METU NORTHERN CYPRUS CAMPUS

MAT 101
MATHEMATICS FOR SOCIAL SCIENCES

LIMITS
Solutions of Selected Problems from Sections 10.1, 10.2, and 10.3

Problem 10.1 (page 457)
Suggested Problems: 1-44

2. a. 2

b. 1

c. 2

4. a. −1

b. does not exist

c. 1

6. $f(-3.1) = -6.1$ $f(-2.9) = -5.9$
$f(-3.01) = -6.01$ $f(-2.99) = -5.99$
$f(-3.001) = -6.001$ $f(-2.999) = -5.999$
estimate of limit: −6

12. $\lim\limits_{t\to 1/3}(5t-7) = 5\left(\frac{1}{3}\right) - 7 = -\frac{16}{3}$

16. $\lim\limits_{t\to -6}\frac{x^2+6}{x-6} = \frac{\lim\limits_{x\to -6}\left(x^2+6\right)}{\lim\limits_{x\to -6}(x-6)} = \frac{(-6)^2+6}{(-6)-6}$

18. $\lim\limits_{z\to 0}\frac{z^2-5z-4}{z^2+1} = \frac{\lim\limits_{z\to 0}(z^2-5z-4)}{\lim\limits_{z\to 0}(z^2+1)}$
$= \frac{0^2-5(0)-4}{0^2+1}$
$= -4$

20. $\lim\limits_{y\to 15}\sqrt{y+3} = \sqrt{\lim\limits_{y\to 15}(y+3)} = \sqrt{15+3} = \sqrt{18}$
$= 3\sqrt{2}$

22. $\lim\limits_{x\to -1}\frac{x+1}{x+1} = \lim\limits_{x\to -1} 1 = 1$

24. $\lim\limits_{t\to 0}\frac{t^3+3t^2}{t^3-4t^2} = \lim\limits_{t\to 0}\frac{t^2(t+3)}{t^2(t-4)} = \lim\limits_{t\to 0}\frac{t-3}{t-4} = -\frac{3}{4}$

28. $\lim\limits_{x\to 0}\frac{x^2-2x}{x} = \lim\limits_{x\to 0}\frac{x(x-2)}{x} = \lim\limits_{x\to 0}(x-2) = -2$

30. $\lim\limits_{x\to -3}\frac{x^4-81}{x^2+8x+15} = \lim\limits_{x\to -3}\frac{(x^2+9)(x^2-9)}{(x+3)(x+5)}$
$= \lim\limits_{x\to -3}\frac{(x^2+9)(x+3)(x-3)}{(x+3)(x+5)}$
$= \lim\limits_{x\to -3}\frac{(x^2+9)(x-3)}{x+5}$
$= -54$

34. $\lim\limits_{x\to 0}\frac{(x+2)^2-4}{x} = \lim\limits_{x\to 0}\frac{x^2+4x}{x} = \lim\limits_{x\to 0}(x+4) = 4$

40. $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\frac{\left[(x+h)^2+(x+h)+1\right]-\left(x^2+x+1\right)}{h} = \lim\limits_{h\to 0}\frac{2xh+h^2+h}{h} = \lim\limits_{h\to 0}(2x+h+1) = 2x+1$

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42. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{[3-(x+h)+4(x+h)^2]-(3-x+4x^2)}{h}$

$=\lim_{h\to 0}\frac{3-x-h+4x^2+8xh+4h^2-(3-x+4x^2)}{h}$

$=\lim_{h\to 0}\frac{-h+8xh+4h^2}{h}$

$=\lim_{h\to 0}\frac{h(-1+8x+4h)}{h}$

$=\lim_{h\to 0}(-1+8x+4h)$

$=-1+8x$

Problem 10.2 (p. 465)
Suggested Problems: 1-58

2. **a.** 0

b. $-\infty$

c. does not exist

d. ∞

e. 2

f. 1

g. 1

4. $\lim_{x\to -1^+}(1-x^2)=0$

8. $\lim_{x\to 2}\frac{7}{x-1}=\frac{\lim_{x\to 2}7}{\lim_{x\to 2}(x-1)}=\frac{7}{1}=7$

