The theory of value and earnings, and an introduction to the Ball-Brown analysis*

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Abstract. The paper develops a simple and parsimonious model that relates earnings and unexpected earnings to market returns. The analysis emphasizes that any model under uncertainty must be consistent with the theory of value, earnings, and dividends under certainty (i.e., Hicksian income theory). An extension of this theory exists such that the model subsumes uncertainty. The Hicksian approach is useful because it embeds key dividend irrelevancy concepts due to Modigliani and Miller (1961), and these can be retained under uncertainty. An interesting empirical proposition can be inferred from the model: earnings, rather than the change in earnings, ought to serve as a premier exploratory variable of returns. This contention is consistent with some recent empirical findings due to Easton and Harris (1991).

Résumé. L'auteur élabore un modèle simple et parcimonieux qui relie les bénéfices, et les bénéfices imprévus, aux rendements du marché. L'analyse met en relief le fait que tout modèle en situation d'incertitude doit être conforme à la théorie de la valeur, des bénéfices et des dividendes en situation de certitude (c'est-à-dire la théorie hicksienne des bénéfices). Cette théorie peut être élargie de telle sorte que le modèle tienne compte de l'incertitude. L'utilité de l'approche hicksienne tient au fait qu'elle englobe les concepts clés de non-pertinence relatifs au dividende que l'on attribue à Modigliani et Miller, et que ces concepts peuvent être appliqués en situation d'incertitude. Ce modèle permet de formuler une proposition empirique intéressante : les bénéfices, plutôt que l'évolution des bénéfices, doivent servir de première variable exploratoire des rendements. Cette affirmation est conforme aux résultats empiriques récemment obtenus pas Easton et Harris.

Introduction

Without exaggeration, it can be said that the Ball-Brown (1968) paper has had an enormous influence on modern empirical accounting research. Their analysis has led to an informational perspective on accounting data. This research paradigm considers earnings and other accounting descriptors to explain market returns (or unexpected returns), and a key concept focuses on unexpected earnings, normalized by the beginning of period stock price, as the primary independent

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variable.[1] As a practical matter, the independent variable unexpected earnings is generally measured by the change in earnings, a procedure that can be traced to the Ball-Brown study.[2]

In spite of the importance of this paradigm, existing theory to a large extent fails to articulate why and how earnings relate to returns. Any useful theory of market value and earnings should provide a role for earnings and contrast this information variable to other kinds of information. The modeling of information and value therefore introduces the problem as to why an information variable ought to be thought of as earnings. In this regard, it becomes particularly important that the model at least distinguishes nontrivially between earnings and dividends. Another theoretical problem concerns the relevance of unexpected earnings as a variable explaining returns. This construct appears to have the status of a "folklore concept" with limited economic content. A more substantive analysis of unexpected earnings requires an *equilibrium model linking value* and earnings through the expectation (or information) dynamics. At the very least, such model must also permit the derivation of the response coefficient for unexpected earnings.

This paper analyzes earnings and value under uncertainty by exploiting a necessary criterion for a valid model: We know how to link earnings, dividends, and value in the special case of certainty. The classical (Hicksian) theory of earnings implies that the value times the risk-free rate equals next-period earnings, regardless of the policy determining the payment of dividends. The theory achieves richness because value also equals the present value of dividends. Earnings can be distinguished from dividends, and there is no need for notions such as "payout ratio" or "dividends are paid out of earnings."

One can examine the theoretical literature on value, earnings, and dividends under uncertainty and check how these models work without uncertainty. The certainty structure criterion is generally not meaningfully satisfied. For example, the valuation model of Beaver, Lambert, and Morse (1980) does not distinguish the primitive earnings variable "x_t" from dividends. Under certainty, x_t equals dividends, and earnings remain constant over time (See Ohlson, 1989a). Similarly, Ohlson's (1983) analysis makes no sense unless one views his primitive variable (also x_t) as proportional to dividends. Neither model captures the substance of earnings measurement adequately, and elementary concepts of *valuation under certainty are either absent or violated.*

The theory development in this paper emphasizes that a thorough analysis of the classical certainty theory of value and earnings (and dividends) logically precedes the more complex uncertainty case (the second and third sections). Provided that one applies the right "twist" in the analysis of the certainty case, a simple and parsimonious uncertainty extension follows without difficulty (fourth section). This "twist" deals with how one derives value from earnings as

1 Ball-Brown (1968) did not normalize their measures of unexpected earnings with initial security price. This normalization procedure has become common (if not standard) during recent years. Christie (1987), in particular, has argued in support of this procedure.

2 Lev (1989) reviews the empirical research dealing with returns/earnings correlations.

opposed to the other way around. (The literature recognizes this problem; see, e.g., Hendriksen, 1977, page 152). Thus, the model is consistent with the results under certainty, yet the introduction of uncertainty permits an informational perspective on earnings as it relates to value. In equilibrium, one obtains the market value as a function of current earnings and dividends, and the realization of these (random) variables explains returns.[3]

Analysis of this simple model yields a *striking result with empirical content.* Returns and unexpected returns are explained by current earnings divided by beginning period price (x_{t+1}/P_t). This result follows because *expected earnings divided by price equals a constant* (which equals the risk-free rate under risk neutrality). That is, the effect of adjusting x_{t+1}/P_t for its expected value—the constant—is irrelevant in a regression context because the regression intercept picks up the constant. The analysis further shows that $(x_{t+1} - x_t)/P_t$ explains returns "almost" as well as x_{t+1}/P_t. The quality of the approximation depends on the degree to which x_t/P_t approximates a constant, which in turn depends on dividends. If dividends equal earnings, then x_t/P_t equals a constant; in this case $(x_{t+1} - x_t)/P_t$ obviously works as well as x_{t+1}/P_t. This analysis provides some justification for the Ball-Brown concepts. Nevertheless, the simple extension of the certainty model results in a x_{t+1}/P_t as the explanatory variable of returns, not $(x_{t+1} - x_t)/P_t$.

The paper expands on the simple model with earnings and dividends as information to show how one can incorporate "other" information into the valuation analysis (section six and Appendix B). Such other information is characterized generically by a scalar variable that influences the prediction of future earnings. This extended model can be used to demonstrate that under certain narrow circumstances, the earnings change variable $(x_{t+1} - x_t)/P_t$ correlates strictly more with returns compared to x_{t+1}/P_t. The (published) literature has not provided such a case to date. This setting underscores that the superiority of $(x_{t+1} - x_t)/P_t$ compared to x_{t+1}/P_t in explaining returns requires earnings and dividends to be insufficient (or imperfect) determinants of value. But there are no general reasons why $(x_{t+1} - x_t)/P_t$ should work better than x_{t+1}/P_t given imperfections in earnings. Thus, the x_{t+1}/P_t variable takes on a premier role as an explanatory earnings variable of returns because it is the correct variable under idealized conditions and the only one consistent with the Hicksian certainty case.

