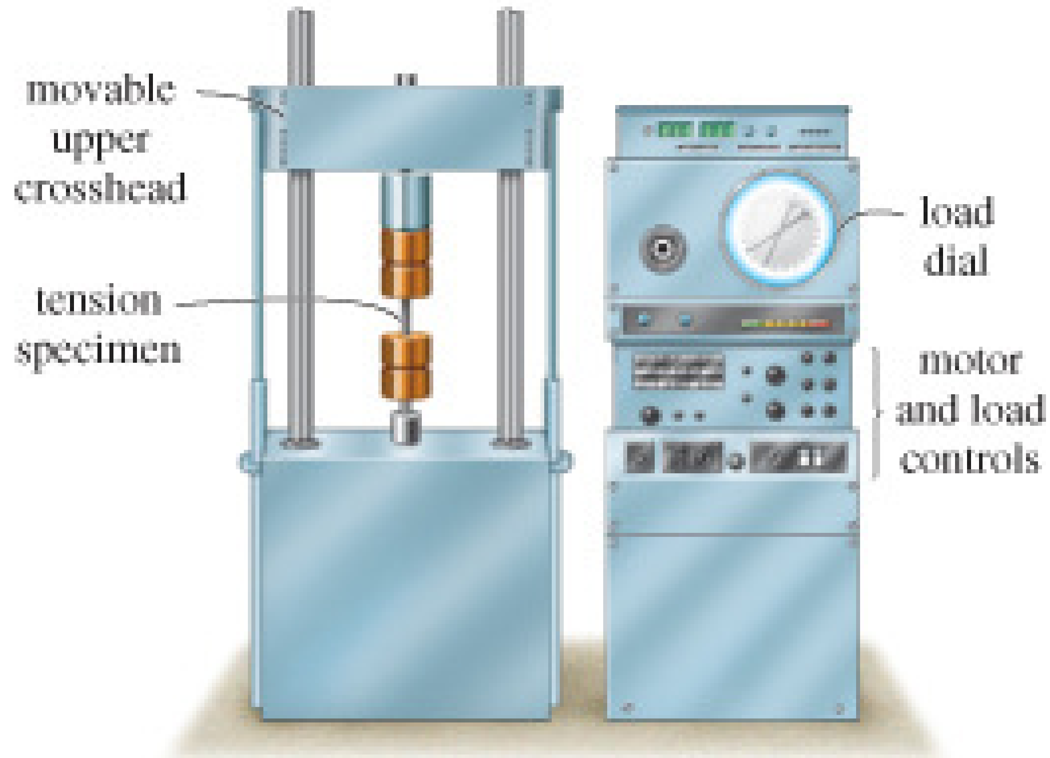
- Specimen of material is made into "standard" shape and size
- Before testing, 2 small punch marks identified along specimen's length
- Measurements are taken of both specimen's initial x-sectional area $A_0$ and gauge-length distance $L_0$; between the two marks
- Seat the specimen into a testing machine shown below



# TENSION & COMPRESSION TEST

**Performing the tension or compression test**

- Seat the specimen into a testing machine shown below




The machine will stretch specimen at slow constant rate until breaking point

At frequent intervals during test, data is recorded of the applied load F.

# STRESS-STRAIN

- A *stress-strain diagram* is obtained by plotting the various values of the stress and corresponding strain in the specimen

## Conventional stress-strain diagram

- Using recorded data, we can determine nominal or engineering stress by

$$\sigma = \frac{F}{A_0}$$

Assumption: Stress is *constant* over the x-section and throughout region between gauge points

