Exploring Bundling Theory with Geometry

John C. Eckalbar

Abstract: The author shows how instructors might successfully introduce students in principles and intermediate microeconomic theory classes to the topic of bundling (i.e., the selling of two or more goods as a package, rather than separately). It is surprising how much students can learn using only the tools of high school geometry. To be specific, one can prove that with independently distributed reservation prices, pure bundling raises both profits and consumers' surplus. The author also explores the topic of mixed bundling (i.e., when both bundles and separate sales are conducted).

Key words: bundling, microeconomics
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One of the real challenges of teaching a quality micro-theory class is to cover all of the basics (like cost curves or elasticity) and include enough eccentric material (location theory or efficiency wage models, for example) to keep things interesting. One of the eccentric topics that I have had some success with lately is the theory of bundling, in which a firm takes two or more goods that might be sold separately and instead sells them together. Examples are everywhere: The textbook is a bundle of chapters, the magazine is a bundle of articles, or the CD a bundle of songs. Cable or satellite TV offerings come in bundles of stations. Hardware is bundled with software, and software is bundled with other software. It is interesting to ask why firms do this and what the impact is on the consumer. It is also interesting to see how much can be learned with fairly simple geometry. Some textbook authors are beginning to cover the topic, although lightly (see Varian 1999 and Besanko and Braeutigam 2002). The standard references are Stigler (1963), Adams and Yellen (1976), Schmalensee (1984), McAfee, McMillan, and Whinston (1989), and Salinger (1995).

I illustrate how the topic might be covered in a principles class and then carry the ideas forward to the level of intermediate theory. I conclude with some comments on possible extensions. (A fairly advanced version, entitled "The Geometry of Bundling," with generalizations and formal proofs is available either as a PDF file or an interactive Mathematica notebook at: http://www.csuchico.edu/~eckalbar/bundling. I henceforth refer to it as GOB.)

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TABLE 1. Traders' Reservation Prices for Two Goods

Trader	Good 1 ($r1$)	Good 2 ($r2$)
A	6	1
B	5	2
C	4	3
D	3	4
E	2	5
F	1	6

Note: $r1$ and $r2$ = reservation price 1 and 2.

I introduce my micro principles classes to the topic of bundling using Table 1. I imagine a monopolist with six potential customers, traders A through F. The monopolist sells two products, goods 1 and 2. I assume that all costs are fixed (FC) with FCi denoting the fixed cost in division i. I ask the class what price the monopolist would charge for good 1 if it sold the product separately, and the students easily determine that the firm would be indifferent between setting the price at \$3 or \$4. In either case division 1 profits would be \$12 − FC1. In the same way, division 2 profits would be \$12 − FC2. Thus, total company profits would be \$24 − FC1 − FC2, if sales are separate.

I then ask if there is anything the firm might do to raise its profits, and someone will likely mention price discrimination because that is the topic generally covered immediately prior to bundling. Always ready to reinforce a recent lesson, I can confirm that under perfect (first-degree) price discrimination, profits would be \$42 − FC1 − FC2. The contrast between price discrimination and conventional single-price tactics reveals the frustrations of a monopolist charging a single price. On the one hand, the seller lets some buyers pay much less than their high reservation prices, and on the other hand, some potential buyers are unwilling to pay the single asking price even though they would contribute to profits if they were allowed to pay less, as long as what they pay exceeds marginal cost.

If the class is stumped to come up with another profit raising ploy, I ask, "What would be trader A's reservation price for a bundle containing goods 1 and 2?" If I assume that the reservation value of the bundle is the sum of the individual reservation values, as I will throughout, then the answer is obvious, \$7. The students quickly see that every trader's reservation value for the bundle is \$7. If the firm only offered bundles containing goods 1 and 2, it would sell six bundles at \$7 each, and its profits would be \$42 − FC1 − FC2, the same as perfect price discrimination. Maybe that is why *Time* magazine has articles on both hockey and ballet.

Somewhere a student is scanning the table to see how I cooked the data to get this result. The answer is not hard to find—there is a perfect negative correlation between the reservation values for the two goods (i.e., traders with high reservation values for one good have low reservation values for the other). It is easy to see that if I rearranged the reservation values to get perfect positive correlation, bundling cannot improve profits. What if the two reservation values were more random?