The paper finally observes (the seventh section) that even if one focuses on book value rather than earnings as a determinant of value, x_{t+1}/P_t again obtains

3 This paper draws on the concepts developed in Ohlson (1989b). While the analytical results in that paper bear on those in the current paper, the issues raised differ. Ohlson (1989b) deals with the integration of earnings, book values, dividends, and other information into a general theory of valuation. In contrast, the current paper emphasizes the importance of the certainty case, how one generalizes to uncertainty, and the empirical implications one can extract from the relatively simple models developed. It should further be noted that for some (uninteresting) reasons, the Ohlson (1989b) paper will never be published. The current paper is basically self-contained, and I have made no attempt to cross-reference specific concepts and results.

as an explanatory variable of returns. The variable $(x_{t+1}-x_t)/P_t$ is of no interest in this context. Thus, one concludes that x_{t+1}/P_t explains returns in simple models. The contention has testable implications. Indeed, Easton and Harris (1991) find that x_{t+1}/P_t dominates $(x_{t+1}-x_t)/P_t$ as an explanatory variable of returns for annual return windows.

The present value of dividends and value under certainty

The initial case dealing with earnings and value relies on a dynamic certainty setting. This model will be shown to have powerful implications, which can be exploited to derive a relation between earnings and security values under uncertainty. Such an uncertainty extension is, of course, necessary to identify theoretical underpinnings for the Ball-Brown empirical analysis.

Many of the certainty results that follow are known and discussed in the literature. However, we structure the logic of the certainty case such that the introduction of uncertainty creates minimal complications. The logic used also guarantees that basic equilibrium requirements are satisfied.

We commence the analysis by relating the value of the security, P_t, to a sequence of certain future dividends, $d_{t+1}, d_{t+2}, \ldots$ Negative d_t are capital contributions, but to keep the language simple, these are also referred to as dividends. Let R_F denote 1 plus the rate of return on a risk-free asset. Given a condition of no-arbitrage (NA), it follows immediately that at all dates t

$$P_t = (P_{t+1} + d_{t+1})/R_F \qquad \text{[NA]} \tag{1}$$

Substituting recursively,

$$P_t = R_F^{-1} d_{t+1} + R_F^{-2}(P_{t+2} + d_{t+2}), \tag{2}$$

$$P_t = R_F^{-1} d_{t+1} + R_F^{-2} d_{t+2} + R_F^{-3}(d_{t+3} + P_{t+3}) \tag{3}$$

In the limit, one obtains (assuming convergence) the equilibrium value as a function of the present value of future dividends—*PVD*:

$$P_t = \sum_{\tau=1}^{\infty} R_F^{-\tau} d_{t+\tau} \qquad \text{[PVD]} \tag{4}$$

Hence, NA implies PVD. Without difficulty, the converse can also be shown,[4] that is, the PVD formula (4) implies the NA condition (1).

Proposition I: NA *and* PVD *are equivalent conditions.*

The equivalence is of importance because it shows the PVD formula as an uncontroversial valuation scheme: NA serves as the weakest possible equilibrium

4 PVD implies NA since $R_F P_t = R_F \sum_{\tau=1} R_F^{-\tau} d_{t+\tau} = \sum_{\tau=1} R_F^{-\tau+1} d_{t+\tau} = d_{t+1} + \sum_{\tau=2} R_F^{\tau+1} d_{t+\tau} = d_{t+1} + P_{t+1}$.

This result, and the converse, can be found in most finance texts. We state proposition I (below) only to highlight a key building block in the logic leading up to the major propositions (primarily III and IV).

requirement. Nevertheless, it may be tempting to argue that the PVD valuation approach is unsatisfactory because the dividend pattern can be viewed as "arbitrary." Dividends planned to be paid at data $t+1$ (say) can be withheld and invested to earn a return of R_F, and this leads to greater dividends at the subsequent date. But this change in dividend policy has no effect on the date t price. More precisely, consider the following "primed" dividend pattern: $d'_{t+1} = 0, d'_{t+2} = d_{t+2} + R_F d_{t+1}, d'_{t+3} = d_{t+3}, d'_{t+4} = d_{t+4}, \ldots$ It follows immediately that $P_t = P'_t$. This well-known concept of *dividend policy irrelevance* is generally attributed to Modigliani and Miller (1961). Their analysis implies that there exist an infinite number of dividend policies leading to the same value, given the availability of investment opportunities that earn the risk-free rate. As will be seen, dividend policy irrelevance relates closely to how one conceptualizes earnings under both certainty and uncertainty.

The dividend policy irrelevance concept perhaps suggests that we should be able to calculate the PV of some other, more "fundamental," variable (e.g., earnings) to derive the equilibrium value, P_t. Such an approach would eliminate the problem associated with the arbitrariness of the dividend stream. The issue will be considered following an analysis of earnings under certainty.

Earnings and values under certainty

The concept of earnings poses no ambiguities given a certain sequence of dividends. One can simply think of the accounting for a savings account as a prototype model. The value of the savings account equals P_t, and interest revenue (i.e., earnings) realized for the period $(t, t+1)$ is determined by $(R_F - 1)P_t$. Hence, if x_{t+1} denotes earnings for $(t, t+1)$, one obtains the Hicksian definition

$$x_{t+1} = (R_F - 1)P_t. \tag{5}$$

From equation (5) and the (NA) condition (1), one infers that

$$x_{t+1} = P_{t+1} + d_{t+1} - P_t. \tag{6}$$

Conversely, (6) and (1) imply (5), so that (5) and (6) are equivalent given the NA condition. This equivalence is, of course, well known.[5]

A couple of subtle and important interrelated methodological issues can be raised concerning expressions (5) and (6).

First, the concept of earnings as defined by expression (5) (or (6)) suffers from an apparent drawback since the present value of future dividends (or P_t) determines next-period earnings, rather than the other way around. As Hendriksen (1977, page 152) observes, to define earnings in terms of current value (or change in value adjusted for dividends) puts "the cart in front of the horse." A more meaningful approach to earnings and value *derives* values *from* earnings, in which case one needs some definition of earnings other than (5) (or (6)).

