

# THE DESIGN OF OPTIMAL DEMAND MANAGEMENT PROGRAMS

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## Abstract

Reliable operation in a deregulated environment and under conditions of uncertainty requires that loads be considered adjustable. This paper assumes that participation in demand management programs is entirely voluntary, and the compensation for participation is an integral part of any demand management program. Another consideration is simplicity: demand management programs must be simple to understand, simple to use and reliable in their response.

This paper considers and describes a variety of voluntary demand management programs, including full interruption, equipment specific partial interruptions, limits in use of current and/or power, and programs that guarantee a certain “relief performance”. It also considers a random mix of provider characteristics and demand characteristics. The paper illustrates that optimality (or near-optimality) requires not one but a small portfolio of incentive programs. It further illustrates that regulatory incentives may be required to “line up” utility optimality with customer optimality.

**Keywords:** Demand management, demand relief, social optimality, interruption, curtailment, system security.

## Introduction

In today’s ever-changing electric power needs, and with the added complications from deregulation, the utilities and the customers are in need of efficient demand management programs. The customers count on the utilities for reliable service, however the utilities are finding it harder to meet the demand and provide reliable service. Some customers have back up generation [3] other customers have demand management programs. If the utilities look into customer outage costs [5] [3] and analyze their load behavior they can design several

different kinds of demand management programs and try to attract selected customers to help in case of emergencies in return for an incentive fee.

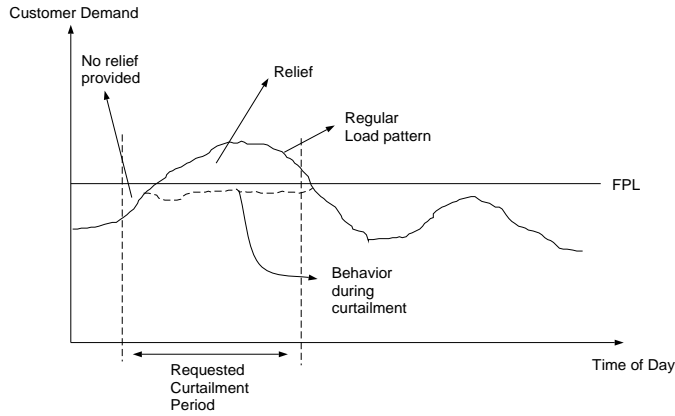
The main theme of the paper is the design of a small set of near-optimal demand management programs which are based on a mix of more than one simple program. The social benefits that are attained by these programs are studied under a variety of conditions, and a general method for designing demand management programs for different types of systems is described. Finally, methods and policies for assuring that the interests of suppliers always align with the interests of the producers are also described.

## 1 Demand Management Programs

Maximum customer benefit derives from minimum cost and sufficient supply availability. At times, lower cost implies risk of potential unavailability of enough power. Customers willing to share in “availability risk” can derive further benefit by way of participating in controlled outage programs. Specifically whenever utilities foresee dangerous loading patterns, there is a need for a rapid reduction in demand either system-wide or at specific locations. The utility needs to get some relief in order to solve its problems quickly and more efficiently without wasting energy. This relief can come from customers who agree to curtail their loads upon request in exchange for an incentive fee.

There are many different ways to implement controlled outages. In this paper we define, consider and compare three types of programs:

- Firm Power Level programs
- Agreed Relief programs
- Equipment Specific Interruption programs



**Fig. 1:** Firm Power Level Program

## 1.1 Firm Power Level Programs

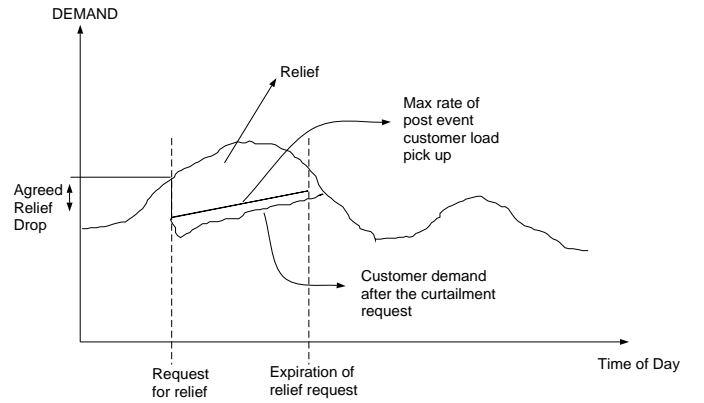
The Firm Power Level program defines a maximum (or firm) power level (FPL) for each customer. During an interruption request the customers in this program are required to reduce their demand to their preagreed FPL or below (see Fig. 1). The means by which this is achieved are unimportant at this point<sup>1</sup>. The levels of FPL which are socially optimal, and the policies that attain this optimality are derived and discussed in the appendix.

### 1.1.1 Observations about FPL programs

- The main problem about this program is the accuracy of the load estimation in the absence of an interruption. The utility will not know the exact amount of load relief obtained unless load estimation is done accurately.
- Depending on the customer demand at the time, if the customer demand is below the contracted threshold the customer might not provide any relief.
- Easy to administer
- Relatively easy to enforce compliance.