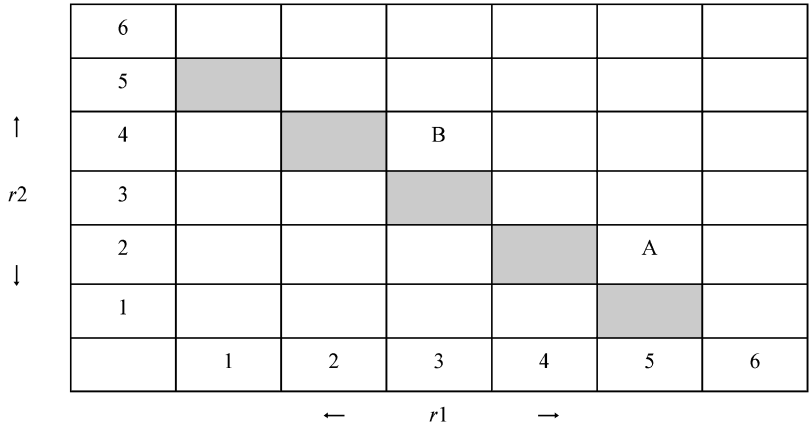


FIGURE 1. Reservation price matrix.

I propose a thought experiment: I have two dice, one red and one green. I have the students come up one at a time and roll the dice. The red one is their reservation price for good 1, and the green one is their reservation value for good 2. Each observation is recorded in Figure 1. For example, student A might have rolled a 5 on the red die and a 2 on the green one, whereas B rolled 3 and 4, respectively. How would the table fill in as more and more students rolled the dice? Someone familiar with statistics will say that each cell in the table is equally probable, so the cells will tend to fill in evenly, which is true. I assume that the table does in fact fill in evenly, and to keep matters simple, I further suppose that there are 36 students, one in each cell. (In a "smart classroom," one with a computer and associated overhead projector, it is easy to simulate thousands of tosses of the dice and immediately display the results.)

Now if the goods were sold separately, the optimal prices would again be $3 or $4, but because demands are six times higher than in the first example, the total profits from separate sales would be $144 − FC1 − FC2. What if the goods were bundled? What would the bundle demand curve look like, and what would be the optimal bundle price?

If each trader's reservation price for the bundle is the sum of his or her individual reservation prices for the two goods, then one trader values the bundle at $12, two value it at $11, and so forth. In Figure 2, I show the resulting bundle demand curve, and a little arithmetic reveals that the optimal bundle price is $6, and profits would be $156 − FC1 − FC2. (All of the traders who would buy the bundle are in or above the shaded cells in Figure 1. I use the set of shaded cells in the discrete analogue of the bundle price lines later in this article.) So even with a uniform distribution of reservation values, bundling profits can exceed profits from separate sales.

In our first example from Table 1, the firm expropriated the entire consumers' surplus when it bundled the products, but in the present case, there is actually a

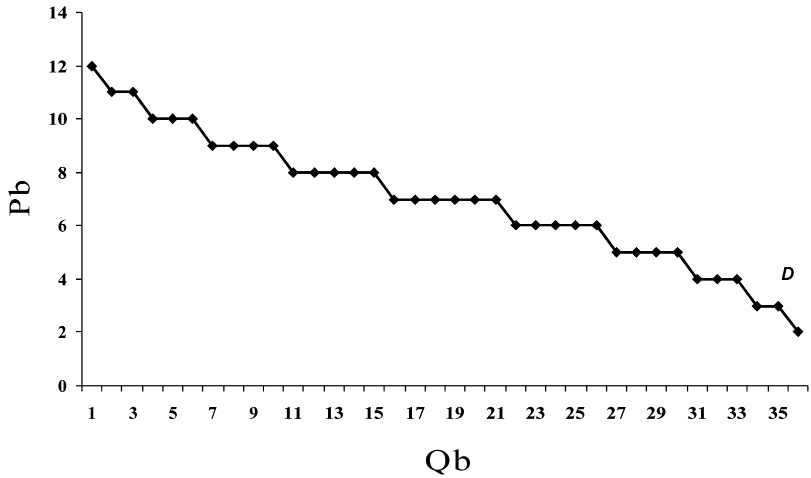


FIGURE 2. A discrete bundle demand curve.

Notes: Pb is bundle price, and Qb is number of bundles demanded.

consumers' surplus of \$56 because one trader would have been willing to pay \$12 for the bundle, and two would have paid \$11, and so on. In fact, if the firm engaged in separate sales and charged \$4 for each good, the consumers' surplus would only total \$36, so in this case, bundling would actually increase the consumers' surplus. Although if the firm charged \$3 each under separate sales, the consumers' surplus would be \$72, and bundling would reduce the consumers' surplus. Note that the firm would charge \$4 rather than \$3 with even the tiniest positive marginal cost for either good, so it is certainly possible for bundling to increase consumers' surplus.

The material up to this point takes about 30 minutes to present to a principles class. If more time is available for the topic, the instructor might include the material on positive marginal cost found in the section on possible extensions. It is surprising how much more one can learn with a little geometry and a lot of patience. What follows is a doable model for an intermediate (or well-motivated principles) class.