5 See, for example, Beaver [1989, Ch. 3]. Beaver refers to (5) as "permanent earnings" and to (6) as "economic earnings." The discussion illustrates the influence of Hicks' concept of earnings on accounting thought.

Second, in any event, it is unclear how (5) and (6) can be generalized to allow for uncertainty. Definition (5) introduces an obvious problem since it requires x_{t+1} to be observable at date t. As an extension of (5), one may consider replacing x_{t+1} with its expected value:

$$P_t(R_F - 1) = E[\tilde{x}_{t+1} | \text{information available at date } t] \tag{7}$$

However, expression (7) is ad hoc and it ought not to serve as a starting point explaining how earnings relate to value and value changes. At worst, expression (7) could violate basic equilibrium conditions since the mapping from available information at date t to P_t must be endogenous. It is unlikely that (7) holds except under narrow assumptions, a point which will be illustrated later. Similarly, to presume that earnings under uncertainty satisfies (6) would seem too arbitrary in an equilibrium framework that derives the earnings-to-value (change) relation. We further note that although (6) and (7) are equivalent under certainty, this conclusion may be false under uncertainty. In sum, the use of (7) (or (6)) as an assumption introduces problems because one may concoct reasonable models of earnings and value violating the condition(s) under uncertainty but not so in the special cases of certainty.

The problem of viewing value as determining earnings rather than deriving from earnings can be circumvented. To develop a sufficiently rich theory of earnings and value, we exploit the conceptually straightforward strategy of deducing useful properties of earnings from its definition (5) and the PVD (or NA) condition, and, after this characterization of earnings, cast the analysis in the reverse. That is, the Hicksian relation (5) is derived rather than assumed. The idea is to come up with some definition of earnings that derives from (5) but does not directly refer to P_t and such that (5) can be derived from this alternative definition.

The proposition below uses PVD and the Hicksian definition (5) of earnings to derive the characterization of earnings that subsequently (proposition III) is assumed. In addition, the proposition expresses the equilibrium value P_t as it relates to *current* earnings and dividends.

Proposition II

Given PVD (or NA) and the definition (5) of earnings, one obtains the earnings dynamics ED

$$x_{t+1} = R_F x_t - (R_F - 1)d_t \tag{ED}$$

and the value function VF

$$P_t = P(x_t, d_t) = \phi x_t - d_t \tag{VF}$$

where

$\phi \equiv R_F/(R_F - 1)$.

Proof: $x_t/(R_F - 1) = P_{t-1} = R_F^{-1}(P_t + d_t) = R_F^{-1}(x_{t+1}/(R_F - 1) + d_t)$, which simplifies to ED. VF follows immediately from ED and (5).

The earnings dynamics ED can be viewed as a characterization of earnings that does not *by itself* refer to value. Given the initialization condition $x_0 = 0$ and any sequence of dividends, $d_0, d_1, d_2, \ldots$, one obtains a sequence of earnings, $x_1, x_2, \ldots$ The ED definition of earnings differs conceptually from the definition (5) because ED defines x_{t+1} in terms of *past* dividends (i.e., $d_0, d_1, \ldots, d_t$), whereas (5) defines x_{t+1} in terms of *future* dividends (i.e., $d_{t+1}, d_{t+2}, \ldots$). Thus, ED is a potentially useful alternative to (5) (and (6)) when one characterizes earnings.

We note a critical difference between (5) and VF. Expression (5) makes no sense under uncertainty since in that case x_{t+1} is unknown at date t. In contrast, VF could, at least in principle, be consistent with an equilibrium even under *un*certainty since x_t is observable at date t. The next section exploits this simple observation.

The relations VF and ED embed the MM dividend payment irrelevancy concept. Note that $P_t + d_t = \phi x_t$ so that current earnings determine the payoff, $P_t + d_t$, and $\partial P_t / \partial d_t = -1$. A dollar of incremental dividends is thus exactly offset by a dollar of reduction in market value. The latter result makes use of $\partial x_t / \partial d_t = 0$ i.e., current dividends do not affect current earnings. Also, $\partial x_{t+1} / \partial d_t = -(R_F - 1)$ i.e., the payment of dividends reduces future earnings.

Although ED derives from (5) (and NA), we emphasize that in a theoretical analysis, one can reverse the process by focusing on the implications of ED as an assumption (in conjunction with other assumptions). At first glance, such an exercise may seem pedantic or even redundant since the certainty model of earnings, dividends, and value is not particularly complex. Why reinvent the wheel using a different starting point? The answer to the question was alluded to previously. We want to identify assumptions for the certainty case such that these can be extended to uncertainty without risking a violation of basic equilibrium conditions. As noted, expression (5) suffers from the drawback that it cannot be applied under uncertainty, and the ad hoc extension (7) of expected earnings and value could be invalid in some settings. The analytical usefulness of ED will not become fully apparent until proposition IV. This proposition extends proposition III by allowing for uncertainty.

The next proposition, III, uses the PVD (or NA) condition and the definition ED of earnings to derive (5) and VF. In contrast to proposition II, (5) is thus derived as opposed to assumed. The conclusion requires only an additional regularity condition linking dividends to earnings. This relation reflects the *dividend policy*. To keep matters simple but without substantive loss of generality, we consider only a class of linear dividends dynamics (DD) models:

$$d_{t+1} = \theta_1 x_t + \theta_2 d_t \qquad \text{[DD]}$$

where $\theta_1 \neq 0$. The last condition reflects that dividends cannot be paid without reference to earnings. Thus, θ_1 and θ_2 are *dividend policy parameters*. The model distinguishes between earnings and dividends: to guarantee that current dividends are not proportional to lagged or current earnings, one puts $\theta_2 \neq 0$.

Most important, given $\theta_1 \neq 0$, the values of the policy parameters do not affect the equilibrium valuation function.[6]

Proposition III

Given (1) PVD (or NA), (2) the earnings dynamics ED, and (3) the dividends dynamics DD, it follows that

$$P_t = \phi x_t - d_t,$$

and

$$P_t(R_F - 1) = x_{t+1}.$$

The relations hold regardless of the dividend policy parameters θ_1 and θ_2.

Proof: See Appendix A.