The study in [6] shows that the largest amount of load relief from this type of program is likely to be generated by large customers with high load factors in the process industries.

<sup>1</sup>One possible method is to use adjustable “current limiters” at the customers load connection points. These limiters would temporarily disconnect and re-connect customers that fail to respond to the contracted request.



**Fig. 2:** Agreed Relief Program with initial relief specified as well as mandated maximum rate of post event customer load pick up

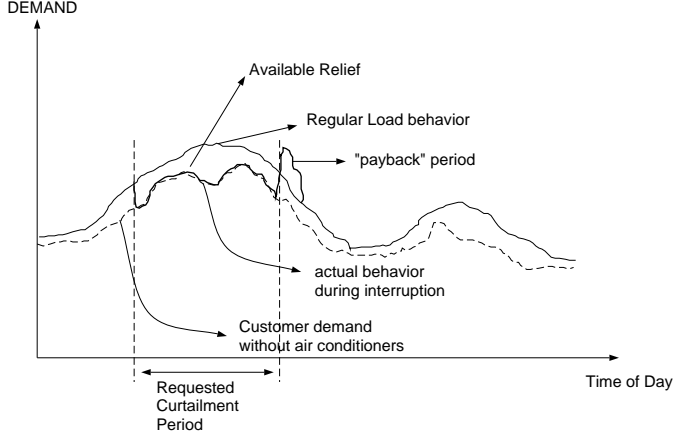
## 1.2 Agreed Relief Programs

The Agreed Relief Program shares some similarities with the FPL program. When customers receive a relief request, they shed a predetermined amount of load from their demand level at the time of the curtailment request and have a sloping upper limit to their demand pick up during the requested curtailment period (see Fig. 2). After the curtailment ends they may resume their typical demand.

The sensitive parameters in this program is the preagreed amount of drop and the slope of the envelope the customer has to stay under. Another option could be implemented so that the envelope is not sloping and the customer just stays under the fixed drop during the curtailment request.

### 1.2.1 Observations about Agreed Relief programs

- This program might provide more predictable relief than the FPL program since the customer has to drop the agreed amount of load (if possible) and stay below a preagreed maximum demand level, where as in the FPL program the customer might have already been under the FPL at the time of the request.
- Administration would be more difficult than the FPL programs.
- This program seems particularly well suited to customers with their own back-up generation capabilities.



**Fig. 3:** *Equipment Specific Interruption Program*

### 1.3 Equipment Specific Interruption Program

This program is different in the sense that there are no parameters to decide upon. However the customer can decide what equipment to shut off upon request. Air conditioners and heaters would be among the best candidates since they put a big burden on the utility. Figure 3 shows a predicted behaviour under such a program.

Once again the issue here is the incentive paid to the customer and the duration of the interruption. Since this involves a specific kind of equipment the duration becomes much more critical.

It is very difficult to forecast the amount of relief that could be obtained from such a program. For example if the program is to turn off air conditioners, there are lots of factors to be considered:

- Temperature
- Sun
- Wind conditions
- Efficiency
- Economy
- Time of day

When the air conditioners are turned off, there will be an initial relief. However when they turn back on, they might cause a spike in demand (a “payback”) if they all come on at the same time. A re-connection management strategy is required.

## 2 Dangerous Situations for a Utility

In a complex system like a power system, there are many factors that could lead to big problems for a utility. Early detection or anticipation of these problems can lead to problem prevention by way of load curtailment. The nature of these problems might be very different from each other, hence each problem needs a different kind of response from the utility and eventually the customer who is under contract to help. We organize those problems that can be helped by utility customer interaction under 4 categories:

1. System-wide generation insufficiency
2. Localized problems
3. Cascading line outages due to overloads or instability
4. Distribution problems

All four problems (but particularly (3)) could eventually lead to a total black out of the system. Each of these problems needs to be handled differently, some need to be addressed in a matter of minutes and some may need attention in a matter of seconds. The utility may need a different plan for each type of problem.

## 3 Detection of most valuable loads in the system

In [7], the authors compute the sensitivity of the loading margin to voltage collapse with respect to arbitrary parameters. If the loads in the system are chosen to be the parameters, then the sensitivity of the loading margin can be computed with respect to each load. Following the formulation in [7], let the power system equations be:

$$f(x, \lambda, p) = 0 \quad (1)$$

where  $x$  is the vector of state variables,  $\lambda$  is the vector of real and reactive load powers, and  $p$  is the vector of load parameters. If a pattern of load increase is specified with a unit vector  $k$ , the point of collapse method, [2],[4] could be applied to yield the left eigenvector  $w$ , then the sensitivity of the loading margin to a change in any load is:

$$L_p = \frac{-\omega f_p}{\omega f_\lambda k} \quad (2)$$

The derivation of the above formula is explained in detail in [7]. Once we have the sensitivity of the loading margin to a change in any of the loads, we could use it to rank the system loads. Let  $L$  be the loading margin of the system. The above formula lets us construct a simple expression relating changes in individual loads ( $\Delta p_1$ ,  $\Delta p_2$ , etc) to changes in the security margin:

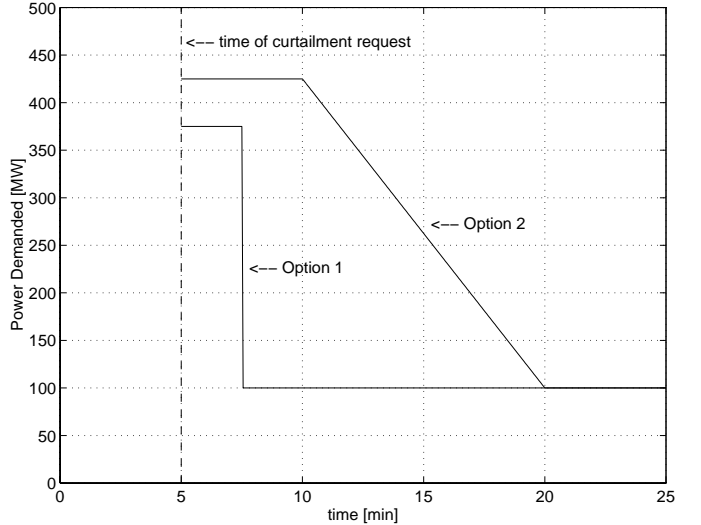
$$\Delta L = L_{p_1} \Delta p_1 + L_{p_2} \Delta p_2 + \dots + L_{p_m} \Delta p_m \quad (3)$$

where  $m$  is the number of loads we are interested in. As equation 3 suggests, the load with the highest sensitivity would help increase the loading margin the most. Hence a ranking of the loads could be obtained from the sensitivity formula and the utility would know which customers are more valuable when it offers contracts for interruptible load.

As mentioned in earlier sections there are many ways of curtailing load. Our goal is to analyze the different kinds of load behaviour after a curtailment request is received, and then match this behaviour with what the utility may need. This will lead to an ability to elaborate and compare the effectiveness of demand management programs that may be offered. It will also lead us to a method for the selection of which program to call upon under specific circumstances. The following section will look into different customer demand behaviour and suggest a different kind of demand management program for each kind.

#### 4 Curtailment options for participating customers

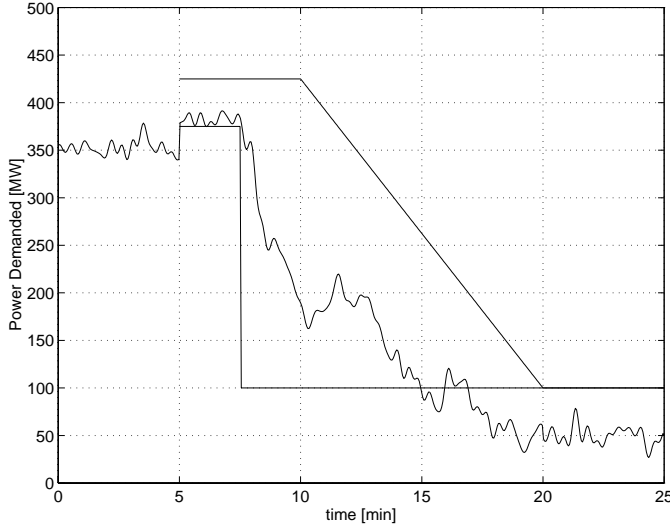
There are many different kinds of customers that could participate in the customer programs that offer load relief in one way or the other. As this paper will illustrate it is not optimal to only offer one kind of option to accomplish curtailment. In order to implement these kind of programs, several parameters must be set and certain rules must be followed. This section explains what kind of curtailment sequences the customers could follow and what options and limitations they have. Figure 4 shows two different “options” customers could sign up for. It should be obvious that customers signing up for option 1 should require a greater incentive since they are providing more immediate relief by curtailing their loads in a much shorter time.



**Fig. 4:** *Curtailment options and their boundaries. Two different FPL or Agreed Relief program options.*

It is assumed that most customers that participate in this kind of program have back up generation or good energy management systems. This provides the likelihood of several scenarios to complete the curtailment, the choice of scenario followed depends on the type of customer and the level of energy management programs they may have. In order to span the different kind of scenarios the following examples are given:

1. Some customers have back up generators that need to be started up by using electric power or have lengthy energy-consuming shut down procedures. This type of customer needs the ability to gradually switch to back up generation to provide relief to the utility. At the time of the curtailment request they may even temporarily draw more power from the network for a short while until they get their generators started and then they start curtailing until they go below the FPL. This is shown in figure 5, the two different options are shown and as it is seen the customer should sign up for option 2 since it violates the boundaries for option 1.
2. Some customers have the ability to instantly or rapidly shut down equipment or have back up generators that could be started immediately but may need some time to get their back up generators to warm up and start pick-