THE CONTINUOUS CASE

I could argue that nearly all economic variables are actually discrete, but I prefer to advance the topic by assuming that prices and quantities are infinitely divisible.

I assume that a monopolist sells two goods to a set of m buyers. The buyers' reservation prices (R) for good 1 are uniformly distributed over the interval $[0, R1]$, and their reservation prices for good 2 are independently drawn from the interval $[0, R2]$. Without loss of generality, I assume $R2 \leq R1$. Let ri denote a

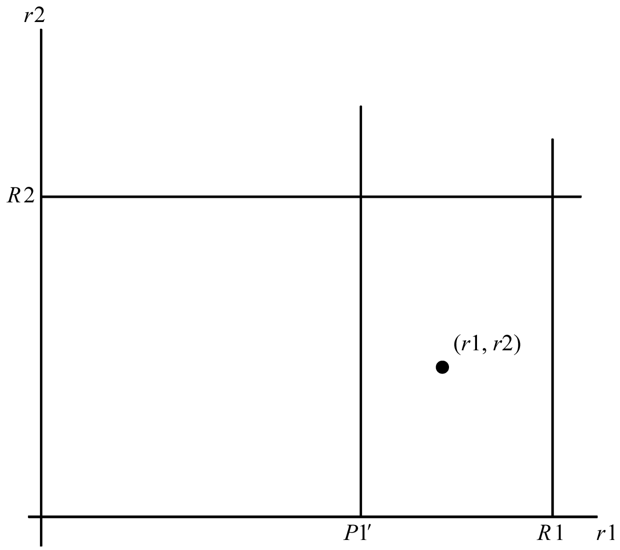


FIGURE 3. Reservation price space.

trader's reservation price for good i. Then a trader can be represented as a point, $(r1, r2)$, in reservation price space as shown in Figure 3. Because the traders are uniformly distributed throughout $R1 \times R2$, any unit area within this space will contain approximately equal numbers of traders when m is large.

Assuming for the moment that the firm sells the products separately, I can compute demand quantities as follows: Suppose $P1 = P1'$ as shown in Figure 3. All the traders whose reservation price points are to the right of $P1'$ will buy good 1. All such traders are in the rectangle of area $(R1 - P1')R2$. If I divide this area by the total area $R1 \cdot R2$, I get the fraction of all traders who choose to buy good 1 when $P1 = P1'$. If I multiply by m, I get the number of buyers for good 1 at $P1 = P1'$. This gives rise to an inverse demand equation for good 1 given by

$$P1 = R1 - \frac{R1}{m} Q1.$$

Assuming for the moment that all costs are fixed, then routine calculations show that if the monopolist sets a single price, the optimum price is $(R1)/2$, and the associated quantity sold will be $m/2$. In this case, profits would be

$$\frac{mR1}{4} - FC1.$$

Finally, the consumers' surplus would be $(mR1)/8$.

Demand, price, profit, and consumers' surplus for good 2 follow in the same way with obvious changes. If the monopolist treats these two products as autonomous divisions, setting a single price for each good and selling them independently, then profits would be

$$\frac{mR1}{4} + \frac{mR2}{4} - FC1 - FC2,$$

and total consumers' surplus would be

$$\frac{mR1}{8} + \frac{mR2}{8}.$$

Now suppose the firm were to offer *only* bundles containing one unit of good 1 and one unit of good 2. The bundles are offered for sale at a price Pb, and separate sales of goods 1 and 2 are discontinued. This is called "pure bundling." What would the demand curve for the bundle look like? If the sum of a trader's reservation values, $r1 + r2$, is greater than or equal to Pb, then the trader will buy the bundle. Equivalently, the trader will buy the bundle if $r2 \geq Pb - r1$. Consider Figure 4; the line $r2 = Pb - r1$ obviously forms the southwest boundary of the shaded area containing willing bundle buyers. It is easy to see that the area shaded is given by

$$\frac{1}{2}(R1 + R2 - Pb)^2.$$

With a little geometry, I can determine the general shape of the bundle demand curve and find how Qb, the quantity of bundle sales, relates to Pb.

Consider Figure 5, which shows many alternative bundle prices. The bundle price line furthest to the right shows that no one will buy the bundle if $Pb \geq R1 + R2$, so the bundle demand curve has a vertical intercept at $R1 + R2$.

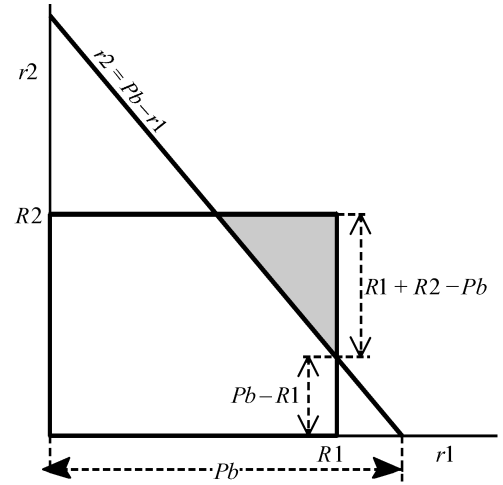


FIGURE 4. Bundle demanders are in the shaded region.

