MAT 101
MATHEMATICS FOR SOCIAL SCIENCES

CURVE SKECTHING
Solutions of Selected Problems from Sections 13.1, 13.2, 13.3, 13.4, and 13.5

Problem 13.1 (page 576)
Suggested Problems: 1-64

2. Decreasing on $(-\infty, -1)$ and $(0, 1)$; increasing on $(-1, 0)$ and $(1, \infty)$; relative minima $(-1, -1)$ and $(1, -1)$; relative maximum $(0, 0)$.

4. Decreasing on $(-\infty, 0)$ and $(0, \infty)$; never increasing; no relative maximum; no relative minimum.

6. $f'(x) = 2x(x-1)^3$

CV: $x = 0, 1$

$+ \quad - \quad +$

$0 \quad 1$

Increasing on $(-\infty, 0)$ and $(1, \infty)$; decreasing on $(0, 1)$; relative maximum when $x = 0$; relative minimum when $x = 1$.

8. $f'(x) = \dfrac{x(x+2)}{x^2+1}$

CV: $x = 0, -2$

$+ \quad - \quad +$

$-2 \quad 0$

Increasing on $(-\infty, -2)$ and $(0, \infty)$; decreasing on $(-2, 0)$; relative maximum when $x = -2$; relative minimum when $x = 0$.

10. $y = x^2 + 4x + 3$

$y' = 2x + 4 = 2(x+2)$

CV: $x = -2$

$- \quad +$

-2

Decreasing on $(-\infty, -2)$; increasing on $(-2, \infty)$; relative minimum when $x = -2$.

14. $y = \dfrac{x^4}{4} + x^3$

$y' = x^3 + 3x^2 = x^2(x+3)$

CV: $x = -3, 0$

$- \quad + \quad +$

$-3 \quad 0$

Decreasing on $(-\infty, -3)$; increasing on $(-3, 0)$ and $(0, \infty)$; relative minimum at $x = -3$.

20. $y = -5x^3 + x^2 + x - 7$

$y' = -15x^2 + 2x + 1 = -(5x+1)(3x-1)$

CV: $-\dfrac{1}{5}, \dfrac{1}{3}$

$- \quad + \quad -$

$-\frac{1}{5} \quad \frac{1}{3}$

Decreasing on $\left(-\infty, -\dfrac{1}{5}\right)$ and $\left(\dfrac{1}{3}, \infty\right)$; increasing on $\left(-\dfrac{1}{5}, \dfrac{1}{3}\right)$; relative minimum when $x = -\dfrac{1}{5}$; relative maximum when $x = \dfrac{1}{3}$.

28. $y = \dfrac{4}{5}x^5 - \dfrac{13}{3}x^3 + 3x + 4$

$y' = 4x^4 - 13x^2 + 3 = \left(4x^2 - 1\right)\left(x^2 - 3\right)$

$= (2x-1)(2x+1)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

CV: $x = \pm\dfrac{1}{2}, \pm\sqrt{3}$

$+ \quad - \quad + \quad - \quad +$

$-\sqrt{3} \quad -\frac{1}{2} \quad \frac{1}{2} \quad \sqrt{3}$

Increasing on $\left(-\infty, -\sqrt{3}\right)$, $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$, $\left(\sqrt{3}, \infty\right)$; decreasing on $\left(-\sqrt{3}, -\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2}, \sqrt{3}\right)$; relative maxima when $x = -\sqrt{3}, \dfrac{1}{2}$; relative minima when $x = -\dfrac{1}{2}, \sqrt{3}$.

30. $y=\sqrt[3]{x}(x-2)$

$y'=\dfrac{2(2x-1)}{3x^{\frac{2}{3}}}$

CV: $x=0, \dfrac{1}{2}$

$-\quad -\quad +$

$0\quad \frac{1}{2}$

Decreasing on $(-\infty, 0)$ and $\left(0, \dfrac{1}{2}\right)$; increasing on $\left(\dfrac{1}{2}, \infty\right)$; relative minimum when $x=\dfrac{1}{2}$; no relative maximum.

34. $y=\dfrac{3x}{2x+5}$

$y'=\dfrac{3(2x+5)-(3x)(2)}{(2x+5)^2}=\dfrac{15}{(2x+5)^2}$

CV: None but $x=-\dfrac{5}{2}$ must be included in the sign chart because it is a point of discontinuity of y.

$+\quad +$

$\boxed{-\frac{5}{2}}$

Increasing on $\left(-\infty, -\dfrac{5}{2}\right)$ and $\left(-\dfrac{5}{2}, \infty\right)$; no relative extremum

38. $y=\dfrac{2x^2}{4x^2-25}$

$y'=\dfrac{(4x^2-25)(4x)-(2x^2)(8x)}{(4x^2-25)^2}$

$=-\dfrac{100x}{(4x^2-25)^2}=-\dfrac{100x}{(2x-5)^2(2x+5)^2}$

CV: $x=0$, but $x=\pm\dfrac{5}{2}$ must be included in the sign chart because they are points of discontinuity of y.