The irrelevance of the DD parameters θ_1 and θ_2 in $P(x_t, d_t)$ is far from obvious on analytical grounds since θ_1 and θ_2 affect the sequence $d_{t+1}, d_{t+2}, \ldots$ given some (x_t, d_t). Nevertheless, despite the dependence of the sequence $d_{t+1}, d_{t+2}, \ldots$ on θ_1 and θ_2, the *present value* of this sequence does *not* depend on θ_1 and θ_2. The irrelevance of θ_1 and θ_2 follows because of the structure of the dynamics that defines earnings. (To be precise, one can show that the function that maps (x_t, d_t) to P_t is independent of θ_1 and θ_2 if, and only if, the linear earnings dynamics satisfies $\partial x_{t+1}/\partial x_t = R_F$.)

Relying on economic intuition, the irrelevance of θ_1 and θ_2 seems less surprising given that $\partial P_t/\partial d_t = -1$. The latter relation entails dividend *payment* irrelevance, and one should therefore also expect dividend *policy* (θ_1 and θ_2) irrelevance. Although these two conditions are technically different, one can indeed demonstrate that any one of the two conditions implies the other: $\partial P_t/\partial d_t = -1$ if, and only if, $P(x_t, d_t)$ does not depend on θ_1 and θ_2.

The economic content implied by assumption ED is thus conveyed by proposition III. One identifies three basic MM precepts by combining the ED definition of earnings with PVD and mild regularity conditions on the dividend policy: (1) The dividend policy parameters (θ_1 and θ_2) do not affect the equilibrium valuation function $P(x_t, d_t)$; (2) $\partial P_t/\partial d_t = -1$, and (3) $\partial x_{t+1}/\partial d_t = -(R_F - 1)$. The three precepts highlight how the model forges tight economic links between current dividends and values—(1) and (2)—and current dividends and future

6 More generally, proposition III does not require a linear dividend policy. For example, a policy $d_{t+1} = \theta_1 x_t + \theta_2 d_t + \theta_3 (\sin(x_t))^2 x_t$ does not change the conclusions if $\theta_1 \geq 0$ or $\theta_3 \geq 0$ and $\theta_1 + \theta_3 \neq 0$. (This can be "verified" through a simple computer program.)

A dividend policy can express d_{t+1} as a function of (x_{t+1}, d_t) rather than (x_t, d_t). For example, the dividend policy $d_{t+1} = \theta'_1 x_{t+1} + \theta'_2 d_t$ equals DD if $\theta_1 = R_F \theta'_1$ and $\theta_2 = -\theta'_1(R_F - 1) + \theta'_2$.

To ensure convergence in the present value calculation, we also need a mild regularity condition on the dividend policy: the maximum characteristic root of the matrix $R_F^{-1}\begin{bmatrix} R_F & -(R_F - 1) \\ \theta_1 & \theta_2 \end{bmatrix}$ *is assumed less than one. (Curiously,* θ_1 *need not be positive.)*

earnings (3). The compelling nature of these ideas suggests that they should be retained even in cases of uncertainty.

The dynamic certainty model of earnings allows for comments concerning *in*valid concepts of valuation.

First, it makes no economic sense to try to equate price with the present value of future earnings. It is not generally true that $P_t = \sum_{\tau=1}^{\infty} R_F^{-\tau} x_{t+\tau}$, unless, of course, $d_t = x_t$. But the latter condition is of little interest since it sharply restricts the dividend policy. Proposition III brings out that the only future earnings of relevance to determine P_t are those that will be earned during the next period. Under certainty, x_{t+1} is known at date t, and one can conclude that $P_t = x_{t+1}/(R_F - 1)$. The irrelevance of $x_{t+2}, x_{t+3}, \ldots$ should be apparent since this sequence depends on the specifics of the sequence $d_{t+1}, d_{t+2}, \ldots$ yet we know that the specifics are of no consequence because of dividend policy irrelevance. Basic MM precepts imply that the present value of the future earnings sequence is unrelated to value. The theoretical reasons for discounting future (expected) earnings, which pervades so much of the accounting/finance literature, are difficult to justify if one grants that dividends should not relate to earnings in a narrow (and empirically implausible) fashion.

Second, it should be apparent that P/E ratios (i.e., P_t/x_t) are of no theoretical interest either. Again, such thinking violates elementary MM precepts since d_t affects P_t but not x_t. To restore some economic meaning to the P/E ratio, one may instead consider the predividend P/E-ratio $(P_t + d_t)/x_t$ or, alternatively, restrict d_t to satisfy $d_t = x_t$, in which case $P_t/x_t = (R_F - 1)^{-1}$.

In summary, this section has shown that one can characterize earnings without referring to value, yet in equilibrium one obtains the usual relation between current value and next-period earnings. Additionally, the current value plus dividends derives as a function of current earnings, i.e., period $(t-1, t)$ earnings, x_t, relate directly to the date t payoff, $P_t + d_t$. The economic intuition that motivates these results follows from basic MM precepts concerning the irrelevance of dividend payment and dividend policy, combined with a weak no arbitrage equilibrium condition. These MM concepts are essential to understand how Hicksian earnings relate to value in a neoclassical framework.

Uncertainty and the Ball-Brown analysis

Proposition III flows from the two central assumptions, (1) the PVD valuation formula (or no arbitrage) and (2) the earnings dynamics ED. Combined with mild regularity conditions on the dividend policy, these two assumptions implied the valuation function $P_t = \phi x_t - d_t$, or $P_t + d_t = \phi x_t$. Although the setting is simple, this analysis shows that returns relate to contemporaneous earnings. In the spirit of a linear regression model, one obtains

$$R_t = \phi(x_t/P_{t-1})$$

where $R_t \equiv (P_t + d_t)/P_{t-1}$. The independent variable x_t/P_{t-1} "explains" returns, R_t, and ϕ specifies the earnings response coefficient. The word "explain" is put within quotation marks because certainty implies no variation in the variables. Variation occurs under uncertainty, however; now it makes sense to say that x_{t+1}/P_t (fully) explains R_{t+1}.

The above considerations set the stage for the obvious question. How do we extend the analysis to incorporate uncertainty? The following assumptions generalize PVD and ED in a straightforward fashion:

$$(a) \quad P_t = \sum_{\tau=1}^{\infty} R_F^{-\tau} E[\tilde{d}_{t+\tau} | x_t, d_t] \qquad \text{[PVED]}$$

$$(b) \quad \tilde{x}_{t+1} = R_F x_t - (R_F - 1) d_t + \tilde{\epsilon}_{t+1} \qquad \text{[UED]}$$

Assumption (a), the present value of expected dividends equals value, obtains under uncertainty in a risk-neutral economy. If risk aversion is present, then we interpret R_F as the risk-free rate plus an adjustment for the risk inherent in the stream of future dividends. This latter scheme of valuation is somewhat heuristic (see Ohlson, 1990), but it also conforms with standard "textbook" finance valuation concepts. Further, note that current earnings and dividends suffice as the conditioning information. No other information affects future expected dividends. This confining assumption will be relaxed later; one can introduce other kinds of valuation relevant information without impairing the substantive conclusions.