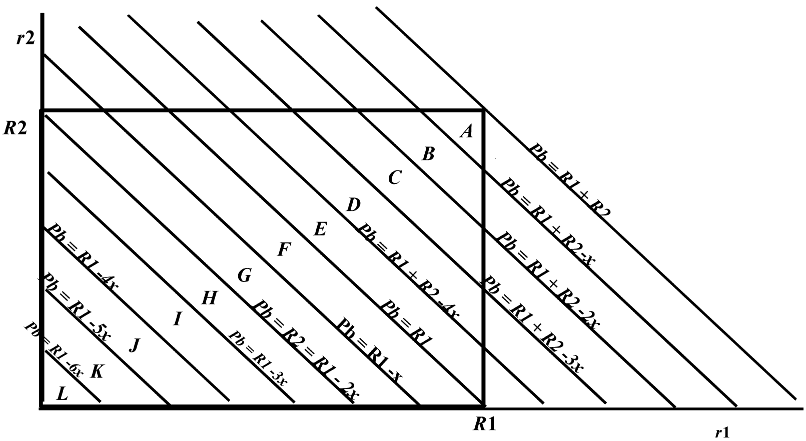


FIGURE 5. Computing the increase in bundle demand as the bundle price falls.

But if the bundle price is lowered to $R1 + R2 - x$, the buyers whose reservation prices put them in triangle A will demand the bundle. Note that if the bundle price is reduced from $R1 + R2 - x$ to $R1 + R2 - 2x$, the quantity demanded will rise further as those traders in trapezoid B begin to demand the bundle. Then if Pb falls to $R1 + R2 - 3x$, buyers in trapezoid C begin to buy. Let g(Z) be the area of region Z. Because g(A) < g(B) < g(C) < g(D) < g(E), the bundle demand curve will be concave from above when $R1 + R2 > Pb \geq R1$.

When $Pb = R1$, the area of the triangle containing the willing bundle buyers is $(R2^2)/2$, so bundle demand will be

$$\frac{R2^2}{2}\left(\frac{m}{R1R2}\right) = \frac{mR2}{2R1}.$$

This marks another point on the bundle demand curve.

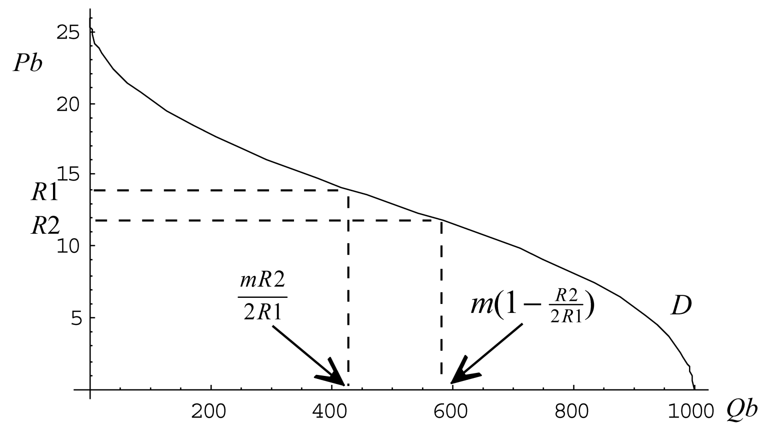
As I reduce Pb in the range between $Pb = R1$ and $Pb = R2$, equal price reductions will give rise to equal increases in the quantity of bundles demanded. For example, g(F) = g(G). This means that bundle demand is linear in the region where Pb is between $R1$ and $R2$.

When $Pb = R2$, the space containing traders who will *not* buy the bundle has an area equal to $(R2^2)/2$. It follows that when $Pb = R2$, $mR2/2R1$ traders elect not to buy the bundle, and the remaining $m(1 - R2/2R1)$ traders will buy the bundle. Armed with the two endpoints of the linear segment of the bundle demand curve, I easily compute that the relevant demand equation for Pb between $R1$ and $R2$ is

$$Pb = \frac{2R1 + R2}{2} - \frac{R1}{m}Qb.$$

Finally, once Pb is below $R2$, each successive equal cut in Pb brings in fewer and fewer additional buyers, because $g(H) > g(I) > \cdots > g(L)$. This forces the demand curve to be concave from below in the region where $Pb < R2$.

