Accuracy of Solution

The accuracy of the solution of a problem depends upon two items:

1) The accuracy of the given data,
2) The accuracy of the computations performed (this item is out of question when a computer or calculator is used suitably)

The solution can not be more accurate than the less accurate of these two items.

Conventions for the presentation of numbers in engineering work:

1) For numbers less than one, a zero is written in front of the decimal point to omit any possible errors due to copy processes or careless reading. e.g. Write 0.345 not .345
2) Use a space, NOT a comma, to divide numbers of three orders of magnitude.
   e.g. Write 4 567.8 instead of 4,567.8 write 0.675 91 instead of 0.675,91

3) For very large or very small numbers use scientific notation, i.e. Write such numbers with powers of ten. By means of scientific notation we can write such numbers compactly and also easly indicate the precision of the numbers. In writing a number in scientific notation we consider one digit and enough number of decimal. The number of decimals depends on the reliability in the measurement and calculation

Scientific notation, examples

- 160→ $1.6 * 10^2$ 14 000→ $1.4 * 10^4$ $0.000567 \rightarrow 5.67 * 10^{-4}$

- 1500$* 250\,000\,000 * 0.0000087 = ? \rightarrow 1.5 * 10^3 * 2.5 * 10^8 * 8.7 * 10^{-6}$
  $=(1.5 * 2.5 * 8.7) * (10^{(3+8-6)}) = 32.625 * 10^5 = 3.3 * 10^6$

* $\frac{150\,000*0.000423}{205\,000*0.000065} = \frac{1.5*10^5 *4.23*10^{-4}}{2.05*10^5 *6.5*10^{-5}} = \frac{(1.5*4.23)*(10^{5-4})}{(2.05*6.5)*(10^{5-5})} = \frac{6.345*10^1}{13.325}$=4.76→4.8

Order of arithmetical operations

- A mathematical calculation that involves more than one operation is performed considering the order of operations:

1. Parentheses
2. Exponents
3. Multiplication, division (from left to right)
4. Addition, substraction (from left to right)

Example 1	Example 2
$5\times(7-3)=5\times4=20$	$24\div(8-5)=24\div3=8$
$7+2^2=7+4=11$	$10-3^2=10-9=1$
$9+12\div3=9+4=13$	$10-6\div2=10-3=7$
$10+2\times3=10+6=16$	$9-4\times2=9-8=1$
$4\times3+5=12+5=17$	$2\times7+8=14+8=22$

Measurements: Accuracy and Precision, Resolution

- Accuracy: is the closeness to the true value of the variable being measured
- Precision: is a measure of reproducibility of measurement. It refers to the repeatability of a measurement, that is, how close successive measurements are to each other

If a measuring device has a high precision it can measure and display the same number with the same digits when the measurement is repeated. But the measurement may not be accurate

If a measuring device has a high resolution it can show more figures.

Resolution gives us the smallest change in measured value to which the instrument respond. The place of last decimal gives us an idea about the resolution. Resolution depends on the sensitivity of the instrument.