$+\quad +\quad -\quad -$

$\boxed{-\frac{5}{2}}\quad 0\quad \boxed{\frac{5}{2}}$

Increasing on $\left(-\infty, -\dfrac{5}{2}\right)$ and $\left(-\dfrac{5}{2}, 0\right)$; decreasing on $\left(0, \dfrac{5}{2}\right)$ and $\left(\dfrac{5}{2}, \infty\right)$; relative maximum at $x=0$.

40. $y=\sqrt[3]{x^3-9x}$

$y'=\dfrac{1}{3}\left(x^3-9x\right)^{-\frac{2}{3}}\left(3x^2-9\right)=\dfrac{\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)}{[x(x+3)(x-3)]^{\frac{2}{3}}}$

CV: $x=\pm\sqrt{3}, 0, \pm 3$

$+\quad +\quad -\quad -\quad +\quad +$

$-3\quad -\sqrt{3}\quad 0\quad \sqrt{3}\quad 3$

Increasing on $(-\infty, -3)$, $\left(-3, -\sqrt{3}\right)$, $\left(\sqrt{3}, 3\right)$, and $(3, \infty)$; decreasing on $\left(-\sqrt{3}, 0\right)$ and $\left(0, \sqrt{3}\right)$; relative maximum when $x=-\sqrt{3}$; relative minimum when $x=\sqrt{3}$.

46. $y=x\ln x$. (Note: $x>0$.)

$y'=1+\ln x$

$y'=0$ when $1+\ln x=0$, $\ln x=-1$, or

$x=e^{-1}=\dfrac{1}{e}$

CV: $x=\dfrac{1}{e}$

$-\quad +$

$0\quad \frac{1}{e}$

Decreasing on $\left(0, \dfrac{1}{e}\right)$; increasing on $\left(\dfrac{1}{e}, \infty\right)$; relative minimum when $x=\dfrac{1}{e}$.

48. $y=x^{-1}e^x$

$y'=x^{-1}e^x-x^{-2}e^x=e^x\left(\dfrac{1}{x}-\dfrac{1}{x^2}\right)=e^x\left(\dfrac{x-1}{x^2}\right)$

CV: $x=1$, but $x=0$ must also be included in the sign chart because it is a point of discontinuity of y.

$-\quad -\quad +$

$\boxed{0}\quad 1$

Increasing on $(1, \infty)$; decreasing on $(-\infty, 0)$ and $(0, 1)$; relative minimum when $x=1$.

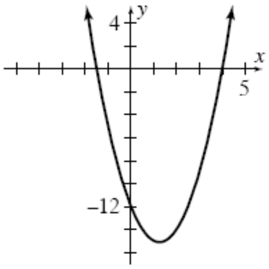
54. $y=2x^2-5x-12=(2x+3)(x-4)$

Intercepts $\left(-\dfrac{3}{2}, 0\right)$, $(4, 0)$, $(0, -12)$

$y'=4x-5=4\left(x-\dfrac{5}{4}\right)$

CV: $x=\dfrac{5}{4}$

Decreasing on $\left(-\infty, \dfrac{5}{4}\right)$; increasing on $\left(\dfrac{5}{4}, \infty\right)$; relative minimum when $x=\dfrac{5}{4}$.



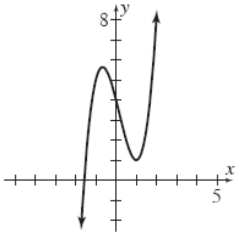
58. $y = 2x^3 - x^2 - 4x + 4$

The x-intercept is not convenient to find.
y-intercept is (0, 4).

$y' = 6x^2 - 2x - 4 = 2(3x+2)(x-1)$

CV: $x = -\frac{2}{3}, 1$

Increasing on $\left(-\infty, -\frac{2}{3}\right)$ and $(1, \infty)$; decreasing on $\left(-\frac{2}{3}, 1\right)$; relative maximum when $x = -\frac{2}{3}$; relative minimum when $x = 1$.



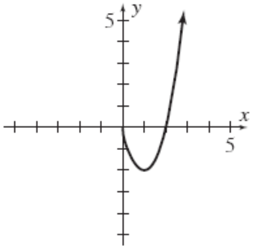
62. $y = \sqrt{x}(x^2 - x - 2) = \sqrt{x}(x-2)(x+1)$

[Note $x \geq 0$.]
Intercepts (0, 0), (2, 0)

$y = x^{5/2} - x^{3/2} - 2x^{1/2}$

$$y' = \frac{5}{2}x^{3/2} - \frac{3}{2}x^{1/2} - 2 \cdot \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}}(5x^2 - 3x - 2)$$
$$= \frac{1}{2\sqrt{x}}(5x+2)(x-1)$$

CV: $x = 0, 1$ ($x \geq 0$)
Decreasing on (0, 1); increasing on (1, ∞); relative minimum when $x = 1$.