Assumption (b), the uncertainty earnings dynamics (EUD), extends the certainty dynamics (ED) by adding serially uncorrelated—and unpredictable—disturbance terms, $\epsilon_t, \epsilon_{t+1}, \ldots$. These disturbance terms are, by construction, identical to *unexpected* earnings. As a matter of theory we justify the UED model because it is *the* simplest possible extension of the ED model.

The UED model conforms broadly with the empirical evidence, which suggests that the time-series behavior of (annual) earnings obey a "random walk with drift" or a "submartingale." If the payout is 100 percent, $x_t = d_t$, then the earnings process satisfies the pure martingale $E[\tilde{x}_{t+1}|x_t] = x_t$. For a payout less than 100 percent, the earnings process has a positive drift with an upper bound on the growth rate given by R_F. That is, $x_t < E[\tilde{x}_t|x_t, d_t] \leq R_F x_t$ provided that $0 \leq d_t < x_t$. The drift clearly depends on the current payout. This adjustment for current dividends is essential to retain a fundamental MM precept: Current dividends lower future expected earnings. A simple borrowing/lending argument motivates the effect. Dividends can be increased by Δd_t through incremental borrowings, but this debt incurs an interest expense of $(R_F - 1)\Delta d_t$ during the subsequent period. This line of reasoning is unambiguous in the case of certainty; and it applies no less under uncertainty when the model precludes any signaling effects due to dividends. Hence UED implies that

$$\partial E[\tilde{x}_{t+1}|x_t, d_t]/\partial d_t = -(R_F - 1). \qquad (8)$$

As a practical empirical matter, the effect of current dividends on the prediction of future earnings may be much more complex than this. But this possibility is irrelevant in a neoclassical model. Condition (8) is not only of analytical importance to generalize the certainty case of valuation but also justifies why the information variable 'x_t' is naturally thought of as earnings. No other attribute of a firm would seem to satisfy this condition.[7]

The UED model of earnings behavior places no restrictions on the variance of unexpected earnings, ϵ_t. For example, the ϵ_ts may be heteroscedastic.

To generalize proposition III to include uncertainty also requires some dividend dynamics. Similar to the extension of ED to UED, consider

$$\tilde{d}_{t+1} = \theta_1 x_t + \theta_2 d_t + \tilde{u}_{t+1}, \qquad \text{[DD]}$$

where the u_ts are unpredictable. The ϵ_ts in UED may at any date correlate with the u_ts, and this correlation reflects the dividend policy beyond the policy parameters $\theta_1(\neq 0)$ and θ_2.

The next proposition extends the results from proposition III by incorporating uncertainty.

Proposition IV

Given (1) the present value of expected dividends formula, PVED, (2) the uncertainty earnings dynamics UED, and (3) the uncertainty dividends dynamics UDD, one obtains

$$P_t = \phi x_t - d_t,$$

and

$$P_t(R_F - 1) = E[\tilde{x}_{t+1} | x_t, d_t].$$

The relations hold regardless of the dividend policy parameters.

We emphasize that the valuation function $P_t = \phi x_t - d_t$ (VF) still applies.[8] The proof of this result is straightforward. The disturbance terms $\tilde{\epsilon}_{t+\tau}$ and $\tilde{u}_{t+\tau}$ enter linearly inside the expectation operator in the PVED formula and in applying this operator, one uses the assumptions $E[\tilde{\epsilon}_{t+\tau}|x_t, d_t] = E[\tilde{u}_{t+\tau}|x_t, d_t] = 0$, all $\tau > 0$. Hence, one can "effectively" put $u_{t+\tau} = \epsilon_{t+\tau} = 0$. But this restriction leads us back to the certainty setting, in which case we know how P_t relates to x_t and d_t (i.e., we use proposition III). The valuation function VF follows.

Concerning the second part of the proposition, VF combined with ED implies that the expected earnings can be used in lieu of next-period earnings when uncertainty is present. This result is noteworthy since any model change from certainty to uncertainty is a delicate issue. Appendix B develops a more general model such that expected earnings divided by price may be stochastic

7 Condition (8) rules out a "cash flow" interpretation of x_t. A firm can issue zero coupon bonds to pay for dividends, in which case current dividends have no effect on expected next-period "cash flow."

8 It also follows that VF combined with the PVED equilibrium condition implies that $E[\tilde{x}_{t+1}|x_t, d_t] = R_F x_t - (R_F - 1)d_t$. This is easily shown. The result is basically due to Ryan (1986), although he regards x_t as an unobservable variable — "permanent earnings" — and x_t is defined by $\phi^{-1}[P_t + d_t]$.

and differ from $R_F - 1$. (The model expands the relevant information set to make $E[\tilde{x}_{t+1} |$ info date $t]$ insufficient to determine P_t).

The fact that x_{t+1}/P_t is the logical explanatory variable of contemporaneous returns, R_{t+1}, raises two questions in view of the conventional wisdom. First, what happened to *un*expected earnings? Second, are there any reasons to believe that $(x_{t+1} - x_t)/P_t$ could work almost as well as x_{t+1}/P_t? The first question is relevant since the empirical literature emphasizes the focal role of unexpected earnings. The variable x_{t+1}/P_t would therefore seem a surprising explanatory variable. In dealing with the first question, one can also answer the second question since $x_{t+1} - x_t$ generally surrogates for unexpected earnings as long as d_t is not significantly larger than x_t and $R_F - 1$ is close to zero. (Of course, this conclusion hinges critically on the empirical validity of the UED assumption.)

The answer to the first question is surprisingly direct. Note that

$$R_{t+1} = \phi\{(x_{t+1}/P_t) - E[\tilde{x}_{t+1}|x_t, d_t]/P_t\} + \phi E[\tilde{x}_{t+1}|x_t, d_t]/P_t$$

$$= \phi\epsilon_{t+1}/P_t + \phi E[\tilde{x}_{t+1}|x_t, d_t]/P_t$$

It follows from proposition IV that the second term equals $\phi(R_F - 1)$; since $\phi = R_F/(R_F - 1)$, one obtains

$$R_{t+1} = \phi\epsilon_{t+1}/P_t + R_F$$

The last expression shows that unexpected earnings (normalized by P_t) does the same job as x_{t+1}/P_t in explaining returns. The apparent paradox is easily resolved once one recognizes that the deflation of $P_{t+1} + d_{t+1}$ by P_t introduces an expectations concept of earnings on the equation's RHS. *This subtle and important effect of price deflation on expectations has not generally been appreciated.* For further development see Easton, Harris and Ohlson (1991).