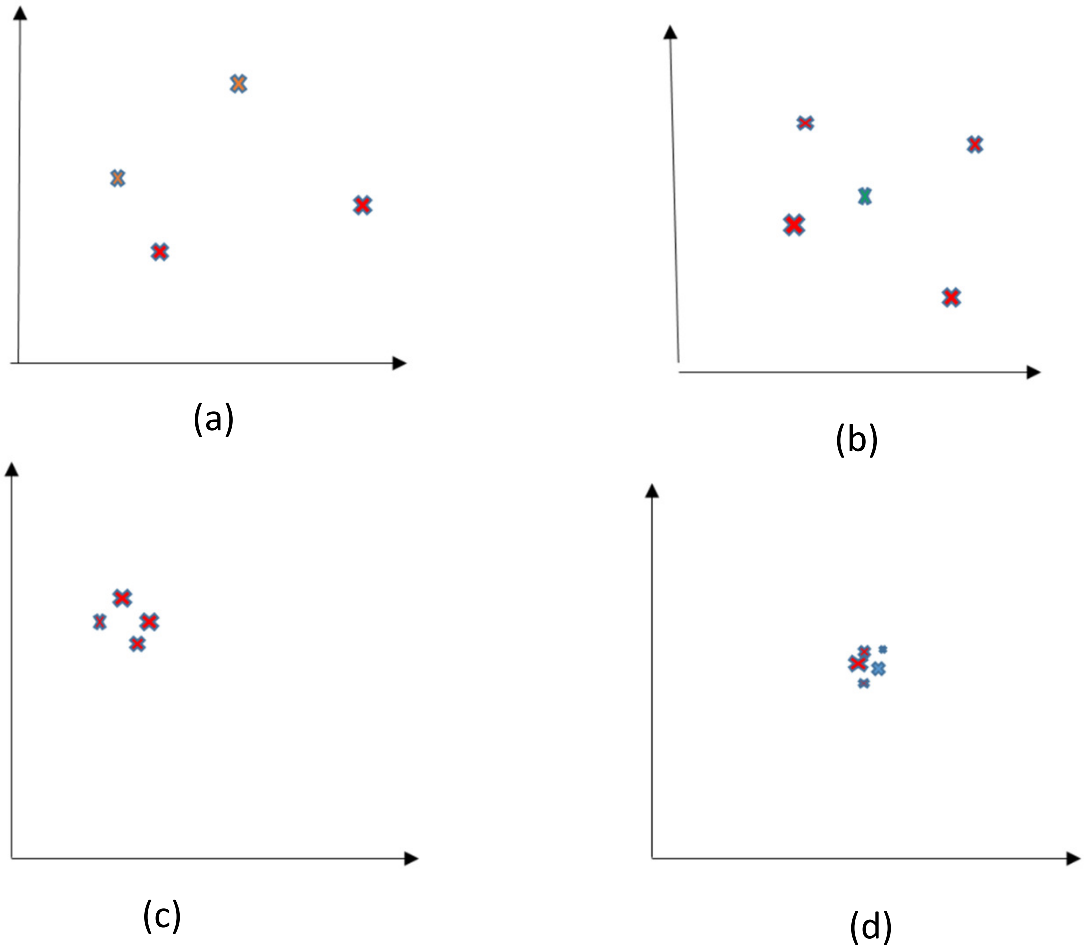
The deviation from the true value of the measured variable is called error.

Errors may come from different sources:

<u>Gross Errors:</u> Come from human errors; misreading of instrumens, incorrect adjustment, computational mistakes

<u>Systematic Errors:</u> Come from shortcomings of the instruments, such as defective or worn out parts, and effect of the environment

<u>Random Errors:</u> Those due to causes which can not be directly established



(a) İnaccurate & imprecise, (b) accurate but imprecise, (c) precise but inaccurate, (d) accurate & precise

Significant Digits (Figures)

- Significant digit, or significant figure, is defined as any digit used in writing a number, except those zeros that are used only for location of the decimal point or those zeros that do not have any nonzero digit on their left
- Guideline to determine the significant figures:
- 1. Non zero digits should always be regarded as being significant:

e.g. 6718→ 4 significant figures

2. Zeros between non-zero digits are significant

e.g. 2.304→ 4 significant figures

3. Zeros before non zero digits serve as place holders and are not significant

e.g. 0.00523→ 3 significant figures

4. Zeros at the end of a whole number are usually not significant

e.g.. 7 600→ 2 significant figures

5. Zeros at the end of a decimal fraction are intended to be significant

e.g. 2.5300→ 5 significant figures

- 6. If numbers are to be multiplied or divided the result has the same number of significant figures as the least precise number has

e.g. $2.50 * 4.621 = 11.5525 \cong 11.6$

3 sig. Fig. 4 sig. Fig. 3 sig. Fig.

7. When numbers in scientific notation are added, substracted or divided the final result can not be more accurate than the original numbers on which it is based.

In multiplication or division the final result should retain as many significant figures as found in the original number with the least significant figures.

e.g. $7.63 * 10^3 * 7.6 * 10^3 = 5.7988 * 10^7 \cong 5.8 * 10^7$

(2.479 h)(60 min/h) = 148.74 min

In this case, the conversion factor is exact (a definition) and could be thought of as having an infinite number of significant figures (60.00000. . ..). Thus, 2.479, which has four significant figures, controls the precision, and the answer is 148.7 min, or 1.487×10^2 min.

8. When numbers are added or substracted, the result should be rounded off such that the place of the last figure should be in the same place relative to the decimal point as the last significant figure of the least accurate number in the sum or difference

e.g. 215.2+1.23=216.43≅ 216.4

9. In addition or substraction of of numbers in scientific notation first write the numbers with the same powers of ten then see the number which has the least number of figures at its decimal part and round the result of the calculation to have the same number of figures at the decimal part.

e.g. $7.63 * 10^4 + 7.62 * 10^3 = 76.3 * 10^3 + 7.62 * 10^3 = (76.3 + 7.62) * 10^3 = 83.92 * 10^3 = 83.9 * 10^3 = 8.39 * 10^4$

10. When rounding off a number ending in 5 or grater than 5 always round upward.

11. For powers and roots, round the answer so that it has the same number of significant figures as found in the number whose power or root is to be taken

e.g. $\sqrt{125.3} = 11.193748 = 11.19$

4 significant fig.

