MATH 120

WEEK 14

(RECITATION QUESTIONS)

1. Determine whether or not $\mathbf{F}(x,y) = (x^3 + 4xy)\mathbf{i} + (4xy - y^3)\mathbf{j}$ is a conservative vector field.If it is, find a function f such that $\mathbf{F} = \nabla f$

2. (a) Find a function f such that $\mathbf{F} = \nabla f$ and

   (b) Use part (a) to evaluate $\int_C \mathbf{F} \bullet d\mathbf{r}$ along the given curve C.

   where

   i) $\mathbf{F}(x,y) = x^3y^4\mathbf{i} + x^4y^3\mathbf{j}$ and $C : \mathbf{r}(t) = \sqrt{t}\mathbf{i} + (1+t^3)\mathbf{j}$, $0 \le t \le 1$

   ii) $\mathbf{F}(x,y,z) = (2xz + y^2)\mathbf{i} + 2xy\mathbf{j} + (x^2 + 3z^2)\mathbf{k}$ and $C : x = t^2,\ \ y = t+1,\ \ z = 2t - 1$, $0 \le t \le 1$

3. (If time permits)Show that the line integral is independent of path and evaluate the integral

$$\int_C (1 - ye^{-x})dx + e^{-x}dy$$

   C is any path from $(0,1)$ to $(1,2)$

4. Find the work done by the force field $\mathbf{F}$ in moving an object from P to Q where

$$\mathbf{F}(x,y) = (y^2/x^2)\mathbf{i} - (2y/x)\mathbf{j};\ \ P(1,1),\quad Q(4,-2)$$

5. Evaluate the line integral $\oint_C y\,dx - x\,dy$ where C is the circle with center the origin and radius 1, by two methods

   a) Directly

   b) Using Green's Theorem

6. Use Green's Theorem to evaluate $\int_C \mathbf{F} \bullet d\mathbf{r}$.(Check the orientation of the curve before applying the theorem.)

$$\mathbf{F}(x,y) = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$$

   , C consists of the arc of the curve $y = \sin x$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$

7. Use Green's Theorem to find the work done by the force $\mathbf{F}(x,y) = x(x+y)\mathbf{i} + xy^2\mathbf{j}$ in moving a particle from the origin along the $x-$axis to $(1,0)$, then along the line segment to $(0,1)$, and then back to the origin along the $y-$axis.