10. $\lim_{t\to\infty}(t-1)^3=\infty$

14. $\lim_{x\to 0^-}2^{1/2}=2^{1/2}$

16. $\lim_{x\to 2^+}\left(x\sqrt{x^2-4}\right)=0$

20. $\lim_{x\to\infty}\frac{-6}{5x\sqrt[3]{x}}=-\frac{6}{5}\lim_{x\to\infty}\frac{1}{x^{4/3}}=-\frac{6}{5}\cdot 0=0$

22. $\lim_{x\to\infty}\frac{2x-4}{3-2x}=\lim_{x\to\infty}\frac{2x}{-2x}=\lim_{x\to\infty}(-1)=-1$

24. $\lim_{r\to\infty}\frac{r^3}{r^2+1}=\lim_{r\to\infty}\frac{r^3}{r^2}=\lim_{r\to\infty}r=\infty$

26. $\lim_{x\to\infty}\frac{5x}{3x^7-x^3+4}=\lim_{x\to\infty}\frac{5x}{3x^7}$

$=\lim_{x\to\infty}\frac{5}{3x^6}=\frac{5}{3}\cdot\lim_{x\to\infty}\frac{1}{x^6}=\frac{5}{3}\cdot 0=0$

28. $\lim_{x\to -\infty}\frac{2}{(4x-1)^3}=\lim_{x\to -\infty}\frac{2}{4^3x^3}$

$=\frac{2}{4^3}\cdot\lim_{x\to -\infty}\frac{1}{x^3}=\frac{2}{4^3}\cdot 0=0$

30. $\lim_{x\to -\infty}\frac{3-2x-2x^3}{7-5x^3+2x^2}=\lim_{x\to -\infty}\frac{-2x^3}{-5x^3}$

$=\lim_{x\to -\infty}\frac{2}{5}=\frac{2}{5}$

34. $\lim_{x\to\infty}\frac{4-3x^3}{x^3-1}=\lim_{x\to\infty}\frac{-3x^3}{x^3}=\lim_{x\to\infty}(-3)=-3$

36. $\lim_{x\to -\infty}\frac{3x-x^3}{x^3+x+1}=\lim_{x\to -\infty}\frac{-x^3}{x^3}=\lim_{x\to -\infty}(-1)=-1$

40. $\lim_{x\to -1}\frac{3x^3-x^2}{2x+1}=\frac{\lim_{x\to -1}\left(3x^3-x^2\right)}{\lim_{x\to -1}(2x+1)}=\frac{-4}{-1}=4$

42. $\lim_{x\to -\infty}-\frac{x^5+2x^3-1}{x^5-4x^2}=\lim_{x\to -\infty}-\frac{x^5}{x^5}$

$=\lim_{x\to -\infty}(-1)=-1$

44. As $x \to -2^+$, then $x \to -2$ and $\sqrt{16-x^4} \to 0$ through positive values. Thus,

$$\lim_{x\to -2^+} \frac{x}{\sqrt{16-x^4}} = -\infty.$$

50. $\lim_{x\to 3^+}\left(-\frac{7}{x-3}\right) = -\infty$

$\lim_{x\to 3^-}\left(-\frac{7}{x-3}\right) = +\infty$

Answer: does not exist.

52. $\lim_{x\to 0^+}\left|\frac{1}{x}\right| = \lim_{x\to 0^+}\frac{1}{x} = \infty$

$\lim_{x\to 0^-}\left|\frac{1}{x}\right| = \lim_{x\to 0^-}\left(-\frac{1}{x}\right) = \infty$