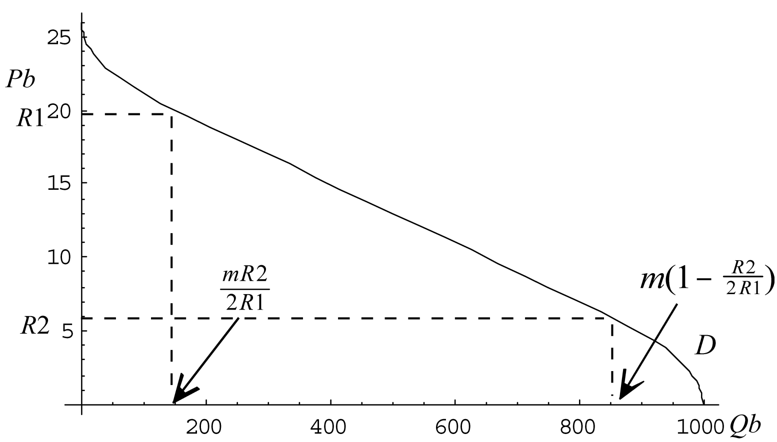
I show two bundle demand curves with different values for $R1$ and $R2$ in Figures 6 and 7. Recall that the demand is linear when Pb is between $R1$ and $R2$, hence each curve has a linear segment between the two horizontal dashed lines. As it turns out, the properties of the equilibrium depend critically upon the ratio



$R1 = 14$; $R2 = 12$; $m = 1000$

FIGURE 6. Bundle demand when *R*1 is slightly greater than *R*2.

Note: $R1$ is the highest reservation price for good 1. $R2$ is the highest reservation price for good 2.



$R1 = 20$; $R2 = 6$; $m = 1000$

FIGURE 7. Bundle demand when *R*1 is much greater than *R*2.

Note: For definitions of $R1$ and $R2$ see note to Figure 6.

of $R2$ to $R1$—bundling a relatively cheap good with a relatively expensive good yields some different properties from bundling two goods of roughly equal value. (For details, see GOB.) For the moment, in the interest of proceeding with minimal complication, I add the assumption that $R2 \leq (2/3)R1$. I will relax that shortly.

My next task is to find the marginal revenue for the bundle demand curve, MR*b*. I am particularly interested in the value of *Qb* at which MR*b* = 0 because that will identify the profit maximizing level of output when marginal cost is zero. It is well known that if a demand curve is linear, then the MR lies half way between the demand curve and the vertical axis at every price. Figure 8 shows a bundle demand curve, *Db*, together with a dashed extension of its linear segment. The horizontal intercept of the dashed extension is at $Qb = m + mR2/2R1$. Under my present assumptions, it follows that the bundle marginal revenue for the linear segment of D will be 0 at $m/2 = mR2/4R1$. Substituting this value into the bundle demand equation, one sees that under the present assumptions, the associated optimal bundle price will be $R1/2 + R2/4$. The MRb under the curved segments of D*b* can be derived from another geometrical fact about MR and D. Specifically, at any given quantity, say Q_0, the associated value of MR (call it MR_0) lies below the accompanying price (P_0) by the amount $-Q_0 \cdot dP/dQ$, where dP/dQ is the slope of the demand curve at Q_0.

I have established that when $R2 \leq (2/3)R1$, the pure bundling profit maximum is characterized by $Qb = m/2 + mR2/4R1$ and $Pb = R1/2 + R2/4$. Armed with these values, I can draw the following conclusions:

1. Recall that under an independent, separate sales regime, the optimal level of output is $m/2$ for each good. The optimum pure bundle quantity involves $mR2/4R1$ additional units of output and sales. As I discuss later, this is better for both producers and customers.

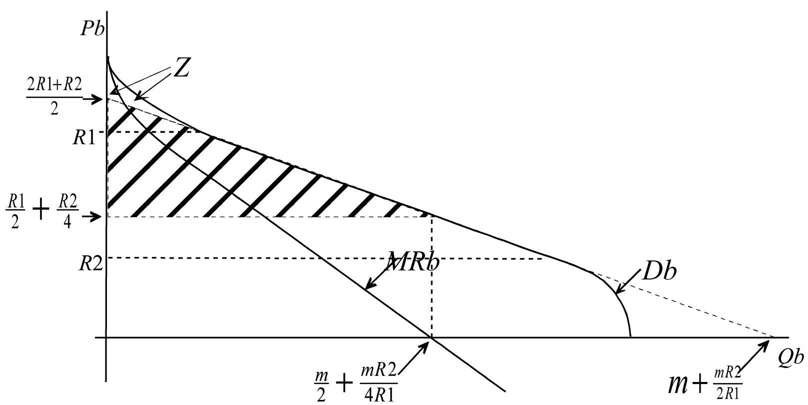


FIGURE 8. Deriving bundle marginal revenue (MR) and consumers' surplus.