## Conventional Stress-Strain Diagram

- Likewise, nominal or engineering strain is found directly from strain gauge reading, or by

$$\varepsilon = \frac{\delta}{L_0}$$

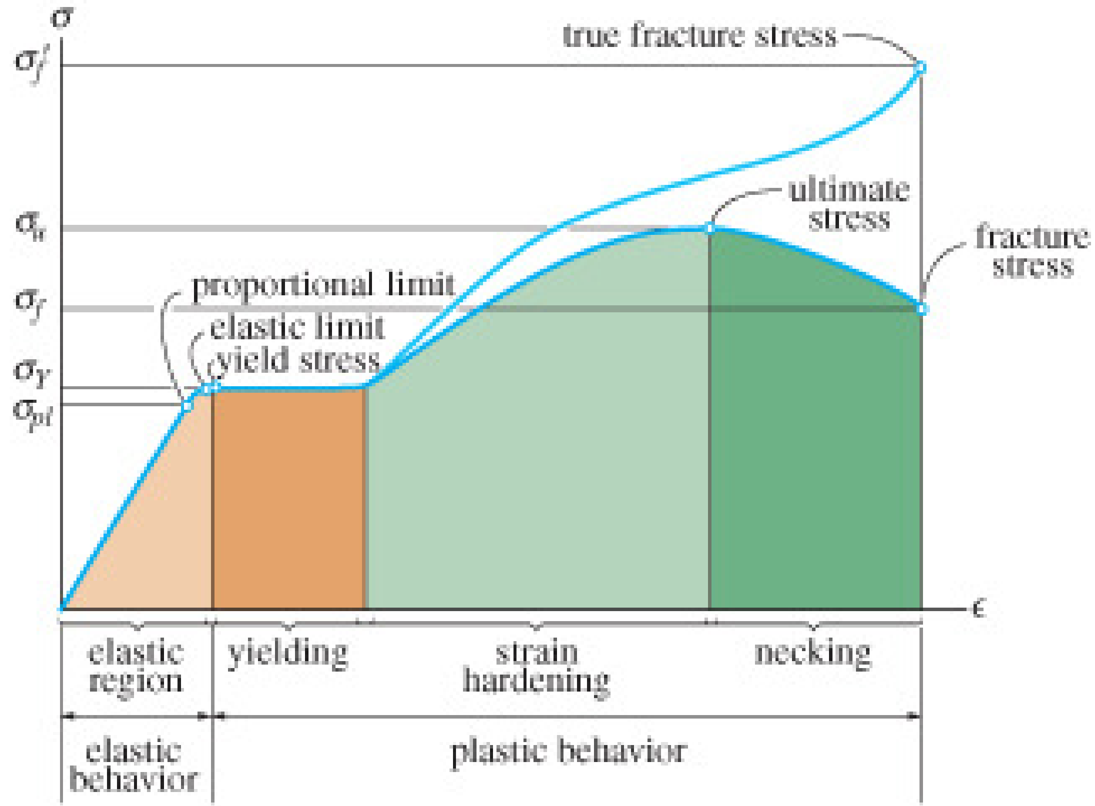
Assumption: Strain is constant throughout region between gauge points

By plotting $\sigma$ (ordinate) against $\varepsilon$ (abscissa), we get a *conventional stress-strain diagram*

# STRESS-STRAIN DIAGRAM

## Conventional stress-strain diagram

- Figure shows the characteristic stress-strain diagram for steel, a commonly used material for structural members and mechanical elements




Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

# Conventional stress-strain diagram

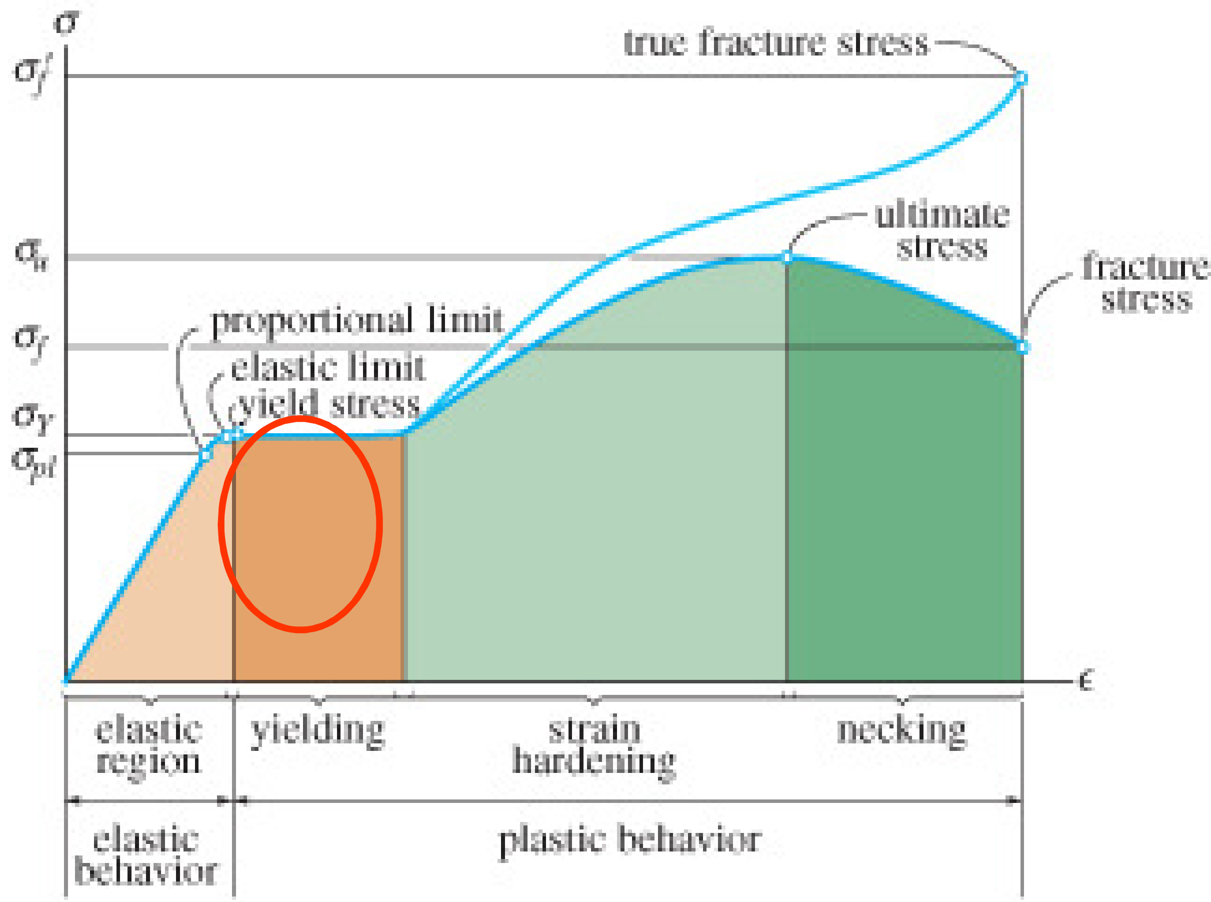
**Elastic behavior.**

A straight line

Stress is proportional to strain, i.e., linearly elastic

Upper stress limit, or *proportional limit*; $\sigma_{pl}$

If load is removed upon reaching elastic limit, specimen will return to its original shape




Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

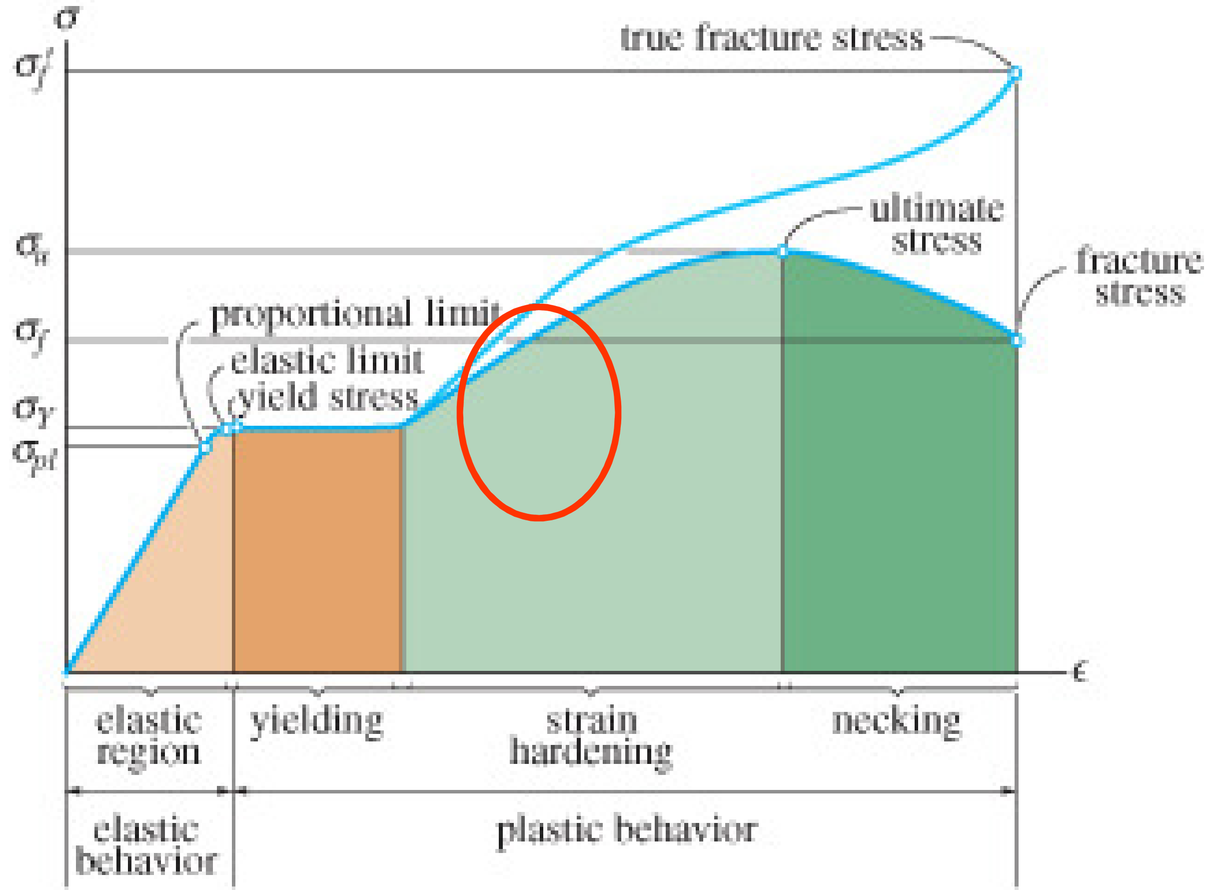
# Conventional stress-strain diagram

**Yielding.**

Material deforms permanently; yielding; plastic deformation

Yield stress, $\sigma_Y$

Once yield point reached, specimen continues to elongate (strain) *without any increase in load*