4 significant fig.

Quantity	Number of Significant Figures
4784	4
36	2
60	1 or 2
600	1, 2, or 3
6.00×10^2	3
31.72	4
30.02	4
46.0	3
0.02	1
0.020	2
600.00	5

Numbers 10 or larger that are not written in scientific notation and that are not counts (exact values) can cause difficulties in interpretation when zeros are present. For example, 2000 could contain one, two, three, or four significant digits; it is not clear which. If you write the number in scientific notation as 2.000×10^3, then clearly four significant digits are intended. If you want to show only two significant digits, you would write 2.0×10^3. It is our recommendation that if uncertainty results from using standard decimal notation, you switch to scientific notation so your reader can clearly understand your intent.

Rounding. General Rule: Round a value to the proper number of significant figures, *increase the last digit retained by 1 if the first figure dropped is 5 or greater.* This is the rule normally built into a calculator display control or a control language.

Examples

a. 23.650 rounds to 23.7 for three significant figures.
b. 0.014 3 rounds to 0.014 for two significant figures.
c. 827.48 rounds to 827.5 or 827 for four and three significant digits, respectively.

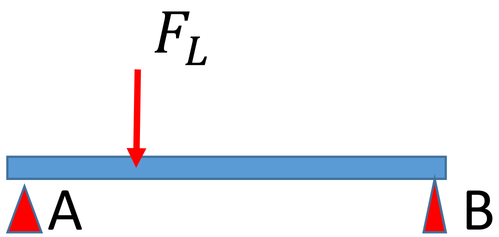
(*Note:* You must decide the number of significant figures before you round. For example, rounding 827.48 to three significant figures yields

Combined Operations. General rule: If products or quotients are to be added or subtracted, perform the multiplication or division first, establish the correct number of significant figures in the intermediate answer, then perform the addition or subtraction, and round to proper significant figures. Note, however, that in calculator or computer applications it is not practical to perform intermediate rounding. **It is normal practice to perform the entire calculation and then report a reasonable number of significant figures.**

Uncertainty in computation

- The solution can not be more accurate than the inputs of the problem:

e.g.



If the loading of a beam is given to be 75000[N] with a possible error of $\pm$100[N] the relative error

100/75000 =0.0013→ 0.13%

So in calculating the reaction force applied by the support B would be meaningless written as 14 322 [N]. Because possible error in the answer may be as large as 0.0013$*$14 322$\cong$ 20[N]

The answer should be written as 14 300$\pm$ 20 [N]

- **Experimental uncertainties should be rounded to one significant figure.**

e.g. Wrong: 52.3 cm ± 4.1 cm
Correct: 52 cm ± 4 cm

***Always round the experimental measurement or result to the same decimal place as the uncertainty.**

e.g. Wrong: 1.237 s ± 0.1 s
Correct: 1.2 s ± 0.1 s

- **Adding and Subtracting Uncertainties**

Work out the total uncertainty when you add or subtract two quantities with their own uncertainties by adding the absolute uncertainties. For example:

e.g. (3.4 ± 0.2 cm) + (2.1 ± 0.1 cm) = (3.4 + 2.1) ± (0.2 + 0.1) cm = 5.5 ± 0.3 cm

e.g. (3.4 ± 0.2 cm) − (2.1 ± 0.1 cm) = (3.4 − 2.1) ± (0.2 + 0.1) cm = 1.3 ± 0.3 cm

$$R = X + Y - Z$$

$$\delta R \approx \delta X + \delta Y + \delta Z$$

$$\delta R = \sqrt{(\delta X)^2 + (\delta Y)^2 + (\delta Z)^2}$$

Example:

Suppose we have measured the starting position as $x_1 = 9.3 \pm 0.2$ m and the finishing position as $x_2 = 14.4 \pm 0.3$ m. Then the displacement is:

$$\Delta x = x_2 - x_1 = 14.4 \text{ m} - 9.3 \text{ m} = 5.1 \text{ m}$$

and the error in the displacement is:

$$(0.2^2 + 0.3^2)^{1/2} \text{ m} = 0.36 \text{ m}$$

***Multiplying or Dividing Uncertainties**

When multiplying or dividing quantities with uncertainties, you add the relative uncertainties together. For example:

(3.4 cm ± 5.9%) × (1.5 cm ± 4.1%) = (3.4 × 1.5) cm^2 ± (5.9 + 4.1)% = 5.1 cm^2 ± 10%

