Q.1 Given that $\boldsymbol{V} = x^2\boldsymbol{i} - (1+2x)\boldsymbol{j} + z\boldsymbol{k}$,

a) Evaluate $\iint \boldsymbol{V} \cdot d\boldsymbol{\sigma}$ over the lateral surface of the cylinder $x^2 + y^2 = 1$ between $0 \leq z \leq 2$. **Ans : 0.**

b) Verify the divergence theorem.

Q.2 Given the electric field $\boldsymbol{E} = 2xy\boldsymbol{i} - y^2\boldsymbol{j} + (z + xy)\boldsymbol{k}$,

a) Find the electric flux $\iint_S \boldsymbol{E} \cdot d\boldsymbol{\sigma}$ through the lateral surface S of the portion of the cylinder $x^2 + y^2 = 9$ bounded by the xz plane, xy plane and the plane $z = 5$.

b) Use divergence theorem to obtain the same result.

Q.3 Given that $\boldsymbol{F} = (yz)\boldsymbol{i} + (xz)\boldsymbol{j} + (xy)\boldsymbol{k}$. Evaluate $\iint_S \boldsymbol{F} \cdot d\boldsymbol{\sigma}$ over the closed surface bounded below by the paraboloid $z = x^2 + y^2$ and above by the $z = 1$ plane.

Q.4 Consider the vector field $\boldsymbol{F} = \ln(x^2 + y^2)(\boldsymbol{i} + \boldsymbol{j})$. Let C be the closed curve bounding the semi-annular region above the x-axis between $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. Compute $\oint_C \boldsymbol{F} \cdot d\mathbf{r}$. *Hint : Use Green's theorem.*

Q.5 a) Use Green's theorem to show that area enclosed by a simple closed curve C is

$$A = \frac{1}{2} \oint x dy - y dx \,.$$

b) Use the result above to find the area bounded by the curve $x^{2/3} + y^{2/3} = a^{2/3}$. (Hint : Take $x = a \cos^n \theta$ and $y = a \sin^n \theta$ for some suitable number n. **Answer:** $\frac{3}{8}\pi a^2$.

Q.6 Given the vector field $\vec{\boldsymbol{V}} = (1+y)z\boldsymbol{i} + (1+z)x\boldsymbol{j} + (1+x)y\boldsymbol{k}$. Use Stokes' theorem to compute $\oint \vec{\boldsymbol{V}} \cdot d\vec{\boldsymbol{r}}$ along a circle of radius 5 that lies in the plane $-3x + 2y + 4z = 1$ and centered at $(1, -2, 2)$.