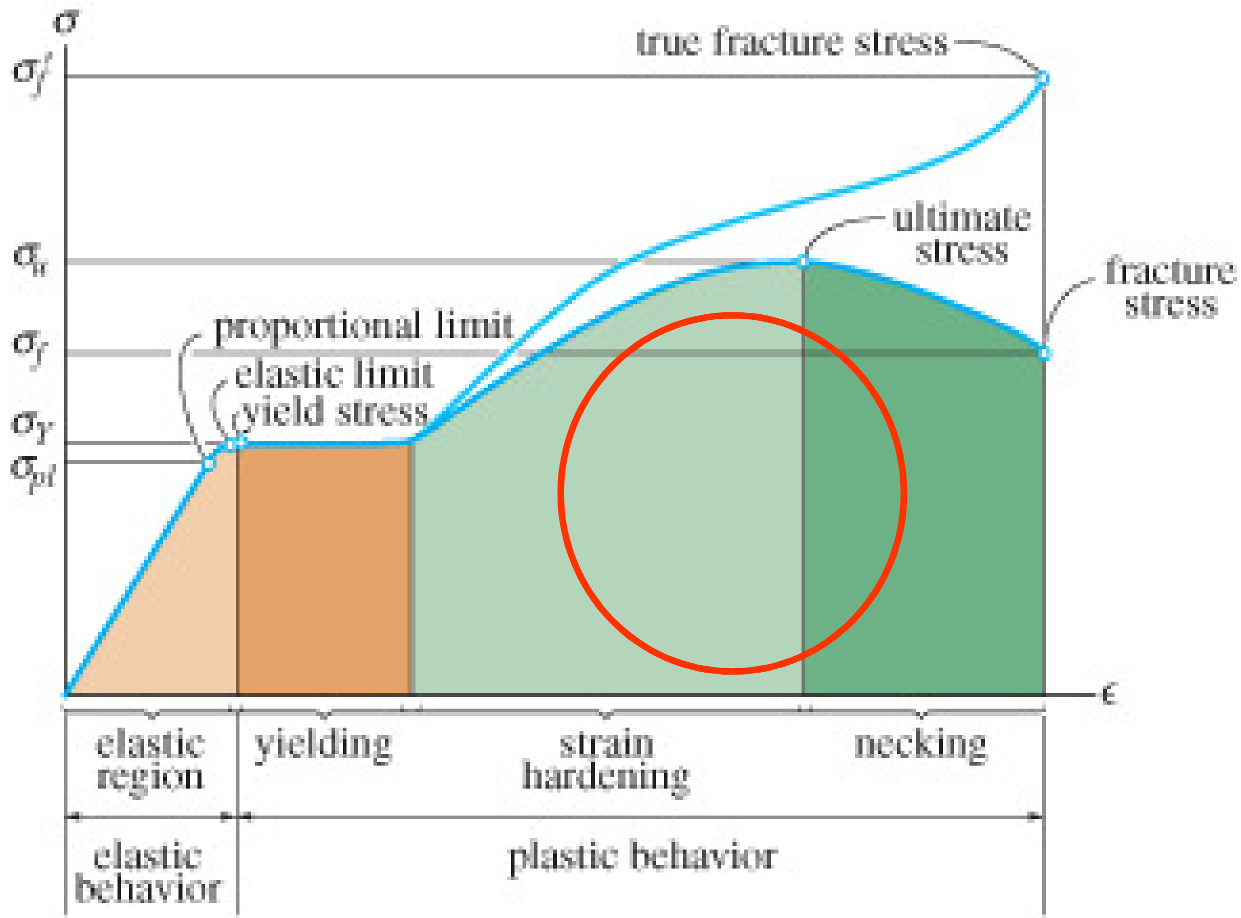

Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

# Conventional stress-strain diagram

**Strain hardening.**

Ultimate stress, $\sigma_u$

While specimen is elongating, its x-sectional area will decrease




Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

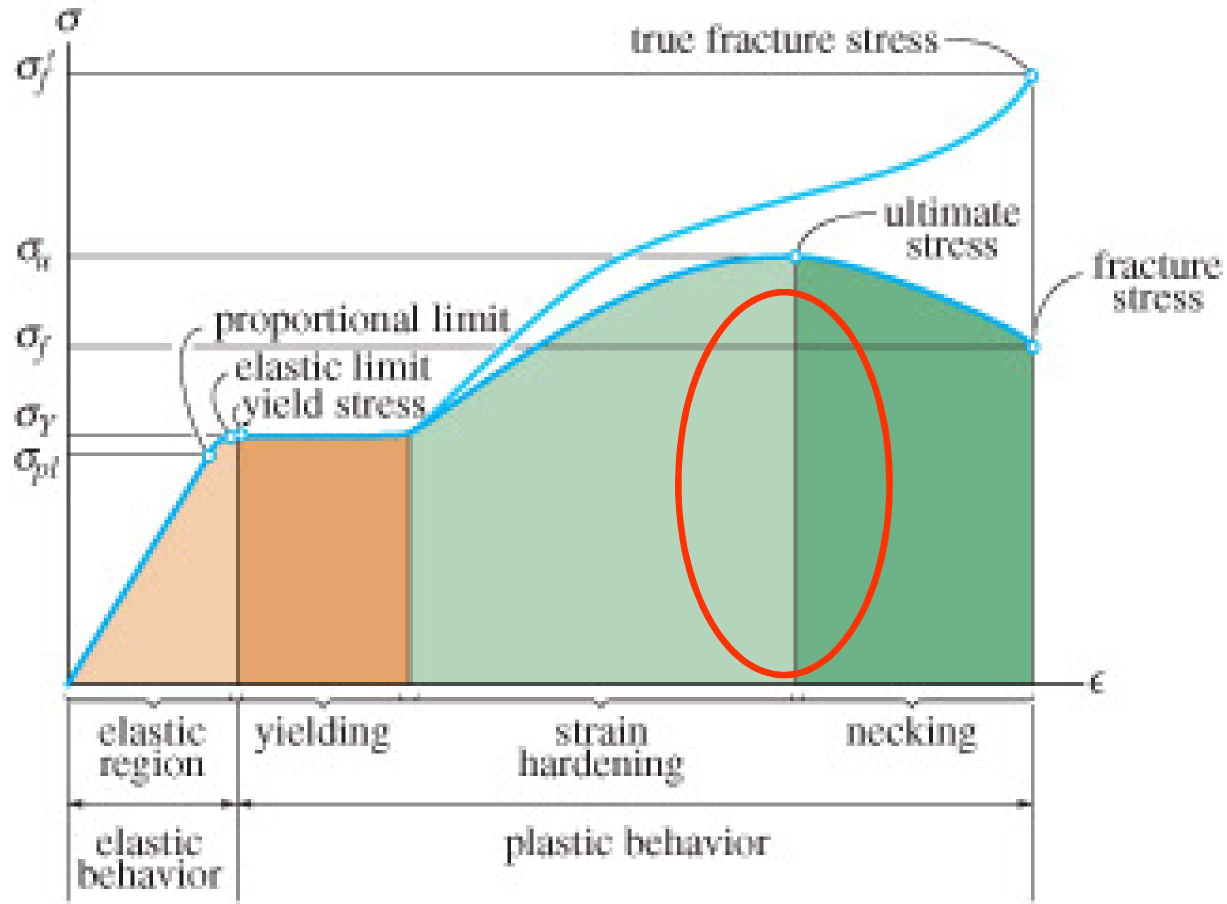
# Conventional stress-strain diagram

**Necking.**

At ultimate stress, x-sectional area begins to decrease in a *localized* region

