MAT 101
MATHEMATICS FOR SOCIAL SCIENCES

MULTIVARIATE CALCULUS
Solutions of Selected Problems from Sections 17.2 and 17.10

Problems 17.2 (page 754)
Suggested Problems: 1-35

2. $f(x, y) = 2x^2 + 3xy$

$f_x(x, y) = 2(2x) + 3(1)y = 4x + 3y$

$f_y(x, y) = 0 + 3x(1) = 3x$

4. $f(x, y) = \ln 2$

$f_x(x, y) = 0$

$f_y(x, y) = 0$

6. $g(x, y) = (x+1)^2 + (y-3)^3 + 5xy^3 - 2$

$$g_x(x, y) = 2(x+1) + 0 + 5(1)y^3 - 0$$
$$= 2(x+1) + 5y^3$$
$$g_y(x, y) = 0 + 3(y-3)^2 + 5x\left(3y^2\right) - 0$$
$$= 3(y-3)^2 + 15xy^2$$

8. $g(w, z) = \sqrt[3]{w^2 + z^2} = \left(w^2 + z^2\right)^{\frac{1}{3}}$

$$g_w(w, z) = \frac{1}{3}\left(w^2 + z^2\right)^{-\frac{2}{3}}(2w) = \frac{2w}{3\left(w^2 + z^2\right)^{\frac{2}{3}}}$$

$$g_z(w, z) = \frac{1}{3}\left(w^2 + z^2\right)^{-\frac{2}{3}}(2z) = \frac{2z}{3\left(w^2 + z^2\right)^{\frac{2}{3}}}$$

10. $h(u,\ v)=\dfrac{8uv^2}{u^2+v^2}$

$$h_u(u,\ v)=8v^2\,\frac{\left(u^2+v^2\right)(1)-u(2u)}{\left(u^2+v^2\right)^2}$$

$$=\frac{8v^2\left(v^2-u^2\right)}{\left(u^2+v^2\right)^2}$$

$$h_v(u,\ v)=8u\frac{\left(u^2+v^2\right)(2v)-v^2(2v)}{\left(u^2+v^2\right)^2}$$

$$=\frac{16u^3v}{\left(u^2+v^2\right)^2}$$

16. $z=(x^2+y^2)e^{2x+3y+1}$

$$\frac{\partial z}{\partial x}=(x^2+y^2)[e^{2x+3y+1}(2)]+e^{2x+3y+1}[2x]=(2x^2+2y^2+2x)e^{2x+3y+1}$$

$$\frac{\partial z}{\partial y}=(x^2+y^2)[e^{2x+3y+1}(3)]+e^{2x+3y+1}[2y]=(3x^2+3y^2+2y)e^{2x+3y+1}$$

18. $z=\ln(5x^3y^2+2y^4)^4=4\ln(5x^3y^2+2y^4)$

$$\frac{\partial z}{\partial x}=4\cdot\frac{1}{5x^3y^2+2y^4}[5(3x^2)y^2+0]=\frac{60x^2y^2}{5x^3y^2+2y^4}=\frac{60x^2y^2}{y^2(5x^3+2y^2)}=\frac{60x^2}{5x^3+2y^2}$$

$$\frac{\partial z}{\partial y}=4\cdot\frac{1}{5x^3y^2+2y^4}[5x^3(2y)+2(4y^3)]=\frac{4(10x^3y+8y^3)}{5x^3y^2+2y^4}=\frac{8y(5x^3+4y^2)}{y(5x^3y+2y^3)}=\frac{8(5x^3+4y^2)}{5x^3y+2y^3}$$

20. $f(r,\ s)=(rs)^{\frac{1}{2}}e^{2+r}$

$$f_r(r,\ s)=(rs)^{\frac{1}{2}}\left[e^{2+r}(1)\right]+e^{2+r}\left[\frac{1}{2}(rs)^{-\frac{1}{2}}(s)\right]=\left[\sqrt{rs}+\frac{s}{2\sqrt{rs}}\right]e^{2+r}$$

$$f_s(r,\ s)=e^{2+r}\left[\frac{1}{2}(rs)^{-\frac{1}{2}}(r)\right]=\frac{re^{2+r}}{2\sqrt{rs}}$$

22. $f(r,\ s)=\left(5r^2+3s^3\right)(2r-5s)$

$$f_r(r,\ s)=\left(5r^2+3s^3\right)[2]+(2r-5s)[10r]=2\left(5r^2+3s^3\right)+10r(2r-5s)$$

$$f_s(r,\ s)=\left(5r^2+3s^3\right)[-5]+(2r-5s)\left[9s^2\right]=-5\left(5r^2+3s^3\right)+9s^2(2r-5s)$$

24. $g(x,\ y,\ z) = 2xy^2z^6 - 4x^2y^3z^2 + 3xyz$

$$g_x(x,\ y,\ z) = 2(1)y^2z^6 - 4(2x)y^3z^2 + 3(1)yz$$
$$= 2y^2z^6 - 8xy^3z^2 + 3yz$$
$$g_y(x,\ y,\ z) = 2x(2y)z^6 - 4x^2(3y^2)z^2 + 3x(1)z$$
$$= 4xyz^6 - 12x^2y^2z^2 + 3xz$$
$$g_z(x,\ y,\ z) = 2xy^2(6z^5) - 4x^2y^3(2z) + 3xy(1)$$
$$= 12xy^2z^5 - 8x^2y^3z + 3xy$$