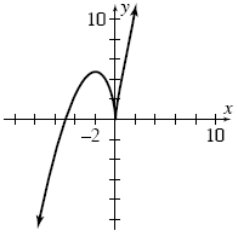


64. $y = x^{\frac{5}{3}} + 5x^{\frac{2}{3}} = x^{\frac{2}{3}}(x+5)$

Intercepts (0, 0), (–5, 0)

$$y' = \frac{5}{3}x^{\frac{2}{3}} + \frac{10}{3x^{\frac{1}{3}}} = \frac{5(x+2)}{3x^{\frac{1}{3}}}$$

CV: $x = 0, -2$
Increasing on (–∞, –2) and (0, ∞); decreasing on (–2, 0); relative maximum when $x = -2$; relative minimum when $x = 0$.



Problem 13.2 (p. 580)
Suggested Problems: 1-12

2. $f(x) = -2x^2 - 6x + 5$ and f is continuous over [–3, 2].

$f'(x) = -4x - 6 = -4\left(x + \frac{3}{2}\right)$

The only critical value on (–3, 2) is $x = -\frac{3}{2}$. We have $f(-3) = 5$, $f\left(-\frac{3}{2}\right) = \frac{19}{2}$, and $f(2) = -15$.

Absolute maximum: $f\left(-\frac{3}{2}\right) = \frac{19}{2}$;

absolute minimum: $f(2) = -15$

4. $f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2$ and f is continuous over [0, 1].

$f'(x) = x^3 - 3x = x\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

There are no critical values on (0, 1), so we only have to evaluate f at the end points: $f(0) = 0$ and $f(1) = -\frac{5}{4}$

Absolute maximum: $f(0) = 0$;

absolute minimum: $f(1) = -\frac{5}{4}$

6. $f(x) = x^{\frac{2}{3}}$ and f is continuous over [–8, 8].

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$.

The only critical value on (–8, 8) is $x = 0$. We have $f(-8) = 4$, $f(0) = 0$, and $f(8) = 4$. Thus there is an absolute maximum when $x = -8$ or $x = 8$, and an absolute minimum when $x = 0$.
Absolute maximum: $f(-8) = f(8) = 4$;
absolute minimum: $f(0) = 0$

8. $f(x) = \frac{7}{3}x^3 + 2x^2 - 3x + 1$ and f is continuous over [0, 3].

$f'(x) = 7x^2 + 4x - 3 = (7x - 3)(x + 1)$

The only critical value on (0, 3) is $x = \frac{3}{7}$. We have $f(0) = 1$, $f\left(\frac{3}{7}\right) = \frac{13}{49}$, and $f(3) = 73$.

Absolute maximum: $f(3) = 73$;

absolute minimum: $f\left(\frac{3}{7}\right) = \frac{13}{49}$

10. $f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3$ and f is continuous over [–2, 3].

$f'(x) = x^3 - x = x(x^2 - 1) = x(x-1)(x+1)$

The critical values of f on (–2, 3) are $x = -1$, 0, 1. We have $f(-2) = 5$, $f(-1) = \frac{11}{4}$, $f(0) = 3$, $f(1) = \frac{11}{4}$ and $f(3) = \frac{75}{4}$.

Absolute maximum: $f(3) = \frac{75}{4}$

Absolute minimum: $f(-1) = f(1) = \frac{11}{4}$

12. $f(x) = \frac{x}{x^2 + 1}$ and f is continuous over [0, 2].

$$f'(x) = \frac{\left(x^2 + 1\right) - x(2x)}{\left(x^2 + 1\right)^2} = \frac{1 - x^2}{\left(x^2 + 1\right)^2}$$

$$= \frac{(1 + x)(1 - x)}{\left(x^2 + 1\right)^2}$$

The only critical value on (0, 2) is $x = 1$. We have $f(0) = 0$, $f(1) = \frac{1}{2}$, and $f(2) = \frac{2}{5}$.

Absolute maximum: $f(1) = \frac{1}{2}$;

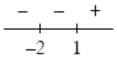
absolute minimum: $f(0) = 0$

Problem 13.3 (p. 586)
Suggested Problems: 1-66

2. $f(x) = \dfrac{x^5}{20} + \dfrac{x^4}{4} - 2x^2$

$f''(x) = (x-1)(x+2)^2$

$f''(x)$ is 0 when $x = 1, -2$. Sign chart for f'':

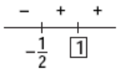


Concave down on $(-\infty, -2)$ and $(-2, 1)$; concave up on $(1, \infty)$. Inflection point when $x = 1$.

4. $f(x) = \dfrac{x^2}{(x-1)^2}$

$f''(x) = \dfrac{2(2x+1)}{(x-1)^4}$

$f''(x) = 0$ when $x = -\dfrac{1}{2}$. Although f'' is not defined when $x = 1$, f is not continuous at $x = 1$. So there is no inflection point when $x = 1$, but $x = 1$ must be considered in concavity analysis. Sign chart of f'':



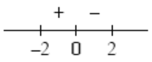
Concave up on $\left(-\dfrac{1}{2}, 1\right)$ and $(1, \infty)$; concave down on $\left(-\infty, -\dfrac{1}{2}\right)$.

