Calculus 119
3rd Week[1]

1. Let's first obtain the formula.

$$x^4 + y^4 = x^2y^2$$
$$\Rightarrow 4x^3 + 4y^3y^{'} = 2xy^2 + x^2 2yy^{'}$$
$$\Rightarrow y^{'} = \frac{2x^3 - xy^2}{x^2y - 2y^3}$$

Why is this result meaningless?
This result meaningless, because the solution set of the equation $x^4 + y^4 = x^2y^2$ consists of a single point as seem below.

$$x^4 + y^4 = x^2y^2 \Rightarrow x^4 - 2x^2y^2 + y^4 + x^2y^2 = 0$$
$$\Rightarrow (x^2 - y^2)^2 + x^2y^2 = 0$$
$$\Rightarrow (x^2 - y^2)^2 = 0 \ and \ x^2y^2 = 0$$

since both of $(x^2 - y^2)^2 \ and \ x^2y^2$ are positive.

$$\Rightarrow x = 0 \ and \ y = 0$$

2. By using implicit differentiation, we obtain the following equality

$$cos(x + 2y)(1 + 2y^{'}) = 2cos(y) + 2x(-sin(y)y^{'})$$

The slope of the tangent line at point (0,0) (i.e origin) is $\frac{\partial y}{\partial x}|_{(0,0)}$. It can be computed by replacing (0,0) with (x,y)

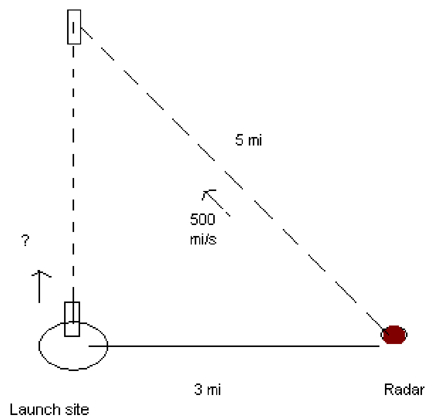
$$\frac{\partial y}{\partial x}|_{(0,0)} = y^{'}(0) = \frac{1}{2}$$

Consequently, The equation is

$$y = \frac{1}{2}x$$

[1]Please email all corrections and suggestions to these solutions to htor@metu.edu.tr. All solutions are available on the web at the url http://www.metu.edu.tr/~htor.

3. The problem can be understood from the below figure.



From Pythagorean Theorem

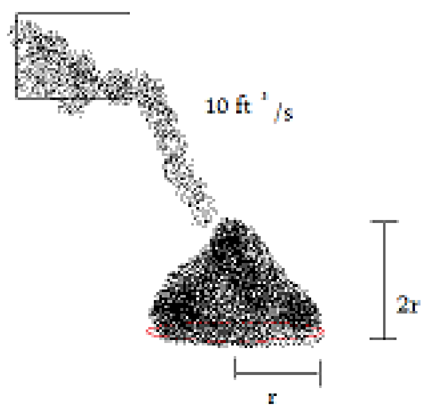
$$3^2 + y^2 = z^2$$

$$\Rightarrow 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\Rightarrow 8\frac{dy}{dt} = 10.5000 \quad since\, y = 4\, when\, z = 5$$

the vertical speed of the rocket$= \dfrac{dy}{dt} = 6250\, mi/s$

4. The problem can be understood from the below figure.



$$\text{The volume of a conic} = V = \frac{1}{3}\pi r^2 h = \frac{2}{3} r^3 \pi$$

$$\Rightarrow V^{'} = 2r^2 \pi r^{'} \quad (1)$$

where $v^{'}$ and $r^{'}$ denote the derivative of the volume and radius with respect to time (second) respectively.
Since the height is 5 ft (i.e the radius is 5/2 ft)

$$\Rightarrow 10 = 2(\frac{5}{2})^2 \pi r^{'} \quad from(1)$$

$$\Rightarrow r^{'} = \frac{4}{5\pi} ft/s$$

5.

$$y = Acos(kt) + Bsi(kt)$$

$$\Rightarrow y^{'} = -Aksin(kt) + Bkcos(kt)$$

$$\Rightarrow y^{''} = -Ak^2 cos(kt) - Bk^2 sin(kt) \quad (2)$$

From (2) and the equation,associated with y, given in the question

$$\frac{\partial^2 y}{\partial t^2} + k^2 y = -Ak^2 cos(kt) - Bk^2 sin(kt) + k^2(Acos(kt) + Bsin(kt))$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} + k^2 y = 0$$

6.

$$x^2 - xy + y^2 = 0$$

$$\Rightarrow 2x - (y + xy^{'}) + 2yy^{'} = 0 \quad By\ using\ implicit\ differentiation$$

$$\Rightarrow y^{'} = \frac{2x - y}{x - 2y} \tag{3}$$

$$\Rightarrow y^{''} = \frac{(2 - y^{'})(x - 2y) - (2x - y)(1 - 2y^{'})}{(x - 2y)^2}$$

$$\Rightarrow y^{''} = \frac{(2 - \frac{2x-y}{x-2y})(x - 2y) - (2x - y)(1 - 2\frac{2x-y}{x-2y})}{(x - 2y)^2} \quad from(3)$$

After arrenging, we obtain the following equation

$$\Rightarrow y^{''} = \frac{6(x^2 - xy + y^2)}{(2x - y)^3}$$

$$\Rightarrow y^{''} = \frac{54}{(2x - y)^3}$$

since $x^2 - xy + y^2 = 9$ from the equation in the question

7. $f(x) \approx L(x) = f(a) + (x - a)f^{'}(a)$ Taking $f(x) = cos(x)$, $a = \frac{\pi}{4}$ and $x = \frac{\pi}{4} - \frac{\pi}{90}$

$$f(\frac{\pi}{4} - \frac{\pi}{90}) \approx L(\frac{\pi}{4} - \frac{\pi}{90}) = f(\frac{\pi}{4}) + ((\frac{\pi}{4} - \frac{\pi}{90} - \frac{\pi}{4})f^{'}(\frac{\pi}{4}))$$

$$\Rightarrow cos(43^o) = f(43^o) = f(\frac{\pi}{4} - \frac{\pi}{90}) \approx cos(\frac{\pi}{4}) - \frac{\pi}{90}(-sin(\frac{\pi}{4}))$$

$$\Rightarrow cos(43^o) \approx \frac{\sqrt{2}}{2} + \frac{\pi}{90}\frac{\sqrt{2}}{2} = 0,9538$$

8. When $h = 0$, $a = g(\dfrac{R}{R+0})^2 = 32$ When $h = 10$, $a = g(\dfrac{R}{R+10})^2 \approx 32(\dfrac{3960}{3970})^2 \approx 31,8389$ So the dicreasing is $32 - 31.8389 = 0,1611$

$$\left.\begin{array}{ll} 32 & 0,1611 \\ 100 & ? \end{array}\right\} \Rightarrow a\ will\ decrease\ by\ 0,5\%$$