

2007-2008 FALL

MATH 119

WEEK 3

RECITATION QUESTIONS

1) Blind application of implicit differentiation to the equation  $x^4 + y^4 = x^2y^2$  leads to the formula

$$\frac{dy}{dx} = \frac{2x^3 - xy^2}{x^2y - 2y^3}$$

Why is this result meaningless ?

2) Write an equation of the tangent line to the curve which is the graph of the equation

$$\sin(x + 2y) = 2x \cos y$$

at the origin. (Ans:  $y = \frac{1}{2}x$ )

3) A rocket that is launched vertically is tracked by a radar station located on the ground 3 *mi* from the launch site. What is the vertical speed of the rocket at the instant that its distance from the radar is 5 *mi* and this distance is increasing at the rate of 5000 *mi/h* ? (Ans: 6250 *mi/h*)

4) Sand is being emptied from a hopper at the rate of 10  $ft^3/s$ . The sand forms a conical pile whose height is always twice its radius. At what rate is the radius of the pile increasing when its height is 5 *ft* ? (Ans:  $\frac{4}{5\pi}$  *ft/s*)

5) Show that if  $A, B,$  and  $k$  are constants and

$$y = A \cos(kt) + B \sin(kt)$$

then  $y$  satisfies the second order differential equation of simple harmonic motion

$$\frac{d^2y}{dt^2} + k^2y = 0$$

6) Find  $y'' = \frac{d^2y}{dx^2}$  if  $y = y(x)$  is defined implicitly by the equation

$$x^2 - xy + y^2 = 9$$

(Ans:  $\frac{d^2y}{dx^2} = -\frac{54}{(2y-x)^3}$ )

7) Use a linear approximation  $L(x)$  to an appropriate function  $f(x)$ , with an appropriate value of  $a$ , to estimate the number  $\cos 43^\circ$

8)(Acceleration of gravity)The acceleration  $a$  of gravity at an altitude of  $h$  miles above the surface of the earth is given by

$$a = g\left(\frac{R}{R+h}\right)^2$$

where  $g \cong 32ft/s^2$  is the acceleration at the surface of the earth, and  $R \cong 3960$  miles is the radius of the earth. By about what percentage will  $a$  decrease if  $h$  increases from 0 to 10 miles ?

(Ans:  $-0.5\%$  approximately )