M E T U
Department of Mathematics

Elementary Number Theory I Final	
Code : *Math 365* Acad. Year : *2017* Semester : *Fall* Instructor : *Küçüksakallı* Date : *January 8, 2018* Time : *13:30* Duration : *135 minutes*	Last Name : Name : Student No. : Signature : 8 QUESTIONS ON 4 PAGES 100 TOTAL POINTS

1	2	3	4	5	6	7	8		

1. (15pts) Find the last three digits of $N = 7^{799}$. In other words, find the remainder of N upon division by 1000.

2. (10pts) Suppose that $n = 79 \cdot 97$. Show that the equation $x^{365} \equiv 2017 \pmod{n}$ has a solution modulo n.

3. (15pts) Suppose that p is an odd prime, and suppose also that $q = 4p + 1$ is prime.

(i) Prove that $\left(\frac{2}{q}\right) = -1$.

(ii) Prove that 2 is a primitive root modulo q.

4. (10pts) You are given that 2 is a primitive root modulo 61. Find all positive integers less than 61 having order 10 modulo 61.

5. (15pts) In this question, fill in the blanks and prove the statements: We have

$$\left(\frac{-3}{p}\right) = \begin{cases} 1 & \text{if } \underline{\hspace{8cm}}, \\ -1 & \text{if } \underline{\hspace{8cm}}. \end{cases}$$

for each prime $p \geq 5$. If a prime number q divides an integer of the form $n^2 + 3$, then $q = 2, 3$ or $q \equiv$ ____________________________________.

6. (10pts) Recall that a Carmichael number is an odd composite number n which satisfies Fermat's little theorem $a^{n-1} \equiv 1 \pmod{n}$ for every a such that $\gcd(a, n) = 1$. Show that $n = 561 = 3 \cdot 11 \cdot 17$ is a Carmichael number.

7. (15pts) Let $n = pqr$ where p, q and r are distinct odd primes. Show that the equation

$$x^2 \equiv 1 \pmod{n}$$

has at least eight distinct solutions modulo n. (Hint: Chinese Remainder Theorem)

8. (10pts) Recall that $\sigma(n)$ denotes the sum of positive divisors of n. Let k be a fixed positive integer. Show that the equation $\sigma(n) = k$ has finitely many solutions.