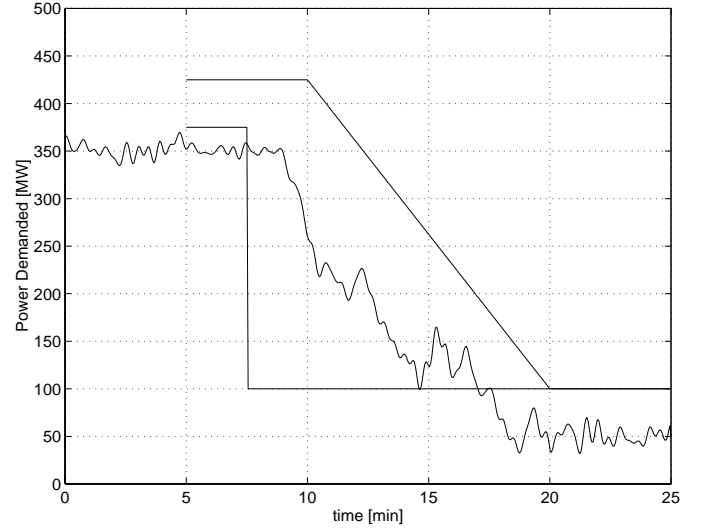
**Fig. 5:** *Curtailment options: customer can shut down slowly and may increase its demand temporarily*

ing up load. This kind of load behaviour is shown in figure 6. This customer needs to sign up for option 2.

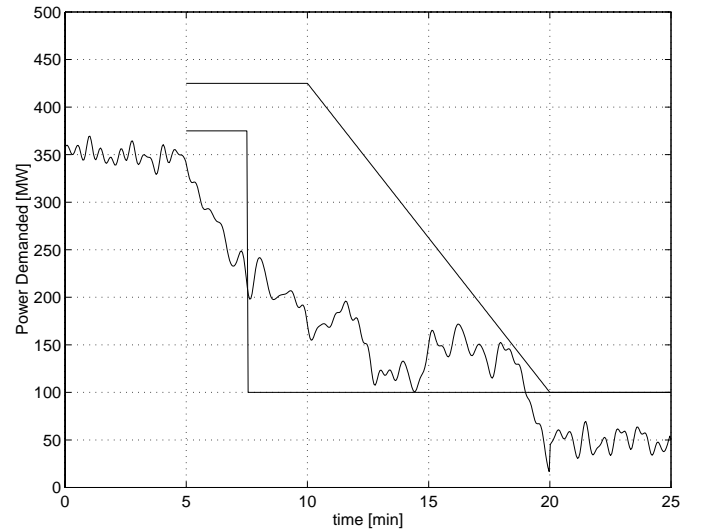
3. There are customers that have stand by back up generators that are already started up ready to pick up load. After they receive the curtailment request they are ready to switch to back up generation right away. The two ways of doing this is shown in figures 7 and 8. The customer who behaves like the one in figure 7 may sign up for option 2, however the customer in figure 8 can also sign up for option 1 if there is a greater incentive.
4. Some customers do not have any kind of back up generation, but can still provide relief by shedding some of their load. The scenario of curtailment they follow depends on how quickly they can shed load, hence they could follow any of the examples shown in figures 6, 7 and 8, and depending on which boundaries they violate they can choose to sign up for the most appropriate curtailment option.

There are other possibilities for customer scenarios. However most customers can probably fit into the categories shown above.

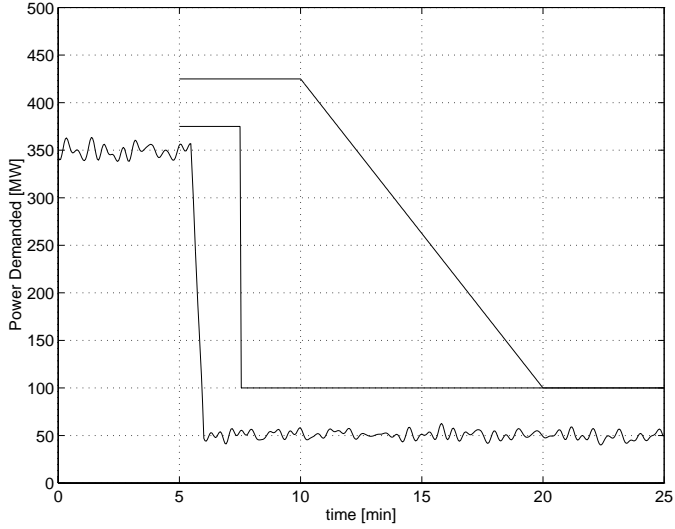
After the customers obey the curtailment request and fall below their FPL or drop the agreed amount of load, they stay below the boundaries for



**Fig. 6:** *Curtailment options: customer can reduce load slowly, but it requires some time to start shedding load.*



**Fig. 7:** *Curtailment options: customer can reduce load gradually but does not need extra time to start lowering load*

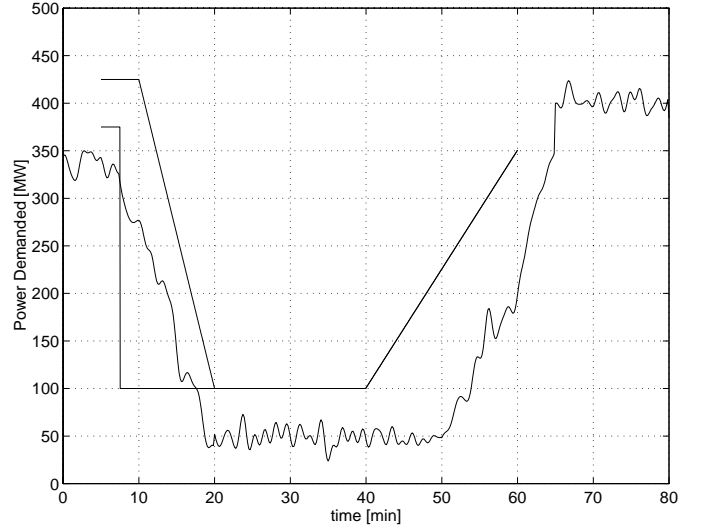


**Fig. 8:** *Curtailment options: customer can reduce the load to the desired level very rapidly without any delay or extra power*

