

Q.1 It is given that $\mathbf{F} = (3x - yz)\mathbf{i} + (z^2 - y^2)\mathbf{j} + (2yz + x^2)\mathbf{k}$,

- Evaluate the surface integral $\oiint_S \mathbf{F} \cdot d\boldsymbol{\sigma}$ over the surface of the sphere $(x - 2)^2 + (y + 3)^2 + z^2 = 9$.
- Verify divergence theorem.

Q.2 Consider the vector field given as $\vec{\mathbf{V}} = (x^2 + 3x)\mathbf{i} + (3y^2 + 3y)\mathbf{j} - 2z(x_3y - 2)\mathbf{k}$.

- Evaluate the surface integral $\oiint_S \mathbf{V} \cdot d\boldsymbol{\sigma}$ over the surface of the unit cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.
- Verify the divergence theorem.

Q.3 Given that $\mathbf{V} = (1 + y)z\mathbf{i} + (1 + z)x\mathbf{j} + (1 + x)y\mathbf{k}$, use Stokes' theorem to evaluate $\oint_C \mathbf{V} \cdot d\mathbf{r}$, where C is a closed curve in the plane $x - 2y + z = 1$. Ans: 0.

Q.4 Consider the vector field $\vec{\mathbf{V}} = (x^2 + z)\mathbf{i} + (y^2 + x)\mathbf{j} + (z^2 + y)\mathbf{k}$ and let C be the curve of intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \sqrt{x^2 + y^2}$.

- Compute $\oint_C \vec{\mathbf{V}} \cdot d\vec{\mathbf{r}}$ along the curve C directly.
- Compute $\oint_C \vec{\mathbf{V}} \cdot d\vec{\mathbf{r}}$ using Stokes' theorem.