Name and Surname:

Math 366 - Spring 2020 - METU

Quiz 1

Question: A Pythagorean triple consists of three integers x, y, and z, such that $x^2 + y^2 = z^2$. For each one of the following, either find a Pythagorean triple with given properties or explain why such a Pythagorean triple cannot exist.

- $x + y + z = 2020$ and $\gcd(x, y, z) = 101$.

Observe that $-5+12+13=20$. Multiplying everything by 101, we obtain such a triple

$(-5 \cdot 101, 12 \cdot 101, 13 \cdot 101)$

- $x + y + z = 2020$ and $\gcd(x, y, z) = 1$.

Consider the primitive triple $\left(2k+1, \frac{(2k+1)^2-1}{2}, \frac{(2k+1)^2-1}{2}\right)$

A small modification can be used to obtain

$\left(2019, -\frac{2019^2-1}{2}, \frac{2019^2+1}{2}\right)$

- $x + y + z = 2020$ and $x, y, z > 0$.

We have $x = d(m^2-n^2)$ with $m>n>0$

$y = d(2mn)$ & $d = \gcd(x,y,z)$

$z = d(m^2+n^2)$

Now $x+y+z = d(2m)(m+n) = 2020$ is not possible

- $101 \mid d \Rightarrow m(m+n) \mid 10 \Rightarrow m=1$ or 2

$\Rightarrow$ a contradiction to $m>n$

at least 101

- $101 \mid m \Rightarrow d(m+n) \mid 10 \Rightarrow$ a condrction to $m \geq 101$

- $101 \mid (m+n) \Rightarrow dm \mid 10 \Rightarrow$ a contradiction to $m \geq 51$

at least 51