M E T U
Department of Mathematics

Group	Discrete Mathematics Midterm 1		List No.
Code : *Math 112* Acad. Year : *2011* Semester : *Spring* Instructor : Bhupal, Küçüksakallı, Okutmuştur, Seven. Date : *March 31, 2011* Time : *17:40* Duration : *90 minutes*	Last Name : Name : Department : Signature :	Student No. : Section :	
	6 QUESTIONS ON 4 PAGES 60 TOTAL POINTS		

1	2	3	4	5	6		

- **Justify your answers!** Correct answers without explanation may receive no credit.

1a. (6pts) How many integers are there between 1000 and 5000 which are divisible by 3 or 7 <u>but not both</u>.

$S_3 = \{1002, 1005, \ldots, 4998\}$ has $\frac{4998-1002}{3} + 1 = 1333$ elements

$S_7 = \{1001, 1008, \ldots, 4998\}$ has $\frac{3997-1001}{7} + 1 = 572$ elements

$S_3 \cap S_7 = \{1008, 1029, \ldots 4998\}$ has $\frac{4998-1008}{21} + 1 = 191$ elements

Thus $(S_3 \cup S_7) - (S_3 \cap S_7)$ has $1333 + 572 - 2 \cdot 191 = 1523$ elements

1b. (6pts) If we pick an <u>odd</u> integer between 1000 and 5000, what is the probability that it is divisible by 7.

There are 2000 odd integers from 1001 to 4999.

We consider the sequence

$1001, 1015, 1029, \ldots, 4991$

There are $\frac{4991-1001}{14} + 1 = 286$ elements.

Thus the probability is $\frac{286}{2000} = \%14.3$

(Very close to $1/7$)

2. (12pts) 10 identical balls should be distributed among 4 children.

- **(4pts)** How many distributions are possible?

$x_1 + x_2 + x_3 + x_4 = 10$ and $0 \le x_i$

There are $\binom{13}{10}$ different solutions to this equation. Therefore there are $\binom{13}{10}$ different distributions.

- **(4pts)** What is the probability that each child gets at least one ball?

Distribute one ball to each child. There are 6 balls left. They can be distributed in $\binom{9}{6}$ ways. Thus the probability is

$$\frac{\binom{9}{6}}{\binom{13}{10}}$$

- **(4pts)** Suppose one child receives 3 balls and each ball can be painted with one of 5 different colors. How many ways can the child paint her balls, if she can choose any color any number of times and the order of choice is not important?

Seperate colors from each by using 4 bars
Then distribute 3 balls

|00| |0|

Color 1 C2 C3 C4 C5

2 (Color 2)
1 (Color 1)

There are $\binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{\cancel{3} \cdot \cancel{2} \cdot 1} = 35$ ways.

3. (9pts) How many ways are there to rearrange the string *abccc*1123 if letters must appear together? (For example, 23*ccbac*11 is acceptable but *ab*11*ccc*23 is not acceptable.)

Consider abccc as a single symbol, say L.
Then L1123 permutes in $\frac{5!}{2!}$ ways since 1's repeat two times. Then permute letters within themselves in $\frac{5!}{3!}$ ways since there are 3 c's.
Product rule implies that there are

$$\frac{5!}{2!} \cdot \frac{5!}{3!} = 1200 \text{ ways}$$

4. (9pts) Suppose a department contains 15 women and 10 men. How many ways are there to form a departmental committee with 6 members if the committee must have more women than men.

There are three possibilitis

6 W	0 M	$\binom{15}{6}$
5 W	1 M	$\binom{15}{5}\binom{10}{1}$
4 W	2 M	$\binom{15}{4}\binom{10}{2}$

Therefore the committee can be formed in

$$\binom{15}{6} + \binom{15}{5}\binom{10}{1} + \binom{15}{4}\binom{10}{2}$$

different ways.

5. (6pts) What is the coefficient of x^6 in the expansion of $(\frac{x}{5} - \frac{3}{x})^{20}$?

Note that $x^{13} \cdot \left(\frac{1}{x}\right)^7 = x^6$. Therefore the coefficient of x^6 is

$$\left(\frac{1}{5}\right)^{13} \cdot (-3)^7 \cdot \binom{20}{13}$$

by Binomial Theorem.

6. (12pts) Suppose that a fair coin is flipped 5 times.

- **(6pts)** What is the probability that there are at least three heads?

There are 3 cases

$$\left.\begin{array}{l} HHHTT \longrightarrow \frac{5!}{3!2!} = 10 \\ HHHHT \longrightarrow \frac{5!}{4!1!} = 5 \\ HHHHH \longrightarrow \frac{5!}{5!} = 1 \end{array}\right\} \longrightarrow 16 \text{ cases}$$

Since the coin is fair, each outcome is of probability $1/32$. Therefore the answer is $16 \cdot \frac{1}{32} = \frac{1}{2}$

- **(6pts)** What is the probability that there are at least three heads or at least three tails?

By Pigeonhole Principle, there are at least three heads or three tails. Therefore it is for sure that there are three flips of the same kind. The answer is 1.