26. $g(r,\ s,\ t,\ u) = rs\ln(2t + 5u)$

$$g_r(r,\ s,\ t,\ u) = (1)s\ln(2t+5u) = s\ln(2t+5u)$$
$$g_s(r,\ s,\ t,\ u) = r(1)\ln(2t+5u) = r\ln(2t+5u)$$
$$g_t(r,\ s,\ t,\ u) = rs\left[\frac{1}{2t+5u}(2)\right] = \frac{2rs}{2t+5u}$$
$$g_u(r,\ s,\ t,\ u) = rs\left[\frac{1}{2t+5u}(5)\right] = \frac{5rs}{2t+5u}$$

28. $z = \sqrt{2x^3 + 5xy + 2y^2}$

$$\frac{\partial z}{\partial x} = \frac{6x^2 + 5y}{2\sqrt{2x^3 + 5xy + 2y^2}}$$
$$\left.\frac{\partial z}{\partial x}\right|_{(0,\ 1)} = \frac{5}{2\sqrt{2}}$$

30. $g(x,\ y,\ z) = \dfrac{3x^2y^2 + 2xy + x - y}{xy - yz + xz}$

$$g_y(x,\ y,\ z) = \frac{(xy - yz + xz)(6x^2y + 2x - 1) - (3x^2y^2 + 2xy + x - y)(x - z)}{(xy - yz + xz)^2}$$
$$g_y(1,\ 1,\ 5) = \frac{(1-5+5)(6+2-1) - (3+2+1-1)(1-5)}{(1-5+5)^2} = 27$$

Problem 17.10 (p. 793)
Suggested Problems: 1-18

2. $\int_1^4\int_0^3 y\,dy\,dx = \int_1^4 \frac{y^2}{2}\Big|_0^3 dx = \int_1^4 \frac{9}{2}dx = \frac{9x}{2}\Big|_1^4 = \frac{27}{2}$

4. $\int_0^2\int_0^3 x^2 dy\,dx = \int_0^2 x^2 y\Big|_0^3 dx = \int_0^2 3x^2 dx = x^3\Big|_0^2 = 8$

6. $\int_{-2}^3\int_0^2 (y^2 - 2xy)\,dy\,dx = \int_{-2}^3 \left(\frac{y^3}{3} - xy^2\right)\Big|_0^2 dx$

$= \int_{-2}^3 \left[\left(\frac{8}{3} - 4x\right) - 0\right] dx = \int_{-2}^3 \left(\frac{8}{3} - 4x\right) dx$

$= \left(\frac{8}{3}x - 2x^2\right)\Big|_{-2}^3 = (8-18) - \left(-\frac{16}{3} - 8\right)$

$= \frac{10}{3}$

8. $\int_0^3\int_0^x \left(x^2 + y^2\right) dy\,dx = \int_0^3 \left(x^2 y + \frac{y^3}{3}\right)\Big|_0^x dx$

$= \int_0^3 \left(x^3 + \frac{x^3}{3}\right) dx = \int_0^3 \frac{4x^3}{3} dx = \frac{x^4}{3}\Big|_0^3 = 27$

10. $\int_1^2\int_0^{x-1} 2y\,dy\,dx = \int_1^2 y^2\Big|_0^{x-1} dx$

$= \int_1^2 (x-1)^2 dx = \frac{(x-1)^3}{3}\Big|_1^2 = \frac{1}{3}$

12. $\int_0^2\int_0^{x^2} xy\,dy\,dx = \int_0^2 \frac{xy^2}{2}\Big|_0^{x^2} dx$

$= \int_0^2 \frac{x^5}{2} dx = \frac{x^6}{12}\Big|_0^2 = \frac{16}{3}$

14. $\int_0^1\int_{y^2}^y y\,dx\,dy = \int_0^1 xy\Big|_{y^2}^y dy = \int_0^1 (y^2 - y^3)\,dy$

$= \left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

16. $\int_0^3\int_{y^2}^{3y} 5x\,dx\,dy = \int_0^3 \frac{5x^2}{2}\Big|_{y^2}^{3y} dy$

$= \int_0^3 \left(\frac{45y^2}{2} - \frac{5y^4}{2}\right) dy$

$= \left(\frac{15y^3}{2} - \frac{y^5}{2}\right)\Big|_0^3 = \frac{405}{2} - \frac{243}{2} = 81$

18. $\int_0^1\int_0^1 e^{y-x} dx\,dy = \int_0^1 -e^{y-x}\Big|_0^1 dy$

$= \int_0^1 (-e^{y-1} + e^y)\,dy = (-e^{y-1} + e^y)\Big|_0^1$

$= (-e^0 + e^1) - (-e^{-1} + e^0) = -1 + e + e^{-1} - 1$

$= -2 + e + e^{-1}$