2. Remember that the optimal independent prices are $R1/2$ and $R2/2$, so when the goods are bundled and sold at $R1/2 + R2/4$, one sees the optimality of the commonly observed pricing strategy of "buy good 1 at the regular price and get good 2 at half off."

To highlight these issues, let us look at profits and consumers' surplus under pure bundling versus separate sales. Recall that with separate sales, total company profits are $mR1/4 + mR2/4 - FC1 - FC2$. With pure bundling, profits are

$$Pb \cdot Qb - FC1 - FC2 = \left(\frac{R1}{2} + \frac{R2}{4}\right)\left(\frac{m}{2} + \frac{mR2}{4R1}\right) - FC1 - FC2$$
$$= \frac{mR1}{4} + \frac{mR2}{4} + \frac{mR2^2}{16R1} - FC1 - FC2,$$

so profits under pure bundling exceed profits from separate sales by $mR2^2/16R1$. That partly explains why bundling is so prevalent—it can raise profits. It is also possible, as I discuss later, that bundling can increase profit by reducing production cost.

Does the higher profit come at the expense of reduced consumers' surplus, as happens with first-degree price discrimination? The answer turns out to be, "No," in the present case. The proof is surprisingly easy to come by using only the geometric tools at hand. First, recall that consumers' surplus under separate sales is given by $mR1/8 + mR2/8$. With pure bundling, the consumers' surplus will be the area under the bundle demand curve from 0 to $m/2 + mR2/4R1$, minus sales revenue. This is a fairly complex integral, because it must be done in two parts, that is, under the nonlinear segment from 0 to $mR2/2R1$ and then under the linear segment from $mR2/2R1$ to $m/2 + mR2/4R1$. (The integral is beyond the ability of the typical intermediate student and is not shown here, but see GOB for a complete treatment.)

The area of the triangle marked in Figure 8 is smaller than the consumers' surplus integral by the amount of the area Z, which is below the bundle demand and above the marked triangle. If I can show that the area of the marked triangle is greater than the consumers' surplus under separate sales, then I can be sure that bundling will increase consumers' surplus. The area of the triangle is

$$\frac{1}{2}\left(\frac{R1}{2} + \frac{R2}{4}\right)\left(\frac{m}{2} + \frac{mR2}{4R1}\right) = \frac{mR1}{8} + \frac{mR2}{8} + \frac{mR2^2}{32R1},$$

which is clearly larger than the total consumers' surplus under separate sales.

It appears to be the case under the current assumptions that pure bundling is a win–win proposition in that both profits and consumers' surplus increase compared with separate sales. Note that these issues relate to pricing decisions by a single firm. If a firm bundles goods, some of which are also sold by competitors, then one must ask about the implications of bundling on effective competition. One might imagine a firm selling a product with a highly inelastic demand and bundling that product at near-zero apparent cost with another product that is also sold by a few competitors. The Microsoft case comes to mind. Students might be encouraged to do a Web search of "bundling + Microsoft." One of the first hits is likely to be an interesting article in *Business Week* (http://www.businessweek.com/magazine/content/01_49/b3760065.htm).

I have been assuming that $R2 \leq (2/3)R1$. If $R2 > (2/3)R1$, then the analysis is more complex, but the same results obtain—pure bundling increases both profits and consumers' surplus. The complexity is simply a result of the following: When $R2 > (2/3)R1$, the profit maximum is found where the nonlinear segment of MRb on the bottom right section of the curve intersects the horizontal axis. This requires some extra fuss to solve nonlinear equations, and it takes more time than I feel justified in an intermediate class. (See GOB for the derivations.) Although the proof is complex, the key fact is that optimal output is quite high under pure bundling, and this can be good for both profits and consumers' surplus. I have shown that when $R2 \leq (2/3)R1$, bundle output is $m/2 + mR2/4R1$, and it can be shown that with $R2 > (2/3)R1$, output is $(2/3)m$. Both values are greater than separate sales output of $m/2$. This is a novel finding that runs counter to speculation (based on discrete examples) by both Stigler (1963) and Adams and Yellen (1976).

EXTENSIONS

I conclude by briefly considering the following extensions: mixed bundling, positive marginal cost, and correlations between reservation prices.

Mixed Bundling

A firm follows a strategy of *mixed bundling* when it offers both the bundle and one or more of the goods separately. Mixed bundling is relatively easy to explore graphically, but some of the proofs are unavailable except with more serious math. I show how mixed bundling works in Figure 9, where the bundle price is Pb, and goods 1 and 2 are offered separately at prices $P1$ and $P2$. Traders in the shaded area, G1 (G2), will buy only good 1 (2), traders in area B will buy the bundle, and all other traders will buy nothing.