The careful reader will also note that the return, R_{t+1}, does not depend on unexpected dividends, u_{t+1}. The model distinguishes between wealth-*generating* uncertainty (ϵ_{t+1}) and wealth-*transferring* uncertainty (u_{t+1}). Due to the MM precepts inherent in the model, only the former factor influences the uncertainty in market returns. (The point has subtle implications: if $\tilde{\epsilon}_t \equiv 0$ while $\mathrm{Var}(\tilde{u}_t) > 0$, then one can show that the sequence of random variables $\sum_{\tau=1}^{T} R_F \tilde{d}_{t+\tau}$ converges almost surely to $\phi x_t - d_t$ as $T \to \infty$. Hence, under the conditions, one can replace PVED with PVD even though earnings and dividends are uncertain!)

We can now address the second question concerning the relevance of $(x_{t+1} - x_t)/P_t$. This variable introduces an error in unexpected earnings, but, as a practical matter, the error may be sufficiently slight to allow the variable to fully correlate with returns. The special case when the payout equals 100 percent ($d_t = x_t$) makes it irrelevant whether one uses $(x_{t+1} - x_t)/P_t$ or x_{t+1}/P_t since x_t/P_t now equals a constant $(R_F - 1)$. The Ball-Brown concept of unexpected earnings conforms broadly with the theory developed here. But we must also note that our simple model provides no apparent reasons why $(x_{t+1} - x_t)/P_t$ should correlate more with returns when compared to x_{t+1}/P_t.

The case when earnings and dividends are insufficient determinants of value

The theoretical model in proposition IV is confining because the results imply that returns are fully explained by x_{t+1}/P_t, or by unexpected earnings, ϵ_{t+1}/P_t. In reality, of course, the correlations are far from perfect.[9] To eliminate these theoretically perfect correlations, one must introduce information other than earnings and dividends that affects equilibrium value. Information that predicts next-period earnings should generally be valuation relevant. Such modeling is possible, but a standard omitted variables problem reduces the model's empirical content.

Appendix B considers assumptions leading to a linear equilibrium model. It is shown that

$$P_t + d_t = \phi x_t + \beta v_t, \tag{9}$$

where the scalar variable v_t represents "other information" available to value the security, and β denotes a valuation parameter that depends on the stochastic dynamics of (x_t, d_t, v_t). The valuation relation is similar to the one without "other information," and x_{t+1}/P_t remains central in explaining returns. However, since v_t cannot generally be observed by the researcher, the omission of nonearnings information in a regression equation leads to the usual estimation bias unless it happens to be uncorrelated with the earnings variable. (The last condition may be satisfied in specific cases, but these must be viewed as confining. This point is discussed in Appendix B.)

One can impose additional structure on the above model to show that under some circumstances $(x_{t+1} - x_t)/P_t$ correlates *strictly* more with R_{t+1} than does x_{t+1}/P_t. This model provides greater justification for the Ball-Brown concepts than the simple proposition IV model. The analysis is straightforward. From (9) one derives a return expression:

$$P_{t+1} + d_{t+1} - P_t = \phi(x_{t+1} - x_t) + d_t + \beta(v_{t+1} - v_t),$$

so that, by dividing with P_t,

$$R_{t+1} = 1 + \phi \Delta x_{t+1}/P_t + d_t/P_t - \beta \Delta v_{t+1}/P_t.$$

Suppose next that (1) the variability of d_t/P_t is immaterial (or zero because $d_t = 0$ for sure at that particular date), and that (2) $\Delta v_{t+1}/P_t$ does not correlate with either $\Delta x_{t+1}/P_t$ or x_{t+1}/P_t. It then follows that the correlation between R_{t+1} and $\Delta x_{t+1}/P_t$ strictly exceeds the one between R_{t+1} and x_{t+1}/P_t. The conclusion is immediate from the conditions since x_{t+1}/P_t cannot contribute to the explanation of R_{t+1} beyond $\Delta x_{t+1}/P_t$; strict dominance occurs since $\Delta x_{t+1}/P_t$ need not correlate perfectly with x_{t+1}/P_t (v_t is a random variable, and thus x_t/P_t is random even if $d_t = 0$ for sure). Appendix B shows that the assumption (2) is feasible given the model's primitives.

9 As Lev (1989) notes, the correlations between annual returns and earnings (changes) can be described as modest.

To appreciate the importance of other information, note that for $v_t \equiv 0$ one obtains the proposition IV model. In that case, $\Delta x_{t+1}/P_t$ and x_{t+1}/P_t both correlate perfectly with each other, and R_{t+1}, assuming that $d_t = 0$ for sure. One concludes that the strict dominance result requires the presence of other information ($\tilde{v}_t \neq 0$) in the valuation function.

The example uses rather specialized assumptions, and one cannot infer that the dominance result obtains under general conditions. Thus, x_{t+1}/P_t is a more basic explanatory variable of returns than is $\Delta x_{t+1}/P_t$ in the following sense: Given the idealized conditions when earnings and dividends are sufficient determinants of value, only the former variable correlates perfectly with returns for unrestricted dividend policies.

The use of book value rather than earnings

A final observation relates to the emphasis on earnings in explaining returns. The empirical literature virtually always includes earnings as a key variable, which is unsurprising given the prominence of earnings in real-world financial analysis. Much more complex is the theoretical question "Why should we expect earnings to explain returns?" Proposition IV deals with the issue by postulating a single primitive information variable, and its labeling as "earnings" was justified only because of expression (8). The reader may find this approach simplistic because the model does not capture the richness of accounting earnings measurement. This criticism must be recognized. It is also clear that the model's simplicity virtually guarantees that the primitive information variable labeled earnings will be valuation relevant.

