

Partial fractions, improper integrals

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Problem 1. Compute the following indefinite integrals.

(a) $\int \frac{x^3 - x}{x^2 - x - 6} dx$

(d) $\int \frac{x^4 - 3x^3 + 4x^2 - 3x + 1}{x^4 + 3x^3 + 4x^2 + 3x + 1} dx$

(b) $\int \frac{x}{x^2 + x + 1} dx$

(e) $\int \frac{4x^3 + 9x^2 + 8x + 3}{x^4 + 3x^3 + 4x^2 + 3x + 1} dx$

(c) $\int \frac{x}{(x^2 + x + 1)^2} dx$

Problem 2. Assuming $\deg f < 8$, analyse

$$\frac{f(x)}{(x^2 + 4x - 5)^2 \cdot (x^2 + 4x + 5)^2}$$

as a sum of partial fractions. (No need to compute the numerators.)

Problem 3. Evaluate the following definite integrals.

(a) $\int_0^{32} \frac{dx}{1 + x^{1/5}}$

(b) $\int_0^{\pi/2} \frac{dx}{2 + \sin x}$

Problem 4. Evaluate, if possible, the following improper integrals.

(a) $\int_{-1}^1 \frac{dx}{x^3}$

(c) $\int_{-\infty}^{\infty} \frac{dx}{|x^3| \cdot e^{1/x^2}}$

(b) $\int_{-\infty}^{\infty} \frac{dx}{x^3 \cdot e^{1/x^2}}$

Problem 5. Determine whether the following improper integrals converge.

(a) $\int_0^{\infty} \frac{\sin(x^3)}{x + e^x} dx$

(c) $\int_1^{\infty} \frac{dx}{x^2 - x^{-2}}$

(b) $\int_2^{\infty} \frac{dx}{x^2 - x^{-2}}$