(3.4 cm ± 5.9%) ÷ (1.7 cm ± 4.1 %) = (3.4 ÷ 1.7) ± (5.9 + 4.1)% = 2.0 ± 10%

$$R = \frac{X \cdot Y}{Z}$$

$$\frac{\delta R}{|R|} \approx \frac{\delta X}{|X|} + \frac{\delta Y}{|Y|} + \frac{\delta Z}{|Z|}$$

$$\delta R = |R| \cdot \sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2 + \left(\frac{\delta Z}{Z}\right)^2}$$

Example:

We have measured a displacement of x = 5.1+-0.4 m during a time of t = 0.4+-0.1 s. What is the average velocity and the error in the average velocity?

$$v = x / t = 5.1 \text{ m} / 0.4 \text{ s} = 12.75 \text{ m/s}$$

and the uncertainty in the velocity is:

$$\delta v = |v| \cdot [\,(\delta x/x)^2 + (\delta t/t)^2\,]^{1/2} = 12.75 \text{ m/s} \cdot [(0.4/5.1)^2 + (0.1/0.4)^2]^{1/2} = 3.34 \text{ m/s}$$

- **Multiplying by a Constant**

*If you're multiplying a number with an uncertainty by a constant factor, the rule varies depending on the type of uncertainty. If you're using a relative uncertainty, this stays the same:

e.g. (3.4 cm ± 5.9%) × 2 = 6.8 cm ± 5.9%

*If you're using absolute uncertainties, you multiply the uncertainty by the same factor:

e.g. (3.4 ± 0.2 cm) × 2 = (3.4 × 2) ± (0.2 × 2) cm = 6.8 ± 0.4 cm

$$R = c \cdot X$$

$$\delta R = |c| \cdot \delta X$$

Example:

If an object is realeased from rest and is in free fall, and if you measure the velocity of this object at some point to be v = - 3.8+-0.3 m/s, how long has it been in free fall?

Answer: we can calculate the time as (g = 9.81 m/s^2 is assumed to be known exactly)

$$t = -v / g = 3.8 \text{ m/s} / 9.81 \text{ m/s}^2 = 0.387 \text{ s}$$

The uncertainty in the fall time is then:

$$\delta t = |-1/g| \cdot \delta v = 0.102 \text{ s}^2/\text{m} \cdot 0.3 \text{ m/s} = 0.03 \text{ s}$$

*** A Power of an Uncertainty**

If you're taking a power of a value with an uncertainty, you multiply the relative uncertainty by the number in the power. For example:

$(5\ \text{cm} \pm 5\%)^2 = (5^2 \pm [2 \times 5\%])\ \text{cm}^2 = 25\ \text{cm}^2 \pm 10\%$

$(10\ \text{m} \pm 3\%)^3 = 1{,}000\ \text{m}^3 \pm (3 \times 3\%) = 1{,}000\ \text{m}^3 \pm 9\%$

$$R = X^n$$

$$\delta R = |n| \cdot \frac{\delta X}{|X|} \cdot |R|$$

A more correct estimation can be performed :

Q is a quantity to be calculated using parameters P_1, P_2..........................

$$P_1 = \underbrace{\bar{P}_1}_{\text{Nominal Value}} \pm \underbrace{(\delta P_1)}_{\text{Uncertainty}} \qquad P_2 = \bar{P}_2 \pm (\delta P_2) \qquad ....$$

Then, the uncertainty in Q can be determined by using the following expression:

$$(\delta Q) = \left[\left(\frac{\partial Q}{\partial P_1} * \delta P_1\right)^2 + \left(\frac{\partial Q}{\partial P_2} * \delta P_2\right)^2 + \cdots \right]^{1/2} \text{evaluated for } \bar{P}_1, \bar{P}_2, \ldots$$

Q= $\bar{Q} \pm (\delta Q)$

- Example: To measure pressure of fluid a U-Tube manometer has been used. Measured values: $\rho = 950 \pm 1 \left[\frac{kg}{m^3}\right],\ h = 0.2 \pm 0.001$

ΔP=ρgh, Calculate the uncertainty of pressure measurement

$$\delta P = \sqrt{(\frac{\partial P}{\partial \rho}\delta\rho)^2 + (\frac{\partial P}{\partial h}\delta h)^2} = \sqrt{(g * h * \delta\rho)^2 + (\rho * g * \delta h)^2}$$

=$\sqrt{(10 * 0.2 * 1)^2 + (950 * 10 * 0.001)^2}$= 9.7 ≅ 10[Pa]

So ΔP=$(950 * 10 * 0.2) \pm 10 = 1900 \pm 10[Pa]$