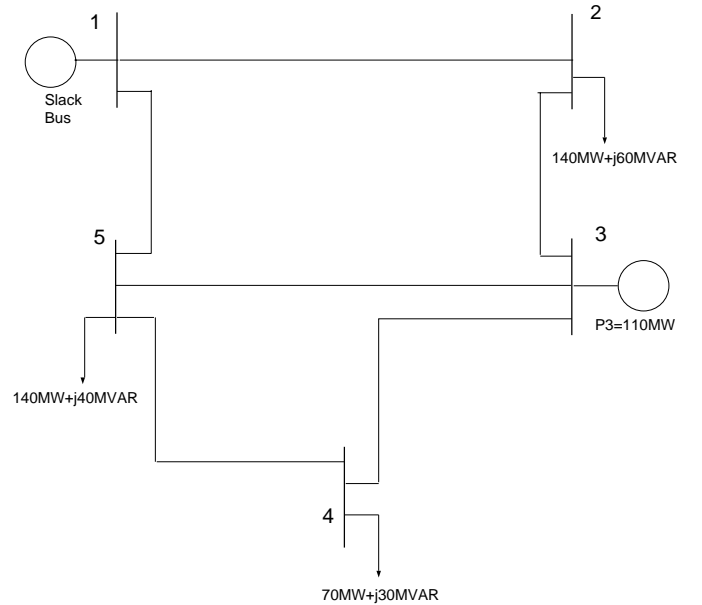
the agreed amount of time. When the curtailment period is over, the utility must set another boundary for the customers as they ramp up to resume their normal load. The ending for both programs have the same boundaries. This is shown in figure 9. The ending of the curtailment request needs to have a slow ramping boundary since the utility does not want any spikes in demand as soon as the curtailment ends. These curtailment options are designed for the demand management programs so that every kind of customer could join and help by providing load relief.

## 5 Example

An example uses a 5 bus system (see fig. 10) with 2 generators and 3 loads, table 1 has the branch data for the system. The system has the generator at bus 1 to be the slack. If we increase the load equally on each load bus and only let the slack pick up the extra load the sensitivity of the loading margin to a change in each load is calculated and shown in table 2. Then another test was run where each load was increased by keeping the other loads constant, and  $\Delta P$  for each load that led to a black out was recorded, see table 3. Both table 2 and table 3 suggests that the most valuable load in the system is the load at bus 4. In other words the load at bus 4 would give the utility the most bang for the buck. It would be optimal to



**Fig. 9:** *A complete scenario for a customer (signed up for option 2) obeying the beginning and ending boundaries of a curtailment request*



**Fig. 10:** *Example 5-bus system*

offer load 4 an option 1 agreed relief program contract (as explained in previous sections). If this is not possible, load 4 should be on either one of the other curtailment programs (Firm Power Level or equipment specific).

If the system gets close to a voltage bifurcation point [1] the utility would want to curtail a guaranteed amount of load, and this would come from the customers enrolled in the agreed relief program, and if they want fast relief they look to their option 1 customers. Hence it is important to determine the ranking of loads before the utility offers curtailment contracts. After the contracts are signed, an algorithm could be developed to check for distance to collapse and suggests the optimum curtailment order of loads to move away from collapse points.

Table 1: Branch Data on the 5-bus system

from	to	R [pu]	X [pu]	charging [pu]
1	2	0.042	0.168	0.0205
1	5	0.031	0.126	0.0155
2	3	0.031	0.126	0.0155
3	4	0.084	0.336	0.0411
3	5	0.053	0.21	0.0255
4	5	0.063	0.252	0.0305

Table 2: Load Sensitivity Information

Load bus	Sensitivity
2	-0.4644
4	-1.9413
5	-0.6246

Table 3: Load margin for each load (while the other two are kept constant)

$\Delta P_2$	315 MW
$\Delta P_4$	130 MW
$\Delta P_5$	290 MW

## 6 Algorithm for designing near-optimal curtailment programs

1. Determine the ranking of loads by computing sensitivities.
2. Offer option 1, agreed relief program contracts to most valuable loads.
  - If it's not accepted, try to sign them up to one of the other kind of contracts.
3. Once the valuable loads are signed up, implement the method in [1] to check for closest bifurcations in the system.
4. If the system is close to a bifurcation point, start calling upon the curtailment customers in the order of their ranking until a safe distance from the voltage bifurcation point is reached. Once the sensitivity of the loading margin to each load is known, the utility could estimate the distance they move away from the bifurcation point (see eqn 3). In the case of agreed relief program customers the curtailed load is known, however the customers who signed up for the other curtailment programs provide relief that is hard to estimate.