One interesting result is that when one of the goods, say good 2, has a relatively low demand curve, specifically $R2 < (1/2)R1$, then only the cheaper good is offered separately. In this case, I can show that the optimum bundle price is $R1/2 + R2/3$, whereas the separate price for good 2 alone will be $(2/3)R2$, and good 1 will not be offered except in the bundle. (See Theorem 2 of GOB.) In Figure 10 I illustrate this case with $R1 = 10$ and $R2 = 3$. Under mixed bundling, the optimal bundle price will be \$6. Good 2 will be offered separately for \$2, and good 1 will not be offered separately. The proof is complex, but the reason is apparent in the geometry of the figure. If the goods were only offered as a bundle at $Pb = 6$, then everyone to the right of the $Pb = 6$ line would buy the bundle. If good 2 were also offered separately at \$2, traders in area B would quit buying the bundle and choose to buy good 2 separately at the lower price. This is bad for the firm. Traders in area A who previously bought nothing would now buy good 2 separately. Because the area of A is large relative to the area of B, there is a net gain to the firm. In contrast, if the firm also experimented in selling good 1

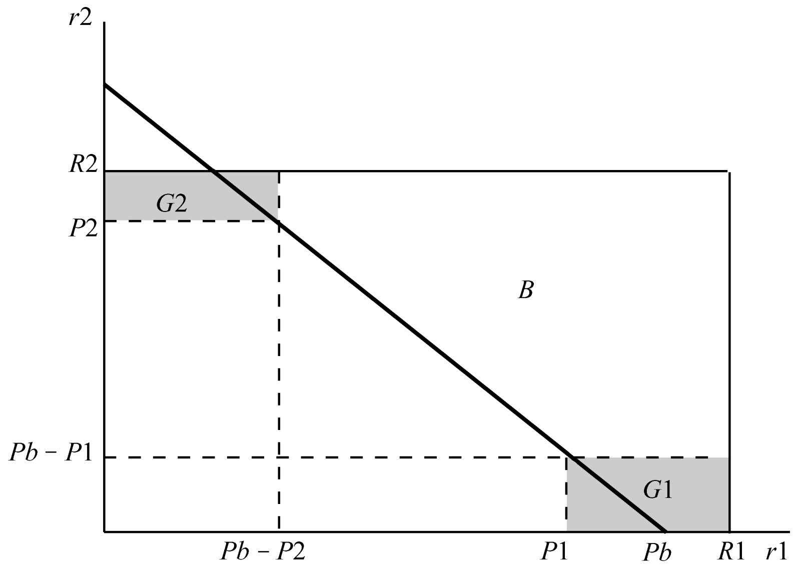


FIGURE 9. Finding buyers of bundle and separate goods under mixed bundling.

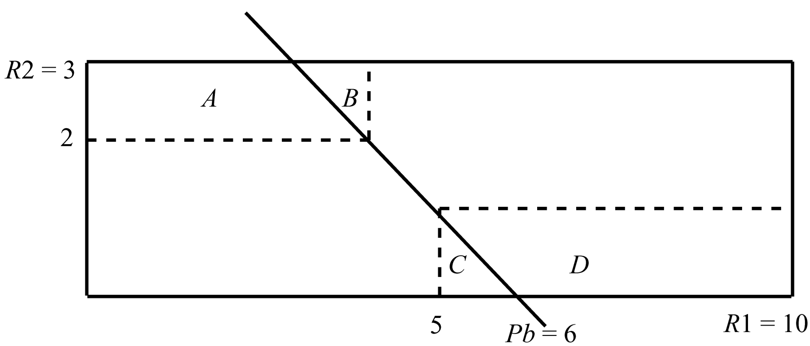


FIGURE 10. Why separate sales are sometimes not allowed.

separately at, say $P1 = \$5$, it would gain from sales to buyers in the small region C, who previously bought nothing, but it would lose revenue from traders in the much larger region D, who would save by switching out of the bundle in favor of good 1 separately. Perhaps that is why one can buy a new guitar with new strings and new strings separately but one cannot buy a new guitar without strings.

It is obvious that mixed bundling will increase profits over pure bundling because pure bundling can be viewed as mixed bundling with the added constraints that $P1 = P2 = Pb$. It can also be shown, although the proof is quite laborious, that mixed bundling results in lower consumers' surplus than do separate sales.

Positive Marginal Cost

The easiest way to see that successful bundling is highly dependent on low marginal cost is to return to the original, simple example from Table 1. To keep the notation as simple as possible, assume that now fixed costs are 0, and the total cost of producing good i is $Ci \cdot Qi$, with $Ci > 0$.