Thus, $\lim_{x\to 0}\left|\frac{1}{x}\right| = \infty.$

56. $f(x) = \begin{cases} x & \text{if } x \le 2 \\ -2+4x-x^2 & \text{if } x > 2 \end{cases}$

a. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(-2+4x-x^2) = 2$

b. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x = 2$

c. $\lim_{x\to 2} f(x) = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = 2$

d. $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}(-2+4x-x^2) = -\infty$

e. $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} x = -\infty$

Problem 10.3 (p. 471)
Suggested Problems: 1-33

2. $f(x) = \frac{x-3}{5x}$; $x = -3$

(i) f is defined at $x = -3$: $f(-3) = \frac{-6}{-15} = \frac{2}{5}$

(ii) $\lim_{x\to -3} f(x) = \lim_{x\to -3}\frac{x-3}{5x} = \frac{2}{5}$, which exists

(iii) $\lim_{x\to -3} f(x) = \frac{2}{5} = f(-3)$

Thus f is continuous at $x = -3$.

4. $f(x) = \frac{x}{8}$; $x = 2$

(i) f is defined at $x = 2$; $f(2) = \frac{2}{8} = \frac{1}{4}$.

(ii) $\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{x}{8} = \frac{2}{8} = \frac{1}{4}$, which exists.

(iii) $\lim_{x\to 2} f(x) = \frac{1}{4} = f(2)$.

Thus f is continuous at $x = 2$.

6. $f(x) = \sqrt[3]{x}$; $x = -1$

(i) f is defined at $x = -1$; $f(-1) = -1$.

(ii) $\lim_{x\to -1} f(x) = \lim_{x\to -1}\sqrt[3]{x} = \sqrt[3]{-1} = -1$, which exists.

(iii) $\lim_{x\to -1} f(x) = -1 = f(-1)$

Thus f is continuous at $x = -1$.

8. Continuous at 2 and –2 because f is a polynomial function (which is continuous everywhere).

10. Continuous at 2 and –2 because f is a rational function and at neither point is the denominator zero.

12. $f(x)=\begin{cases}\frac{1}{x} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$

Because $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{1}{x}=\infty$, $\lim_{x\to 0} f(x)$ does not exist. Thus f is discontinuous at $x = 0$. At $x = -1$, f is defined; $f(-1) = -1$.

$\lim_{x\to -1} f(x)=\lim_{x\to -1}\frac{1}{x}=-1$. Since $\lim_{x\to -1} f(x)=-1=f(-1)$, f is continuous at $x = -1$.

Answer: Discontinuous at 0, continuous at -1.

14. f is a polynomial function $\left[f(x)=\frac{2}{5}+\frac{3}{5}x-\frac{1}{5}x^2\right]$.

18. None, because h is a polynomial function.

20. The denominator of this rational function is zero only when $x = \pm 2$. Thus f is discontinuous only at $x = \pm 2$.

22. None, because f is a polynomial function.

24. $x^2+x=0$, $x(x + 1) = 0$, $x = 0$ or -1. Discontinuous at 0 and -1.

28. $x^4-1=0$, $\left(x^2+1\right)\left(x^2-1\right)=0$,

$\left(x^2+1\right)(x+1)(x-1)=0$, $x = \pm 1$.

Discontinuous at ± 1.

30. $f(x)=\begin{cases}2x+1 & \text{if } x\geq -1\\ 1 & \text{if } x<-1\end{cases}$

For $x < -1$, $f(x) = 1$, which is a polynomial and hence continuous. For $x > -1$, $f(x) = 2x + 1$, which is a polynomial and hence continuous. Because $\lim_{x\to -1^-} f(x)=\lim_{x\to -1^-} 1=1$ and $\lim_{x\to -1^+} f(x)=\lim_{x\to -1^+}(2x+1)=-1$, $\lim_{x\to -1} f(x)$ does not exist.

Thus f is discontinuous at $x = -1$.

32. $f(x)=\begin{cases}x-3 & \text{if } x>2\\ 3-2x & \text{if } x<2\end{cases}$

For $x < 2$, $f(x) = 3 - 2x$, which is a polynomial and hence continuous. For $x > 2$, $f(x) = x - 3$, which is a polynomial and hence continuous. Because f is not defined at $x = 2$, it is discontinuous there.