As a result, a constriction or “neck” tends to form in this region as specimen elongates further

Specimen finally breaks at fracture stress, $\sigma_f$




Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

# Conventional stress-strain diagram

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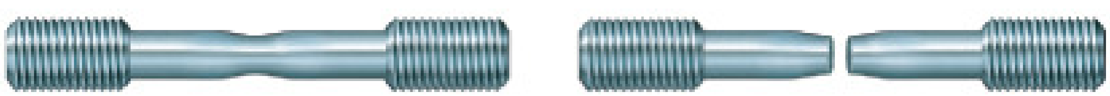
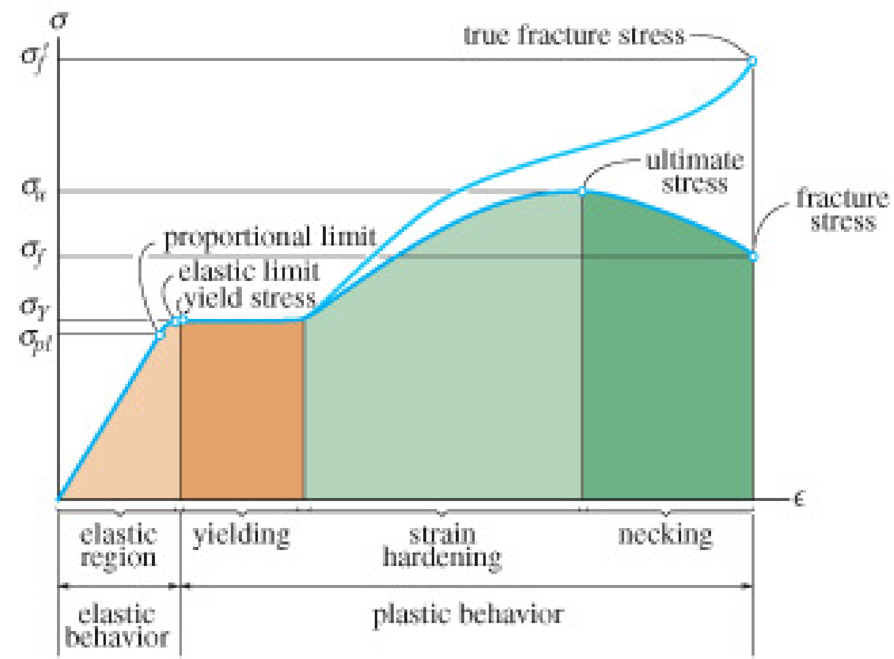


