MATH 120

RECITATION QUESTIONS

(WEEK 9)

1. Let
$$f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$
Show that $D_{\vec{u}}f(0,0)$ exists in every direction $\vec{u} = \langle cos\theta, sin\theta \rangle$, $0 \leq \theta \leq 2\pi$, but the formula $D_{\vec{u}}f(0,0) = \nabla f(0,0) \cdot \vec{u}$ is true only for $\vec{u} = \pm\vec{i}$ or $\vec{u} = \pm\vec{j}$.

2. Determine the points and the directions for which the directional derivative of $f(x,y) = 3x^2 + y^2$ has its largest value if (x,y) is restricted to the points on the circle $x^2 + y^2 = 1$.

3. Let $f(x,y)$ be a function having a directional derivative in every direction at $P_\circ = (a,b)$. It is also given that for $\vec{u} = \frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$ and $\vec{v} = \frac{1}{\sqrt{2}}(\vec{i} + \vec{j})$, the directional derivatives are $D_{\vec{u}}f(a,b) = \frac{1}{\sqrt{2}}$, and $D_{\vec{v}}f(a,b) = 2$. What is the maximum rate of increase of f at (a,b) ?

4. Find and classify all critical points of $f(x,y) = x^2y + xy^2 + 3xy$.

5. Find the minumum and the maximum values of $f(x,y) = 3x^2 + 3y^2 + 2xy + 1$ on the closed disk $x^2 + y^2 \leq 1$.

6. Find the maximum and the minumum values of $f(x,y,z) = x^2 + yz - 5$ on the sphere $x^2 + y^2 + z^2 = 1$.

7. Find an equation of the tangent line at $P_\circ = (1,-1,1)$ to the curve of intersection of the surfaces $x^2 + y^2 = 2$ and $y^2 + z^2 = 2$.

8. Use Lagrange multipliers to determine the point $P_\circ$ on the plane $2x - y + 3z = 5$ at which $f(x,y,z) = 4x^2 + y^2 + 3z^2$ has a local extremum. Is the local extremum you have found an absolute extremum?

9. Find the absolute extrema of the function $f(x,y) = xye^{-x-y}$ on the triangular region with vertices $(0,0)$, $(0,4)$, and $(4,0)$.

10. Express the number 12 as the sum of three positive integers x , y , and z such that xy^2z^3 is a maximum.