Given the limitations of the proposition IV model, it is noteworthy that one can use a different model of value in which earnings (divided by initial price) still explain returns. Consider the case in which a firm's *book value* serves as an indicator of its market value. Although this model is obviously crude, it would seem reasonable if a firm's specific asset and liability carrying values approximate their market values. (As a prototype, the reader can think of an unlevered investment fund that uses market valuation for its assets, "marketable securities.") This valuation model thus hypothesizes that the stock variable "book value" can serve as a better indicator of value than the flow variable "earnings." The book value approach also implies that returns are logically explained by earnings divided by initial price (x_{t+1}/P_t). This follows since $P_{t+1} + d_{t+1} - P_t$ approximates the change in book value plus dividends, and the latter variable equals earnings (assuming regular "clean surplus" accounting—see Ohlson, 1991, for a fuller development). Earnings affect value because they increase the book value, and $R_F - 1$ times book value estimates next-period earnings.[10]

10 Note that $P_t(R_F - 1) = E[\tilde{x}_{t+1} |$ book value at $t]$. This property of the book value model is essentially the same as the one implied by the (proposition IV) earnings model. Only the conditioning information sets differ. On the book value model, the unexpected earnings response coefficient equals 1 rather than $R_F/(R_F - 1)$. This point illustrates that the UED process cannot describe the earnings process of an investment fund.

In this case, it is immediate that the use of x_t in $(x_{t+1} - x_t)/P_t$ introduces noise in any explanation of returns: x_t is a poor estimator of next-period earnings if earnings have only transitory elements. In sum, x_{t+1}/P_t works in theory as an explanatory variable of returns even in a setting in which book value rather than earnings determines value. The relevance of x_{t+1}/P_t in explaining returns cannot be underrated.

Conclusion

Analysis of the simple model of this paper yields a striking result with empirical content. Returns and unexpected returns are explained by current earnings divided by beginning-period price. This result follows because expected earnings divided by price equals a constant. That is, the effect of adjusting current earnings divided by beginning period price for its expected value—the constant—is irrelevant in a regression context because the regression intercept picks up the constant. The analysis further shows that the change in earnings divided by price explains returns almost as well as current earnings divided by beginning period price. The quality of the approximation depends on the degree to which the earnings price ratio approximates a constant, which in turn depends on dividends. If dividends equal earnings, then the current earnings to price ratio equals a constant; in this case, the change in earnings divided by beginning-period price obviously works as well as earnings divided by beginning-period price. This analysis provides some justification for the Ball-Brown concepts. Nevertheless, a simple extension of the certainty model results in earnings divided by beginning-period price as the explanatory variable of returns, not the change in earnings divided by beginning-period price.

The paper expands on this simple model with earnings and dividends as information to show how one can incorporate other information into the valuation analysis. Such other information is characterized generically by a scalar variable that influences the prediction of future earnings. This extended model can be used to demonstrate that under certain narrow circumstances, the change in earnings variable correlates strictly more with returns compared to earnings divided by beginning-period price. The published literature has not provided such a case to date. This study underscores that the superiority of the change in earnings to price compared with earnings to price in explaining returns requires

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Another difference between the book value model and the one implied by proposition IV can be inferred by focusing on expression (6). This expression obviously holds for the book value model, while the assumptions of proposition IV imply (6) if, and *only* if, there is certainty.

The book value model of value eliminates the possibility of goodwill, P_t minus book value at date t. By contrast, the proposition IV model implies that

$$\text{goodwill}_{t+1} = \text{goodwill}_t + \epsilon_{t+1}/(R_F - 1).$$

(i.e., goodwill follows a pure martingale). (We assume that the change in book value equals $x_t - d_t$.)

earnings and dividends to be insufficient (or imperfect) determinants of value. But there are no general reasons why the change in earnings to price should work better than earnings to price given imperfections in earnings. Thus, the earnings to price variable takes on a premier role as an explanatory earnings variable of returns because it is the correct variable under idealized conditions and the only one consistent with the Hicksian certainty case.

Appendix A: Proof of proposition III

We hypothesize a linear solution $P_t = \beta_1 x_t + \beta_2 d_t$ where β_1 and β_2 possibly depend on θ_1, θ_2, and R_F. Given the NA condition (and proposition I), one obtains

$$R_F P_t = R_F \beta_1 x_t + R_F \beta_2 d_t = P_{t+1} + d_{t+1} = \beta_1 x_{t+1} + (\beta_2 + 1) d_{t+1}$$

$$= \beta_1 (R_F x_t - (R_F - 1) d_t) + (\beta_2 + 1)(\theta_1 x_t + \theta_2 d_t).$$

Since x_t/d_t generally depends on t, the coefficients associated with x_t (and d_t) on the RHS and the LHS of the above NA condition must be equal. Hence,

$$R_F \beta_1 = \beta_1 R_F + (\beta_2 + 1)\theta_1 \tag{a}$$

$$R_F \beta_2 = -\beta_1 (R_F - 1) + (\beta_2 + 1)\theta_2. \tag{b}$$

Given that $\theta_1 \neq 0$, it follows from (a) that $\beta_2 = -1$. From (b) it follows that $\beta_1 = R_F/(R_F - 1)$. Thus, VF obtains. The rest is trivial.

Appendix B: The effects of other information

Let v_t be scalar variable summarizing other valuation relevant information, and let $z_t \equiv (x_t, d_t, v_t)$ denote the complete information vector. The information dynamics of z_t is given by

$$\begin{aligned}
&(i) \quad && \tilde{x}_{t+1} = R_F x_t - (R_F - 1) d_t + \gamma_1 v_t + \tilde{\epsilon}_{1t+1} \\
&(ii) \quad && \tilde{d}_{t+1} = \theta_1 x_t + \theta_2 d_t \qquad + \theta_3 v_t + \tilde{\epsilon}_{2t+1} \\
&(iii) \quad && \tilde{v}_{t+1} = \qquad\qquad\qquad\quad + \gamma_2 v_t + \tilde{\epsilon}_{3t+1},
\end{aligned} \tag{A1}$$

where the $\tilde{\epsilon}_{1t+1}$ are unpredictable disturbance terms. The dynamics A1 generalizes the proposition IV model since one obtains the latter model by putting $v_t \equiv \epsilon_{3t} \equiv 0$ for all t. Without loss of generality, assume $\gamma_1 \geq 0$. Thus one interprets $v_t > 0$ (<0) as the case of above (below) normal growth in the expected earnings. However, since v_t is random, above or below normal growth in expected earnings may change from one date to the next. As will become apparent in the next paragraph, the valuation relevance of other information derives from its relevance in predicting future earnings. That is, P_t depends on v_t if and only if $\gamma_1 \neq 0$. Further, it is understood that $\partial E[\tilde{x}_{t+1}|z_t]/\partial d_t = -(R_F - 1)$, which captures the usual MM precept. Also, ϵ_{1t+1} represents unexpected earnings, and $\theta_1, \theta_2, \theta_3$ are dividend policy parameters. The disturbance terms may correlate at any given date. (As a special case, earnings follow an IMA(1,1) type of process with parameter $-\gamma_1$ if $\epsilon_{1t+1} = \epsilon_{3t+1}$ and $\gamma_2 = 0$. The latter conditions combined with $x_t = d_t$ imply a strict IMA(1,1) process).