Necking

(a)

Failure of a ductile material

(b)




Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

**Percent elongation** is the specimen's fracture strain expressed as a percent

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0}(100\%)$$

- **Percent reduction in area** is defined within necking region as

$$\text{Percent reduction in area} = \frac{A_0 - A_f}{A_0}(100\%)$$

# HOOKE'S LAW

Most engineering materials exhibit a *linear relationship* between stress and strain with the elastic region

Discovered by Robert Hooke in 1676 using springs, known as *Hooke's law*

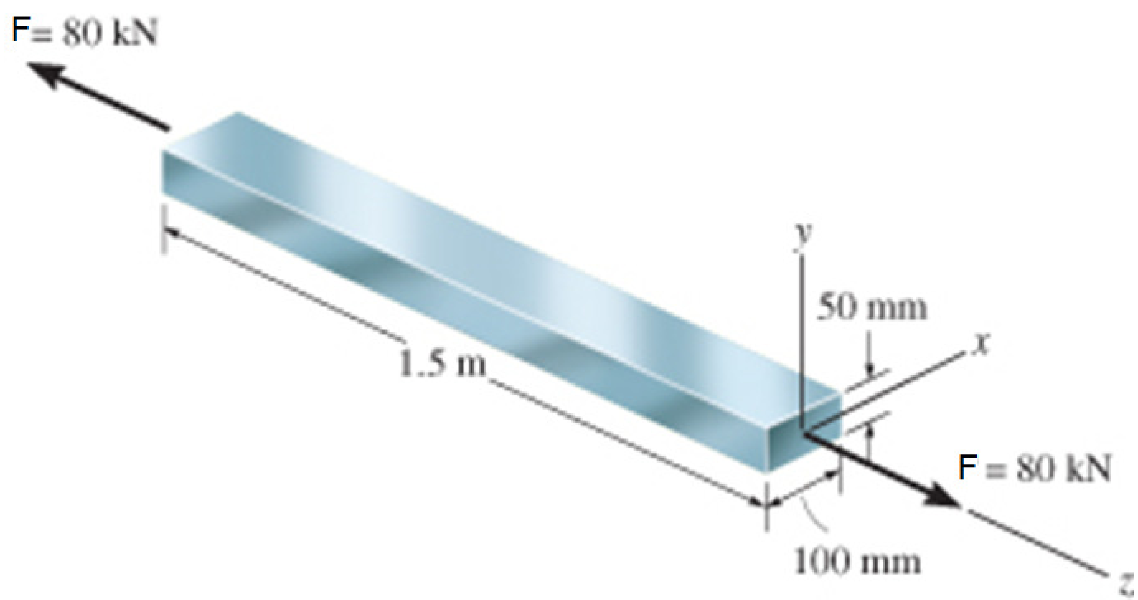
$$\sigma = E\varepsilon$$

- *E* represents the constant of proportionality, also called the *modulus of elasticity* or *Young's modulus*
- *E* has units of stress, i.e., pascals, MPa or GPa.