## 7 Conclusion

Existing bottlenecks in transmission often make it hard for utilities to buy power from neighboring utilities. Hence when the utility needs load relief or more local generation, the best place to get it is often from the customers themselves. The appendix shows that by performing an optimal incentive analysis we can modify free market programs so that “utility optimal conditions” and “customer optimal incentives” coincide with “socially optimal” behavior, without explicit mandates to do so. The optimal incentive analysis was done only on the FPL program. Optimality is not attainable with a single program. Rather, it requires a portfolio of demand management programs. Since every customer has a different load characteristic, different demand management programs need to be designed to include different kinds of customers to maximize the amount of load relief. If the right loads are contracted with the designed demand management programs (with an optimized incentive cost), a utility could increase the safety

margin of the power system by spending the least amount of money and thus minimize the occurrence of outages. A further observation (implicit in the appendix) is that social optimality may require “rules” that help “line up” consumer interests with utilities interests.

### Acknowledgement

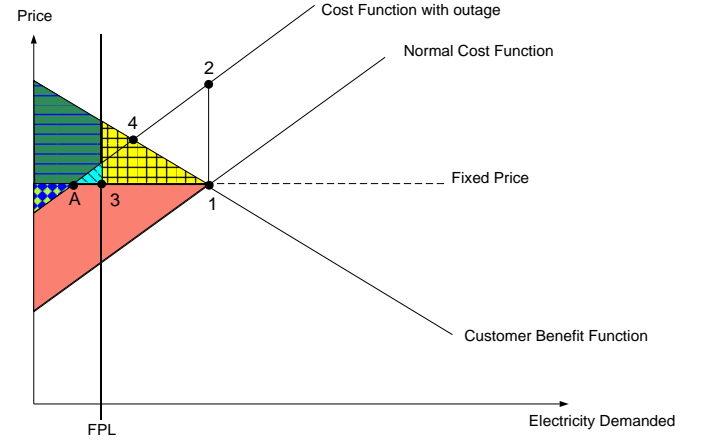
We acknowledge the support of Energy Center of Wisconsin.




### Appendix: The Derivation of an Optimal FPL level



Setting the FPL (Firm Power Level) for each participating customer plays a critical role in this program. Figure 11 shows how the customer and producer benefits would be affected by the level of FPL. Things are ideal at operating point 1 as everything seems to be in equilibrium. However, a contingency (or an outage) could cause the utility cost function to shift up. Since the price stays fixed the customer will not change its demand level causing the utility to operate at point 2 with an undesirable loss and a higher risk for more contingencies. When this happens, the utility can call upon the customer to lower their load to the preagreed FPL level, which is shown in Fig. 11 as operating point 3.




Careful study of Fig. 11 shows that the optimal level of FPL for the utility should be point A and the optimal level of FPL for the customer should be point 1. Hence a criterion should be set to determine a new optimum. This new point should be no other than the societal optimum which is shown as point 4 in Fig. 11. Societal benefit is defined to be the sum of customer and utility benefits, and the societal optimum should occur at the new equilibrium of the customer benefit function and the shifted utility cost function. These observations will also be proven mathematically in this appendix. Figure 12 shows the graph and the information that will be used to perform a mathematical analysis of the FPL program.

It can be seen that  $\lambda$  = normal equilibrium price,  $D_e$  = equilibrium demand (optimal FPL level for the customer),  $h$  = optimal FPL level for the utility, and  $D_{e2}$  is the new equilibrium demand after the shift in the utility cost function. This new







Operating Point 1 (Ideal Situation): Customer Benefit =  +  + 

Producer Benefit =  + 

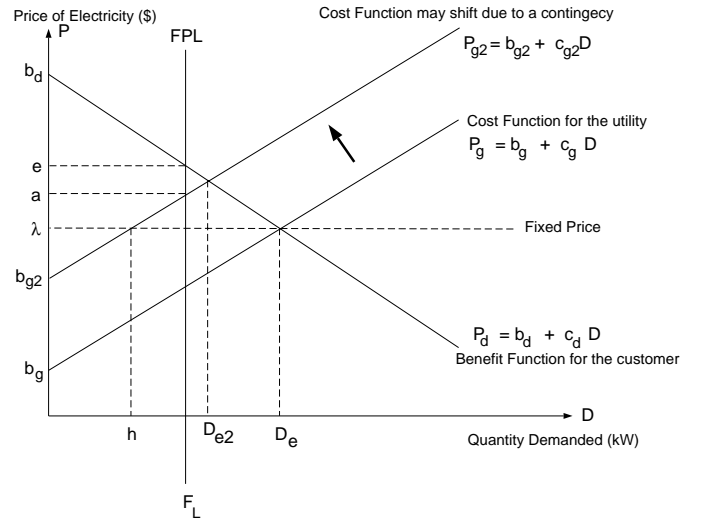
Operating Point 2 (with outage): Customer Benefit =  +  + 

Producer Benefit =  - (A12 triangle)

Operating Point 3 (with outage and FPL): Customer Benefit =  + 

Producer Benefit =  - 

**Fig. 11: Customer and Producer Benefits**



**Fig. 12: Firm Power Level Analysis**



equilibrium is predicted to be the socially optimum level of FPL, which we will soon show mathematically. For the purposes of formulation FPL is represented by  $F_L$ .