Combined with the present value formula of expected dividends, it can be shown (Ohlson, 1989b) that the information dynamics A1 implies the valuation function

$$P_t = \phi x_t - d_t + \beta v_t \tag{9}$$

where $\beta = \phi\gamma_1/(R_F - \gamma_2)$. As in proposition IV, the earnings multiplier ϕ equals $R_F/(R_F - 1)$, and the dividend policy parameters have no effect on the valuation function.

Using the above valuation function, one derives that

$$P_t = R_F^{-1}E[\tilde{P}_{t+1} + \tilde{d}_{t+1}|z_t] = R_F^{-1}E[\phi\tilde{x}_{t+1} + \beta\tilde{v}_{t+1}|z_t]$$

$$= \frac{E[\tilde{x}_{t+1}|z_t]}{R_F - 1} + \frac{\beta\gamma_2}{R_F} v_t,$$

so that

$$P_t(R_F - 1) = E[\tilde{x}_{t+1}|z_t] + [\gamma_1\gamma_2/(R_F - \gamma_2)]v_t.$$

The above expression shows that $P_t(R_F - 1)$ now generally differs from $E[\tilde{x}_{t+1}|z_t]$, and thus expression (7) does not generally hold. The only interesting exception occurs when $\gamma_2 = 0$ (i.e., the v_ts are serially uncorrelated). This point illustrates the danger of relating the market value to next-period expected earnings a priori without reference to the underlying information dynamics.

With respect to the relation between unexpected returns, $(P_{t+1} + d_{t+1})/P_t - R_F$, and unexpected earnings, ϵ_{1t+1}, one shows without difficulty that

$$\begin{aligned}(P_{t+1} + d_{t+1})/P_t - R_F &= \phi\epsilon_{1t+1}/P_t + \beta[v_{t+1} - E[\tilde{v}_{t+1}|z_t]]/P_t \\ &= \phi\epsilon_{1t+1}/P_t + \beta[v_{t+1} - \gamma_2 v_t]/P_t \\ &= \phi\epsilon_{1t+1}/P_t + \beta\epsilon_{3t+1}/P_t.\end{aligned}$$

The equation implies that unexpected earnings, $\tilde{\epsilon}_{1t+1}$, cannot fully explain returns if $\gamma_1 \neq 0$ and $\mathrm{Var}(\tilde{\epsilon}_{3t+1}) > 0$. The omission of ϵ_{3t+1} in a regression model that includes a perfect measure of unexpected earnings would lead not only to an imperfect R^2 but also to a bias in the estimated coefficient associated with unexpected earnings when ϵ_{1t+1} and ϵ_{3t+1} are correlated. The presence of other information (v_{t+1}) clearly complicates the matter of explaining returns.

As noted in the body of the paper, one can develop a special case of the above model such that the correlation between $\Delta x_{t+1}/P_t$ and R_{t+1} strictly exceeds the one between x_{t+1}/P_t and R_{t+1}. That is, the change in earnings works better than the levels of earnings as an explanatory variable for returns. Suppose that (a) $d_t = 0$, and restrict the information dynamics to satisfy (b) $\gamma_2 = 1$ and (c)

$\text{corr}[\tilde{\epsilon}_{1t+1}, \tilde{\epsilon}_{3t+1}|z_t] = 0$, any z_t. From the valuation function (9), one readily derives the return expression

$$R_{t+1} = 1 + \phi\Delta x_{t+1}/P_t + d_t/P_t + \beta\Delta v_{t+1}/P_t$$

$$= 1 + \phi\Delta x_{t+1}/P_t + \beta\epsilon_{3t+1}/P_t$$

and where the last equality follows from (a) and (b). Note next that given restriction (c), it follows that $\text{corr}[\Delta\tilde{x}_{t+1}/P_t, \tilde{\epsilon}_{3t+1}/P_t|z_t] = 0$ for all z_t, and thus one obtains $\text{corr}[\Delta\tilde{x}_{t+1}/\tilde{P}_t, \tilde{\epsilon}_{3t+1}/\tilde{P}_t] = 0$ as well. Similarly, one shows that $\text{corr}[\tilde{x}_{t+1}/\tilde{P}_t, \tilde{\epsilon}_{3t+1}/\tilde{P}_t] = 0$, and, in general, $\text{corr}[\Delta\tilde{x}_{t+1}/\tilde{P}_t, \tilde{x}_{t+1}/\tilde{P}_t] < 1$. (The last inequality obtains if and only if $\text{Var}(\tilde{v}_{t+1}) > 0$.) Thus, it follows immediately that $\text{corr}[\tilde{R}_{t+1}, \Delta\tilde{x}_{t+1}/\tilde{P}_t] > \text{corr}[\tilde{R}_{t+1}, \tilde{x}_{t+1}/\tilde{P}_t]$ given assumption (c).

One can also relate the earnings variable $\Delta x_{t+1}/P_t$ to unexpected earnings. Using A1(i), A1(iii), and (9), one easily derives that $\Delta x_{t+1}/P_t = \alpha + \epsilon_{1t+1}/P_t$, where ϵ_{1t+1} equals unexpected earnings relative to the information (x_t, v_t), and $\alpha \equiv \gamma_1/\beta = (R_F - 1)^2/R_F$. Hence, the constant α aside, $\Delta x_{t+1}/P_t$ measures unexpected earnings normalized by P_t. But the error due to α is, of course, irrelevant in a regression context. That is, $\text{corr}[\Delta\tilde{x}_{t+1}/\tilde{P}_t, \tilde{R}_{t+1}] = \text{corr}[\tilde{\epsilon}_{1t+1}/\tilde{P}_t, \tilde{R}_{t+1}]$, and the earnings response coefficient equals ϕ.

The superiority of $\Delta x_{t+1}/P_t$ vs. x_{t+1}/P_t as an explanatory variable of returns has been derived under specialized conditions. One cannot infer that $\Delta x_{t+1}/P_t$ is intrinsically superior (or inferior, given the presence of other information). Which of the two variables works the best is likely to depend on all aspects of the information dynamics, in particular γ_2 and $\text{cov}[\tilde{\epsilon}_{1t+1}, \tilde{\epsilon}_{3t+1}|z_t]$. This general dependency makes a resolution of the specification issue very complex.

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