Inflection point when $x = \dfrac{1}{2}$

6. $f(x) = x\sqrt{4-x^2}$

$f''(x) = \dfrac{2x\left(x^2-6\right)}{\left(4-x^2\right)^{\frac{3}{2}}}$

Note that the domain of f is $[-2, 2]$. $f''(x)$ is 0 only when $x = 0$; f'' is not defined when $x = \pm 2$, which are the endpoints of the domain of f. The only possible point of inflection occurs when $x = 0$. Sign chart for f'':



Concave up on $(-2, 0)$; concave down on $(0, 2)$. Inflection point when $x = 0$.

14. $y = -\dfrac{x^4}{4} + \dfrac{9x^2}{2} + 2x$

$y' = -x^3 + 9x + 2$

$y'' = -3x^2 + 9 = -3(x^2 - 3)$

$= -3\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$

Possible inflection points when $x = \pm\sqrt{3}$.

Concave down on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$; concave up on $\left(-\sqrt{3}, \sqrt{3}\right)$; inflection points when $x = \pm\sqrt{3}$.

18. $y = -\dfrac{5}{2}x^4 - \dfrac{1}{6}x^3 + \dfrac{1}{2}x^2 + \dfrac{1}{3}x - \dfrac{2}{5}$

$y' = -10x^3 - \dfrac{1}{2}x^2 + x + \dfrac{1}{3}$

$y'' = -30x^2 - x + 1 = -(5x+1)(6x-1)$

Possible inflection points when $x = -\dfrac{1}{5}, \dfrac{1}{6}$.

Concave down on $\left(-\infty, -\dfrac{1}{5}\right)$ and $\left(\dfrac{1}{6}, \infty\right)$;

22. $y = x^6 - 3x^4$

$y' = 6x^5 - 12x^3$

$y'' = 30x^4 - 36x^2 = 30x^2\left(x^2 - \dfrac{6}{5}\right)$

$= 30x^2\left(x - \sqrt{\dfrac{6}{5}}\right)\left(x + \sqrt{\dfrac{6}{5}}\right)$

Possible inflection points when $x = 0, \pm\sqrt{\dfrac{6}{5}}$.

Concave up on $\left(-\infty, -\sqrt{\dfrac{6}{5}}\right)$ and $\left(\sqrt{\dfrac{6}{5}}, \infty\right)$; concave down on $\left(-\sqrt{\dfrac{6}{5}}, 0\right)$ and $\left(0, \sqrt{\dfrac{6}{5}}\right)$.

Inflection points when $x = \pm\sqrt{\dfrac{6}{5}}$.

26. $y = \dfrac{4x^2}{x+3}$

$$y' = \frac{(x+3)(8x) - 4x^2(1)}{(x+3)^2} = \frac{4\left(x^2+6x\right)}{(x+3)^2}$$

$$y'' = \frac{(x+3)^2(4)(2x+6) - 4(x^2+6x)(2)(x+3)}{(x+3)^4}$$

$$= \frac{72}{(x+3)^3}$$

No possible inflection point, but we must include $x = -3$ in the concavity analysis. Concave down on $(-\infty, -3)$; concave up on $(-3, \infty)$.

30. $y = e^x - e^{-x}$

$y' = e^x + e^{-x}$

$y'' = e^x - e^{-x}$

Setting $y'' = 0$ gives $e^x = e^{-x}$ or, equivalently, $x = 0$. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$; inflection point when $x = 0$.

34. $y = \dfrac{x^2+1}{3e^x}$

$$y' = \frac{3e^x(2x) - \left(x^2+1\right)3e^x}{9e^{2x}} = \frac{2x - \left(x^2+1\right)}{3e^x}$$

$$= \frac{2x - x^2 - 1}{3e^x}$$

$$y'' = \frac{3e^x(2-2x) - \left(2x - x^2 - 1\right)3e^x}{9e^{2x}}$$

$$= \frac{(2-2x) - \left(2x - x^2 - 1\right)}{3e^x}$$

$$= \frac{x^2 - 4x + 3}{3e^x} = \frac{(x-1)(x-3)}{3e^x}$$

Possible inflection points when $x = 1, 3$.
Concave up on $(-\infty, 1)$ and $(3, \infty)$; concave down on
$(1, 3)$; inflection point when $x = 1, 3$.

36. $y = x^2 + 2$

Intercept $(0, 2)$

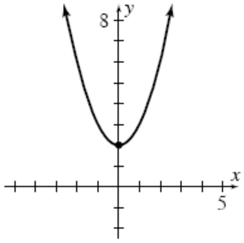
$y' = 2x$

CV: $x = 0$

Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; relative minimum at $(0, 2)$.

$y'' = 2$

No possible inflection point. Concave up on $(-\infty, \infty)$. Symmetric about the y-axis.