# EXAMPLE

Bar is made of A-36 steel and behaves elastically.

Determine change in its length



Normal stress in the bar is

$$\sigma_z = \frac{F}{A} = 16.0(10^6)\ \text{Pa}$$

From tables, $E_{st}$ = 200 GPa, strain in $z$-direction is

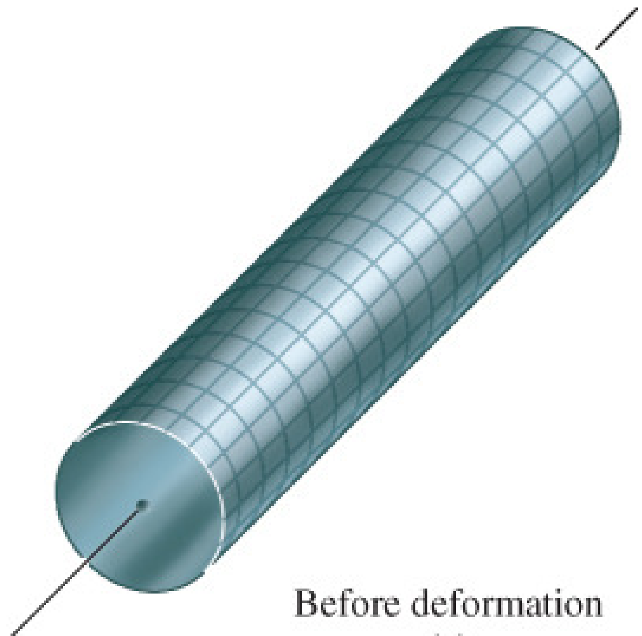
$$\varepsilon_z = \frac{\sigma_z}{E_{st}} = 80(10^{-6})\ \text{mm/mm}$$

Axial elongation of the bar is,

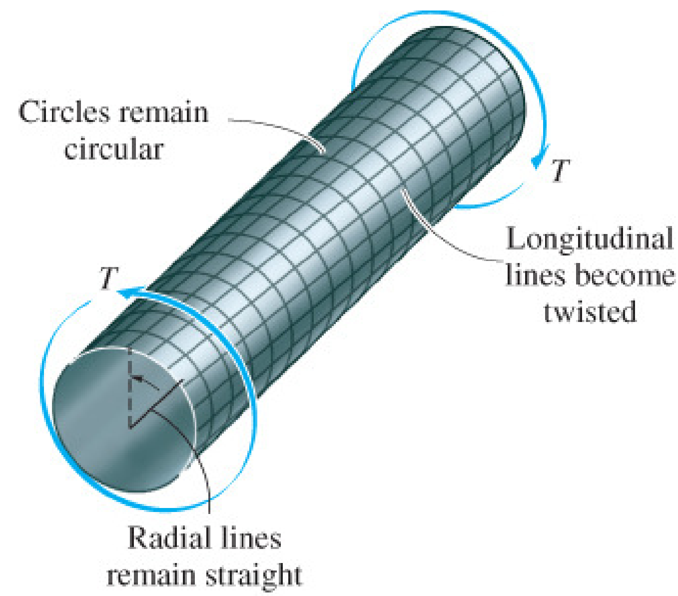
$$\delta_z = \varepsilon_z L_z = [80(10^{-6})](1.5\ \text{m}) = -25.6\ \mu\text{m/m}$$

# TORSION

- Torsion is a moment that twists/deforms a member about its longitudinal axis
- By observation, if angle of rotation is *small*, *length of shaft* and its *radius remain unchanged*



Before deformation
(a)




After deformation
(b)

$$\tau_{max} = \frac{Tc}{J}$$

Torque on shaft determined from $P = T\omega$,

$$\omega = \frac{2\pi n}{60}$$

$\tau_{max}$ = max. shear stress in shaft, at the outer surface

$T$ = resultant internal torque acting at x-section, from method of sections & equation of moment equilibrium applied about longitudinal axis

$J$ = polar moment of inertia at x-sectional area

$c$ = outer radius pf the shaft