Let,

Normal generation marginal cost function:

$$P_g = b_g + c_g D \quad (4)$$

Outage generation marginal cost function:

$$P_{g2} = b_{g2} + c_{g2} D \quad (5)$$

Normal customer marginal benefit function:

$$P_d = b_d + c_d D \quad (6)$$

and some other defined points in Fig. 12,

$$e = b_d + c_d F_L \quad (7)$$

$$a = b_{g2} + c_{g2} F_L \quad (8)$$

At normal equilibrium:

$$b_d + c_d D_e = b_g + c_g D_e \quad (9)$$

This gives us:

$$D_e = \frac{b_g - b_d}{c_d - c_g} \quad (10)$$

and similarly,

$$D_{e2} = \frac{b_{g2} - b_d}{c_d - c_{g2}} \quad (11)$$

and,

$$\lambda = b_g + c_g \frac{b_g - b_d}{c_d - c_g} \quad (12)$$

or,

$$\lambda = b_d + c_d \frac{b_g - b_d}{c_d - c_g} \quad (13)$$

From Fig. 12 geometry tells us the customer benefit is:

$$B_d = \frac{1}{2}[(b_d - \lambda) + (e - \lambda)]F_L \quad (14)$$

if equation 7 is substituted into equation 14, the customer benefit is shown to be a quadratic in  $F_L$ :

$$B_d = \frac{1}{2}c_d F_L^2 + (b_d - \lambda)F_L \quad (15)$$

In order to find the optimal point for the customer the derivative is equated to zero:

$$\frac{dB_d}{dF_L} = c_d F_L + (b_d - \lambda) = 0 \quad (16)$$

if equation 13 is substituted for  $\lambda$  we get:

$$F_L = \frac{b_g - b_d}{c_d - c_g} = D_e \quad (17)$$

So we just proved that with the existing conditions the best FPL level for the customer is to stay at  $D_e$  as predicted.

Similarly the producer benefit is:

$$B_g = \frac{1}{2}[(\lambda - b_{g2}) + (\lambda - a)]F_L \quad (18)$$

and following the same procedure as above using equation 8 into equation 18

$$B_g = -\frac{1}{2}c_{g2} F_L^2 + (\lambda - b_{g2})F_L \quad (19)$$

$$\frac{dB_g}{dF_L} = -c_{g2} F_L + (\lambda - b_{g2}) = 0 \quad (20)$$

substituting  $\lambda = b_{g2} + c_{g2}h$  yields  $F_L = h$  for optimal results for the producer. This also lies at the expected level. Now let's do an analysis for the society to get the societally optimum level of FPL.

Let's define the societal benefit  $B_s$  as:

$$B_s = B_g + B_d \quad (21)$$

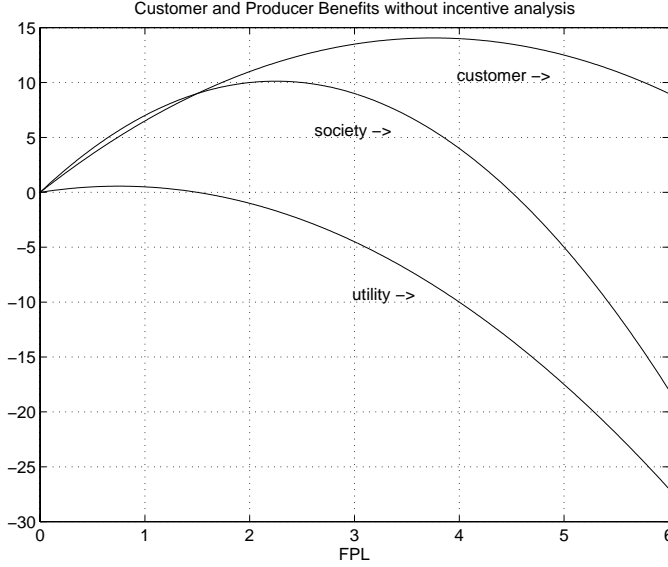
so,

$$\frac{dB_s}{dF_L} = \frac{dB_g}{dF_L} + \frac{dB_d}{dF_L} = 0 \quad (22)$$

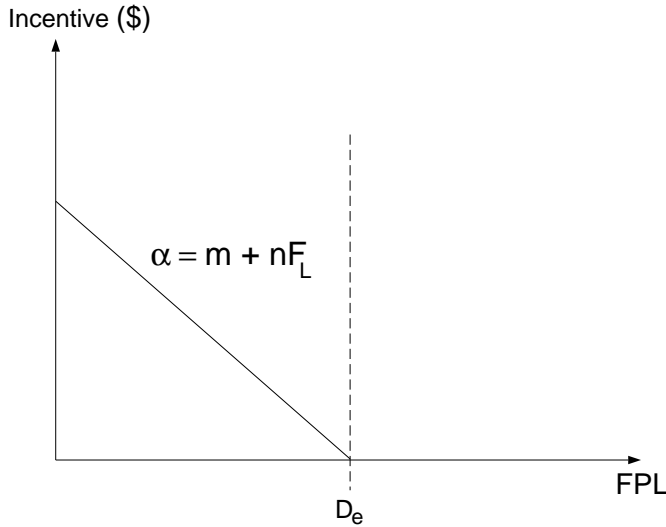
and this yields:

$$F_L = \frac{b_{g2} - b_d}{c_d - c_{g2}} = D_{e2} \quad (23)$$

This makes sense since the societal optimum is expected to be at the new equilibrium. Since this is different from the customer and producer optimums (see Fig. 13), we have to find a way to force the customers and the producers to work at this societally optimal point. One way to do this is to have the producer pay the customer an incentive, formulated in such a way that all three optimums will line up at the societally optimal level. As depicted in Fig 14, the incentive gets more and more expensive for the utility to lower the FPL for a certain customer. Let the incentive function be:



**Fig. 13:** *Benefit Functions for the customer and the producer: The optimum level of relief for a utility is different from the optimum level of relief from a customer's perspective. Neither is optimal*



**Fig. 14:** *Incentive Fee*

$$\alpha = m + nF_L \quad (24)$$

where  $n < 0$  and it should be that  $\alpha = 0$  when  $F_L = D_e$  hence:

$$m = -nD_e \quad (25)$$

Now the new benefit function for the customer becomes:

$$B_d = \frac{1}{2}c_d F_L^2 + (b_d - \lambda)F_L + (m + nF_L) \quad (26)$$

If the new optimum for the customer is worked out in terms of  $n$  (have to substitute equation 13 for  $\lambda$ ) we obtain:

$$F_L = D_e - \frac{n}{c_d} \quad (27)$$

Similarly the benefit function for the producer changes to:

$$B_g = -\frac{1}{2}c_{g2} F_L^2 + (\lambda - b_{g2})F_L - (m + nF_L) \quad (28)$$

which yields a new optimum for the producer in terms of  $n$  (after substituting  $\lambda = b_{g2} + c_{g2}h$ ):

$$F_L = h - \frac{n}{c_{g2}} \quad (29)$$

So we have to chose the right value of  $n$  (the slope of the incentive function) to put both optimums to the same level. When we equate equation 27 to 29:

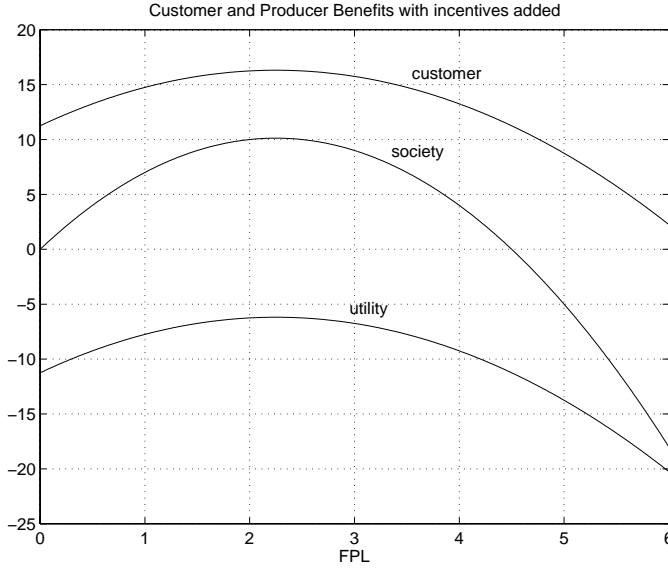
$$n = \frac{c_d c_{g2} (D_e - h)}{c_{g2} - c_d} \quad (30)$$

This value of  $n$  guarantees that the optimum level of FPL for the customer and the producer will be the same and we will now prove that this level will also be the societal optimum we calculated before. So if we just substitute equation 30 into equation 27:

$$D_e - \frac{n}{c_d} = D_e - \frac{c_{g2}(D_e - h)}{c_{g2} - c_d} \quad (31)$$

$$= \frac{D_e(c_{g2} - c_d) - c_{g2}(D_e - h)}{c_{g2} - c_d} \quad (32)$$

$$= \frac{-D_e c_d + c_{g2} h}{c_{g2} - c_d} \quad (33)$$



**Fig. 15:** *Benefit Functions for the customer and the producer, with incentives added based on incentive analysis. Now the socially optimal level coincides with what is optimal to both utilities and customers*

using  $c_{g2}h = \lambda - b_{g2}$  and  $\lambda = b_d + c_d D_e$

$$= \frac{-D_e c_d + \lambda - b_{g2}}{c_{g2} - c_d} \quad (34)$$

$$= \frac{b_d - b_{g2}}{c_{g2} - c_d} \quad (35)$$

$$= \frac{b_{g2} - b_d}{c_d - c_{g2}} \quad (36)$$

$$= D_{e2} \quad (37)$$

we can see that for this calculated value of  $n$ ,  $F_L$  indeed equals  $D_{e2}$  (the societal optimum). All three optimums are shown to line up in Fig. 15 after the incentive function is added to the analysis. Hence the utility gets the load relief it needs while keeping the customer happy financially.

Similar FPL programs, without optimization, was run by several utilities in the U.S. and the results were studied in [6].

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