38. $y = x - x^2 + 2 = -(x-2)(x+1)$

Intercepts $(2, 0)$, $(-1, 0)$, and $(0, 2)$

$y' = 1 - 2x$

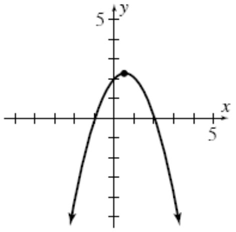
CV: $x = \dfrac{1}{2}$

Increasing on $\left(-\infty, \dfrac{1}{2}\right)$; decreasing on $\left(\dfrac{1}{2}, \infty\right)$;

relative maximum at $\left(\dfrac{1}{2}, \dfrac{9}{4}\right)$

$y'' = -2$

No possible inflection point. Concave down on $(-\infty, \infty)$.



40. $y = x^3 - 25x^2 = x^2(x-25)$

Intercepts: $(0, 0)$ and $(25, 0)$

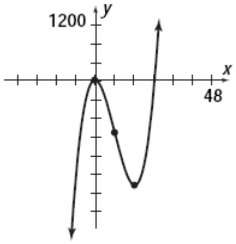
$y' = 3x^2 - 50x = 3x\left(x - \dfrac{50}{3}\right)$

CV: $x = 0, \dfrac{50}{3}$

Increasing on $(-\infty, 0)$ and $\left(\dfrac{50}{3}, \infty\right)$; decreasing on $\left(0, \dfrac{50}{3}\right)$; relative maximum at $(0, 0)$; relative minimum at $\left(\dfrac{50}{3}, -\dfrac{62,500}{27}\right)$.

$y'' = 6x - 50 = 6\left(x - \frac{25}{3}\right)$

Possible inflection point when $x = \frac{25}{3}$. Concave down on $\left(-\infty, \frac{25}{3}\right)$; concave up on $\left(\frac{25}{3}, \infty\right)$; inflection point at $\left(\frac{25}{3}, -\frac{31,250}{27}\right)$.



46. $y = -x^3 + 2x^2 - x + 4$

Intercept (0, 4)

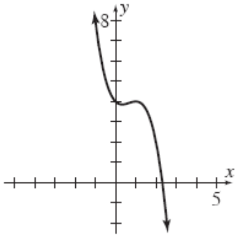
$y' = -3x^2 + 4x - 1 = -(3x-1)(x-1)$

CV: $x = \frac{1}{3}, 1$

Decreasing on $\left(-\infty, \frac{1}{3}\right)$ and $(1, \infty)$; increasing on $\left(\frac{1}{3}, 1\right)$; relative minimum at $\left(\frac{1}{3}, \frac{104}{27}\right)$; relative maximum at (1, 4)

$y'' = -6x + 4 = -6\left(x - \frac{2}{3}\right)$

Possible inflection point when $x = \frac{2}{3}$. Concave up on $\left(-\infty, \frac{2}{3}\right)$; concave down on $\left(\frac{2}{3}, \infty\right)$; inflection point at $\left(\frac{2}{3}, \frac{106}{27}\right)$



50. $y = \frac{x^5}{100} - \frac{x^4}{20} = \frac{x^4}{100}(x-5)$

Intercepts (0, 0), (5, 0)

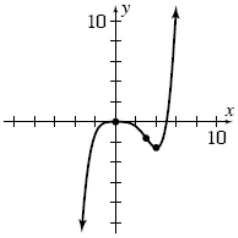
$y' = \frac{x^4}{20} - \frac{x^3}{5} = \frac{x^3}{20}(x-4)$

CV: $x = 0$ and $x = 4$

Increasing on (–∞, 0) and (4, ∞); decreasing on (0, 4); relative maximum at (0, 0); relative minimum at (4, –2.56).

$y'' = \frac{x^3}{5} - \frac{3x^2}{5} = \frac{x^2}{5}(x-3)$

Possible inflection points when $x = 0$ and $x = 3$. Concave down on (–∞, 0) and (0, 3); concave up on (3, ∞); inflection point at (3, –1.62).



54. $y = 3x^5 - 5x^3 = 3x^3\left[x^2 - \frac{5}{3}\right]$

$= 3x^3\left(x + \sqrt{\frac{5}{3}}\right)\left(x - \sqrt{\frac{5}{3}}\right)$

Intercepts (0, 0) and $\left(\pm\sqrt{\frac{5}{3}}, 0\right)$

Symmetric about the origin.

$y' = 15x^4 - 15x^2 = 15x^2(x+1)(x-1)$

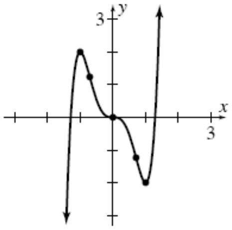
CV: $x = 0$ and $x = \pm 1$

Increasing on (–∞, –1) and (1, ∞); decreasing on (–1, 0) and (0, 1); relative maximum at (–1, 2); relative minimum at (1, –2).

$y'' = 60x^3 - 30x = 60x\left[x + \frac{\sqrt{2}}{2}\right]\left[x - \frac{\sqrt{2}}{2}\right]$

Possible inflection points at $x = 0$ and $x = \pm\frac{\sqrt{2}}{2}$.