Profits from separate sales for good i, πi, are shown in Table 2. It is easy to see from the table that, if $0 < Ci < 2$, the firm will maximize profits by charging \$4 and selling 3 units. If $2 < Ci < 4$, the firm will charge \$5 and sell 2 units. If $4 < Ci < 6$, the firm will charge \$6 and sell one unit. The firm can make a profit with separate sales as long as $Ri > Ci$ (i.e., as long as the highest reservation price exceeds the marginal cost).

The situation is quite different under pure bundling. Recall that with the data in Table 1, all traders have a reservation price of \$7 for the bundle. If I assume that the marginal cost of the bundle is $C1 + C2$, so there are no economies or diseconomies of production with bundling, then pure bundling cannot pay if $C1 + C2 > 7$, whereas separate sales could easily pay if $C1 + C2 > 7$. For example, if the marginal costs for both goods are equal to \$4.50, then it costs \$9 to make the bundle, and no one is willing to pay that much for the bundle. But if the firm sells the goods separately and charges \$6 for each good, it will sell one of each for a combined profit of \$3. It is obvious from this that high marginal cost will dissuade firms from bundling.

To keep matters relatively simple for the moment, assume that $C1 = C2$. If so, it is easy to show that pure bundling dominates separate sales only if $Ci < 2.75$. Suppose for a moment that $Ci = 2.50$. If the firm did not bundle, it would sell each good for 5, and it would sell two units of each good. Total profit would be $20 - (4)(2.50) = 10$. If the firm instead elected to offer only a bundle at 7, it would take in 42 in revenue and have expenses of 30, for a profit of 12. Thus, pure bundling would dominate separate sales if both marginal costs were equal to 2.50.

TABLE 2. Profits with Positive Marginal Cost

Pi	*Qi*	*TRi*	πi
6	1	6	$6 - Ci$
5	2	10	$10 - 2Ci$
4	3	12	$12 - 3Ci$
3	4	12	$12 - 4Ci$
2	5	10	$10 - 5Ci$
1	6	6	$6 - 6Ci$

Notes: *Pi* is price of *i*; Q*i* is quantity of *i*; *TRi* is total revenue of *i*; and πi is profit of *i*.

The following are two comments on this marginal cost discussion:

1. In the pure bundling case, traders E and F buy the bundle containing good 1, even though their reservation values for good 1 are lower than the marginal cost of good 1. Similarly, traders A and B buy the bundle even though $r2 < C2$ for them. Of course, it is socially suboptimal to have traders taking possession of goods when the traders value the goods below marginal cost. Mixed bundling could play an interesting role in this case. Suppose the firm were to set $Pb = 7$, and $P1 = P2 = 4.75$, selling either bundles or separate goods. Now traders A and B would elect to buy only good 1, traders E and F would buy only good 2, and traders C and D would buy the bundle. The firm's profits would rise from 12 under pure bundling to 13 under mixed bundling, and consumers' surplus would rise from 0 under pure bundling to 3 under mixed bundling. Thus, mixed bundling could offer a clear Pareto improvement.
2. When I constructed Table 1, I assumed that reservation values are perfectly negatively correlated, and this is the most favorable situation for bundling. Elsewhere (Eckalbar 2003, 20), I have been able to show that when reservation prices are uniformly and independently distributed, pure bundling ceases to pay when marginal cost exceeds about 19 percent of the value of the demand curve intercept, so bundling does belong in the realm of low marginal cost (MC).

With respect to cost, if there are economies in the production of the bundle, meaning that the marginal cost of the bundle is less than $C1 + C2$, then this will encourage bundling over separate sales. Examples of this are easy to find such as putting more than one program (like the operating system and the Web browser) on one CD, or more than one article in a magazine. This could well be the most common reason for bundling.

Correlations Between Reservation Prices

If reservation prices were perfectly positively correlated, then all traders' reservation price points would lie on the up-sloping diagonal running from (0, 0) to ($R1$, $R2$). It is easy to show in this case that optimal pure bundling yields exactly the same values for profits, sales, and consumers' surplus as does optimal separate sales. If reservation values are perfectly negatively correlated, then one can show that pure bundling raises profits, but consumers' surplus is increased only if $R2 < (3/5)R1$ (see GOB, pp. 25–26).

Finally, if the bundled goods are complements (substitutes), then it makes sense to assume that a trader's reservation price for the bundle is greater than (less than) $r1 + r2$, and this would tend to make bundling more (less) likely. (See GOB and an interesting paper by Venkatesh and Kamakura [2003].)

CONCLUSION

I have shown that a little geometry and a lot of patience are sufficient to illuminate a good deal about bundling. This strikes me as a worthwhile story to tell to micro students. I find it especially attractive in view of the fact that closed-form solutions, rather than just qualitative results, are comparatively easy to obtain.

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