Q1. Find the line element ds^2, the scale factors, the vector $d\boldsymbol{s}$, the volume and area elements and the unit vectors $\boldsymbol{u}$ in the elliptic cylindrical coordinates

$$x = a\cosh u\cos v\,, \quad y = a\sinh u\sin v \quad z = z\,.$$

Q2. Find the line element ds^2. the scale factors, the vector $d\boldsymbol{s}$, the area element and the unit vectors $\boldsymbol{u}$ in the Bipolar coordinates

$$x = \frac{a\sinh u}{\cosh u + \cos v}\,, \quad y = \frac{a\sin v}{\cosh u + \cos v}$$

Q3. Consider the rotational parabolic coordinates defined by

$$x = uvw\,, \quad y = uv\sqrt{1-w^2}\,, \quad z = \frac{1}{2}(u^2 - v^2)\,.$$

a) Find the unit tangent vectors $\hat{\boldsymbol{e}}_u$, $\hat{\boldsymbol{e}}_v$, $\hat{\boldsymbol{e}}_w$ along the coordinate curves.
b) Show that the system of coordinates (u, v, w) is orthogonal.
c) Find the line and volume elements.
d) Obtain the gradient operator and the Laplacian.

Q4. In cylindrical coordinates compute

$$\boldsymbol{\nabla}\cdot\boldsymbol{u}_r\,, \quad \boldsymbol{\nabla}\times\boldsymbol{u}_r\,, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u}_\theta\,, \quad \boldsymbol{\nabla}\times\boldsymbol{u}_\theta$$

Q5. In spherical coordinates compute

$$\boldsymbol{\nabla}\cdot\boldsymbol{u}_r\,, \quad \boldsymbol{\nabla}\times\boldsymbol{u}_r\,, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u}_\theta\,, \quad \boldsymbol{\nabla}\times\boldsymbol{u}_\theta\,, \quad \boldsymbol{\nabla}\times\boldsymbol{u}_\phi$$

Q6. Find the flux of the magnetic field $\boldsymbol{B} = r\boldsymbol{u}_r - 2z\boldsymbol{u}_z$ through the curved surface of a

a) a half cylinder of radius 3 bounded by the $z = 3$ and $z = 7$ planes and the xz-plane. Verify the divergence theorem

b) a quarter cylinder of radius 3 bounded by the $z = 3$ and $z = 7$ and the xz- and yz-planes. Verify the divergence theorem.

Q7. A force field is given in spherical coordinates as $\boldsymbol{F} = \frac{2P\cos\theta}{r^3}\boldsymbol{u}_r + \frac{P\cos\theta}{r^3}\boldsymbol{u}_\theta$. Compute $\oint_C \boldsymbol{F}\cdot d\mathbf{r}$ for a unit circle on the plane $\theta = \frac{\pi}{2}$. Is there a potential associated to $\boldsymbol{F}$.