Concave down on $\left(-\infty, -\frac{\sqrt{2}}{2}\right)$ and $\left(0, \frac{\sqrt{2}}{2}\right)$; concave up on $\left(-\frac{\sqrt{2}}{2}, 0\right)$ and $\left(\frac{\sqrt{2}}{2}, \infty\right)$; inflection points at $\left(\frac{\sqrt{2}}{2}, -\frac{7\sqrt{2}}{8}\right)$, (0, 0), and $\left(-\frac{\sqrt{2}}{2}, \frac{7\sqrt{2}}{8}\right)$.



58. $y=(x-1)^2(x+2)^2$

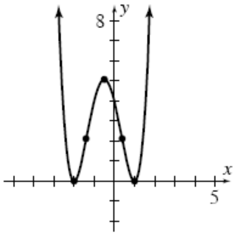
Intercepts (0, 4), (1, 0), (–2, 0)

$y'=(x-1)^2[2(x+2)]+(x+2)^2[2(x-1)]$

$= 2(x-1)(x+2)(2x+1)$

CV: $x=-2,\ -\frac{1}{2},\ 1$

Decreasing on $(-\infty, -2)$ and $\left(-\frac{1}{2}, 1\right)$; increasing on $\left(-2, -\frac{1}{2}\right)$ and $(1, \infty)$; relative maximum at $\left(-\frac{1}{2}, \frac{81}{16}\right)$; relative minima at (–2, 0) and (1, 0); $y'=2(2x^3+3x^2-3x-2)$, so $y''=6\left(2x^2+2x-1\right)$. Setting $y''=0$ and using the quadratic formula gives possible inflection points at $x=\frac{-1\pm\sqrt{3}}{2}$. Concave up on $\left(-\infty, \frac{-1-\sqrt{3}}{2}\right)$ and $\left(\frac{-1+\sqrt{3}}{2}, \infty\right)$; concave down on $\left(\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right)$; inflection points when $x=\frac{-1\pm\sqrt{3}}{2}$



60. $y=(x+1)\sqrt{x+4}$ [Note: $x\geq -4$]

Intercepts (0, 2), (–1, 0) and (–4, 0)

$$y'=(x+1)\cdot\frac{1}{2\sqrt{x+4}}+\sqrt{x+4}\,(1)$$

$$=\frac{1}{2\sqrt{x+4}}[(x+1)+2(x+4)]$$

$$=\frac{3(x+3)}{2\sqrt{x+4}}$$

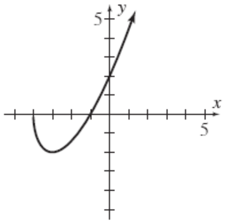
CV: $x = -3, -4$

Decreasing on (–4, –3); increasing on (–3, ∞); relative minimum at (–3, –2)

$$y''=\frac{3}{2}\cdot\frac{\sqrt{x+4}\,(1)-(x+3)\cdot\frac{1}{2\sqrt{x+4}}}{\left(\sqrt{x+4}\right)^2}$$

$$=\frac{3}{4}\cdot\frac{2(x+4)-(x+3)}{(x+4)^{3/2}}=\frac{3(x+5)}{4(x+4)^{3/2}}$$

No possible inflection point. Concave up on (–4, ∞).



Problem 13.4 (p. 589)
Suggested Problems: 1-14

2. $y = 5x^2 + 20x + 2$

$y' = 10x + 20$

CV: $x = -2$

$y'' = 10$

$y''(-2) = 10 > 0$

Thus there is a relative minimum when $x = -2$. Because there is only one relative extremum and *f* is continuous, the relative minimum is an absolute minimum.

4. $y = 3x^2 - 5x + 6$

$y' = 6x - 5$

CV: $x = \frac{5}{6}$

$y'' = 6$

$y''\left(\frac{5}{6}\right) = 6 > 0$

Thus there is a relative minimum when $x = \frac{5}{6}$.

Because there is only one relative extremum and *f* is continuous, the relative minimum is an absolute minimum.

6. $y = x^3 - 12x + 1$

$y' = 3x^2 - 12 = 3(x+2)(x-2)$

CV: $x = \pm 2$

$y'' = 6x$

$y''(-2) = -12 < 0 \Rightarrow$ relative maximum when $x = -2$

$y''(2) = 12 > 0 \Rightarrow$ relative minimum when $x = 2$

8. $y = x^4 - 2x^2 + 4$

$y' = 4x^3 - 4x = 4x(x+1)(x-1)$

CV: $= 0, \pm 1$

$y'' = 12x^2 - 4$

$y''(0) = -4 < 0 \Rightarrow$ relative maximum when $x = 0$

$y''(1) = 8 > 0 \Rightarrow$ relative minimum when $x = 1$

$y''(-1) = 8 > 0 \Rightarrow$ relative minimum when $x = -1$

12. $y = \frac{55}{3}x^3 - x^2 - 21x - 3$

$y' = 55x^2 - 2x - 21 = (5x+3)(11x-7)$

CV: $x = -\frac{3}{5}, \frac{7}{11}$

$y'' = 110x - 2$

$y''\left(-\frac{3}{5}\right) = -68 < 0 \Rightarrow$ relative maximum when $x = -\frac{3}{5}$

