Calculus 119
9th Week[1]

1. Use the subsutitution $t = \frac{3}{2-x}$. Since $t = \frac{3}{2-x} \to +\infty$ when $x \to 2^-$,

$$\lim_{x\to 2^-} e^{\frac{3}{2-x}} = \lim_{t\to +\infty} e^t = \infty$$

2.

$$y' = \frac{d}{dx}(e^{k\tan\sqrt{x}}) = e^{k\tan\sqrt{x}}\frac{d}{dx}(k\tan\sqrt{x}) = e^{k\tan\sqrt{x}}k(\sec^2\sqrt{x})\frac{d(\sqrt{x})}{dx}$$

$$= ke^{k\tan\sqrt{x}}\sec^2\sqrt{x}\frac{1}{2\sqrt{x}} = \frac{k}{2\sqrt{x}}e^{k\tan\sqrt{x}}\sec^2\sqrt{x}$$

3. By using subsutitution $u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}dx$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = \int e^u(2du) = 2e^u + c = 2e^{\sqrt{x}} + c$$

4.

$$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))}$$

Let's find $f^{-1}(4)$
$f^{-1}(4) = x \Rightarrow f(x) = 4 \Rightarrow 3 + x + e^x = 4 \Rightarrow x = 0$ and
$f'(x) = 1 + e^x \Rightarrow f'(0) = 2$ so,

$$(f^{-1})'(4) = \frac{1}{f'(0)} = \frac{1}{2}$$

5.

$$2\ln x = \ln 2 + \ln(3x-4) \Rightarrow \ln x^2 = \ln 2 + \ln(3x-4) \Rightarrow \ln x^2 - \ln(3x-4) = \ln 2$$

$$\Rightarrow \ln\frac{x^2}{3x-4} = \ln 2 \Rightarrow \frac{x^2}{3x-4} = 2 \Rightarrow x^2 - 6x + 8 = 0 \Rightarrow x = 4 \quad or \quad x = 2$$

[1] Please email all corrections and suggestions to these solutions to htor@metu.edu.tr. All solutions are available on the web at the url http://www.metu.edu.tr/~htor.

6.

$$\lim_{x\to\infty}[\ln(2+x)-\ln(1+x)] = \lim_{x\to\infty}\ln(\frac{2+x}{1+x})$$

Note that as $x \to \infty$, $\frac{2+x}{1+x} \to 1$

$$\Rightarrow \lim_{x\to\infty}[\ln(2+x)-\ln(1+x)] = \ln 1 = 0$$

7.

$$x^y = y^x \Rightarrow \ln x^y = \ln y^x \Rightarrow y\ln x = x\ln y$$

Using implicite derivative,

$$y'\ln x + y\frac{1}{x} = \ln y + x\frac{y'}{y} \Rightarrow y' = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$$

8.

$$f(x) = x\ln x \Rightarrow f'(x) = \ln x + x\frac{1}{x} = \ln x + 1$$

Critical numbers;

$$f'(x) = 0 \Rightarrow \ln x + 1 = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1} \text{is a C.N}$$

The domain of ln is $(0, -\infty)$, so $D(f) = (0, -\infty)$

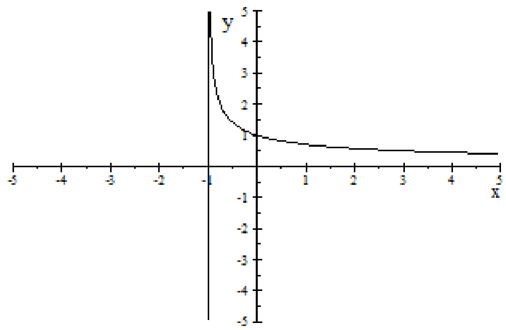
	0	$\frac{1}{e}$	∞
$f'(x)$	$-$		$+$
$f(x)$	$\searrow$		$\nearrow$

$f(\frac{1}{e}) = \frac{1}{e}\ln\frac{1}{e} = -\frac{1}{e}$. Thus, $(\frac{1}{e}, f(\frac{1}{e})) = (\frac{1}{e}, -\frac{1}{e})$ absolute minimum point.

9. By using subsititution $u = \ln x \Rightarrow du = \frac{1}{x}dx$

$$\int_e^6 \frac{dx}{x\ln x} = \int_{\ln e}^{\ln 6}\frac{du}{u} = (\ln u)|_1^{\ln 6} = \ln(\ln 6) - \ln 1 = \ln(\ln 6)$$

10. The graph of $y = \frac{1}{\sqrt{x+1}}$ is the following



$$Volume = \int_0^1 \pi(\frac{1}{\sqrt{x+1}} dx = \int_0^1 \pi \frac{1}{x+1} dx = \pi \ \ln(x+1)|_0^1 = \pi \ln 2 - \ln 1 = \pi \ln 2$$