$y''\left(\frac{7}{11}\right) = 68 > 0 \Rightarrow$ relative minimum when $x = \frac{11}{7}$

Problem 13.5 (p. 598)
Suggested Problems: 1-50

2. $y = f(x) = \dfrac{x+1}{x}$

When $x = 0$ the denominator is zero but the numerator is not. Thus $x = 0$ is a vertical asymptote. $\lim\limits_{x\to\infty} \dfrac{x+1}{x} = \lim\limits_{x\to\infty} \dfrac{x}{x} = \lim\limits_{x\to\infty} 1 = 1$. Similarly $\lim\limits_{x\to-\infty} f(x) = 1$. Thus $y = 1$ is a horizontal asymptote.

6. $y = f(x) = 1 - \dfrac{2}{x^2} = \dfrac{x^{\angle} - 2}{x^2}$

When $x = 0$ the denominator is zero but the numerator is not. Thus $x = 0$ is a vertical asymptote. $\lim\limits_{x\to\infty}\left(1 - \dfrac{2}{x^2}\right) = 1 - 0 = 1.$ Similarly $\lim\limits_{x\to-\infty} f(x) = 1,$ so $y = 1$ is a horizontal asymptote.

8. $y = f(x) = \dfrac{x}{x^2-4} = \dfrac{x}{(x-2)(x+2)}$

Vertical asymptotes: $x = 2$, $x = -2$.

$\lim\limits_{x\to\infty} \dfrac{x}{x^2-4} = \lim\limits_{x\to\infty} \dfrac{x}{x^2} = \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$. Similarly, $\lim\limits_{x\to-\infty} f(x) = 0$. Thus $y = 0$ is a horizontal asymptote.

12. $f(x) = \dfrac{x^3}{5}$ is a polynomial function, so there are no horizontal or vertical asymptotes.

16. $f(x) = \dfrac{x^2-1}{2x^2-9x+4} = \dfrac{x^2-1}{(2x-1)(x-4)}$

Vertical asymptotes are $x = \dfrac{1}{2}$ and $x = 4$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2}{2x^2} = \lim\limits_{x\to\infty} \dfrac{1}{2} = \dfrac{1}{2}$,and

$\lim\limits_{x\to-\infty} f(x) = \dfrac{1}{2}$. Thus $y = \dfrac{1}{2}$ is a horizontal asymptote.

18. $y = f(x) = \dfrac{x^2+4x^3+6x^4}{3x^2}$

Observe that both the numerator and the denominator are zero when $x = 0$. For $x \neq 0$, we have

$$f(x) = \frac{x^2}{3x^2}(1+4x+6x^2) = \frac{1}{3}(1+4x+6x^2).$$

Thus f is a polynomial function for $x \neq 0$. Hence there are neither horizontal nor vertical asymptotes.

24. $f(x) = 12e^{-x}$

$\lim\limits_{x\to\infty} f(x) = 0$ and $\lim\limits_{x\to-\infty} f(x) = +\infty$. Thus $y = 0$ is a horizontal asymptote. There is no vertical asymptote because $f(x)$ neither increases nor decreases without bound around any fixed value of x.

26. $y = \dfrac{2}{2x-3}$

Intercept: $\left(0, -\dfrac{2}{3}\right)$

Vertical asymptote is $x = \dfrac{3}{2}$.

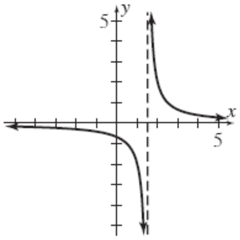
$\lim\limits_{x\to\infty} y = 0 = \lim\limits_{x\to-\infty} y,$ so $y = 0$ is a horizontal asymptote.

$$y' = -\frac{4}{(2x-3)^2}$$

CV: None, but $x=\dfrac{3}{2}$ must be considered in the inc. dec. analysis. Decreasing on $\left(-\infty, \dfrac{3}{2}\right)$ and $\left(\dfrac{3}{2}, \infty\right)$.

$$y''=\frac{16}{(2x-3)^3}$$

No possible inflection point, but $x=\dfrac{3}{2}$ must be considered in the concavity analysis. Concave down on $\left(-\infty, \dfrac{3}{2}\right)$; concave up on $\left(\dfrac{3}{2}, \infty\right)$.



32. $y=\dfrac{1}{x^2+1}$

Intercept (0, 1)

Symmetric about the y-axis.

$\lim\limits_{x\to\infty}\dfrac{1}{x^2+1}=0=\lim\limits_{x\to-\infty}\dfrac{1}{x^2+1}$, so $y = 0$ is a horizontal asymptote.

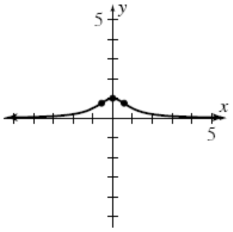
$$y'=\frac{-2x}{\left(x^2+1\right)^2}$$

CV: $x = 0$

Increasing on $(-\infty, 0)$; decreasing on $(0, \infty)$; relative maximum at (0, 1)

$$y''=\frac{2\left(3x^2-1\right)}{\left(x^2+1\right)^3}$$

Possible inflection points at $x=\pm\dfrac{1}{\sqrt{3}}$. Concave up on $\left(-\infty, -\dfrac{1}{\sqrt{3}}\right)$ and $\left(\dfrac{1}{\sqrt{3}}, \infty\right)$; concave down on $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$; inflection points at $\left(\pm\dfrac{1}{\sqrt{3}}, \dfrac{3}{4}\right)$



36. $y = \dfrac{x^3+1}{x}$

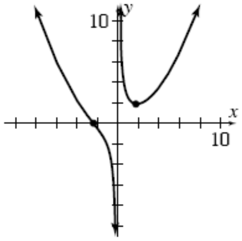
Intercept: (–1, 0)
Vertical asymptote is $x = 0$. Because the degree of the numerator is greater than the degree of the denominator, no horizontal asymptote exists.
Since $y = x^2 + x^{-1}$,

$$y' = 2x - x^{-2} = 2x - \frac{1}{x^2} = \frac{2x^3-1}{x^2}.$$

CV: $x = \sqrt[3]{\frac{1}{2}}$, but $x = 0$ must be included in inc.-dec. analysis. Decreasing on (–∞, 0) and $\left(0, \sqrt[3]{\frac{1}{2}}\right)$; increasing on $\left(\sqrt[3]{\frac{1}{2}}, \infty\right)$; relative minimum at $\left(\sqrt[3]{\frac{1}{2}}, 3\sqrt[3]{\frac{1}{4}}\right)$.

$$y'' = 2 + 2x^{-3} = 2 + \frac{2}{x^3} = \frac{2\left(x^3+1\right)}{x^3}$$

Possible inflection point when $x = -1$, but $x = 0$ must be included in concavity analysis. Concave up on (–∞, –1) and (0, ∞); concave down on (–1, 0); inflection point at (–1, 0).



42. $y = \dfrac{3x}{(x-2)^2}$

Intercept (0, 0)
Vertical asymptote at $x = 2$

$$\lim_{x\to\infty} \frac{3x}{x^2-4x+4} = \lim_{x\to\infty} \frac{3x}{x^2} = \lim_{x\to\infty} \frac{3}{x} = 0 \text{ and}$$

$$\lim_{x\to-\infty} \frac{3x}{x^2-4x+4} = 0, \text{ so } y = 0 \text{ is the only}$$

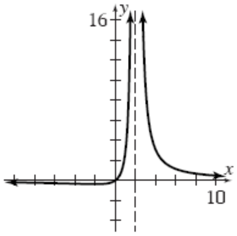
horizontal asymptote.

$$y' = \frac{-3(x+2)}{(x-2)^3}$$

CV: $x = -2$, but $x = 2$ must be included in the inc.-dec. analysis. Decreasing on (–∞, –2) and (2, ∞); increasing on (–2, 2); relative maximum at $\left(-2, -\frac{3}{8}\right)$

$$y'' = \frac{6(x+4)}{(x-2)^4}$$

Possible inflection point when $x = -4$, but $x = 2$ must be included in the concavity analysis. Concave down on (–∞, –4); concave up on (–4, 2) and (2, ∞); inflection point at $\left(-4, -\frac{1}{3}\right)$.



44. $y = \dfrac{3x^4 + 1}{x^3}$

No intercepts

Symmetric about the origin.

Vertical asymptote is $x = 0$. $\dfrac{3x^4 + 1}{x^3} = 3x + \dfrac{1}{x^3}$ so

$y = 3x$ is an oblique asymptote.

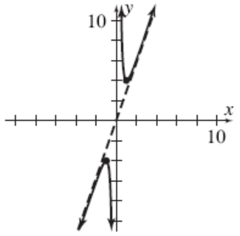
Since $y = 3x + x^{-3}$,

$$y' = 3 - 3x^{-4} = 3 - \frac{3}{x^4} = \frac{3\left(x^2+1\right)(x+1)(x-1)}{x^4}$$

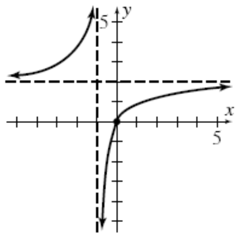
CV: ±1, but $x = 0$ must be considered in the inc.-dec. analysis. Increasing on $(-\infty, -1)$ and $(1, \infty)$; decreasing on $(-1, 0)$ and $(0, 1)$; relative maximum at $(-1, -4)$; relative minimum at $(1, 4)$.

$$y'' = \frac{12}{x^5}$$

No possible inflection point, but $x = 0$ must be included in the concavity analysis. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$.



48.



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