METU NORTHERN CYPRUS CAMPUS

MAT 101
MATHEMATICS FOR SOCIAL SCIENCES

INTEGRATION 3
Solutions of Selected Problems from Sections 14.7 and 14.10

Problems 14.7 (page 657)
Suggested Problems: 1-43

2. $\int_2^4 (1-e)\,dx = (1-e)\,x\Big|_2^4$
$= 4(1-e) - 2(1-e) = 2(1-e)$

6. $\int_{-1}^1 (4-9y) = \left(4y - \frac{9y^2}{2}\right)\Bigg|_{-1}^1 = -\frac{1}{2} - \left(-\frac{17}{2}\right)$

12. $\int_1^2 \frac{x^{-2}}{2}\,dx = -\frac{x^{-1}}{2}\Bigg|_1^2 = -\frac{1}{2x}\Bigg|_1^2$
$= -\frac{1}{4} - \left(-\frac{1}{2}\right) = \frac{1}{4}$

16. $\int_9^{36}\left(\sqrt{x}-2\right)dx = \left(\frac{2}{3}x^{\frac{3}{2}} - 2x\right)\Bigg|_9^{36} = 72 - 0 = 72$

18. $\int_1^8\left(x^{\frac{1}{3}} - x^{-\frac{1}{3}}\right)dx = \left(\frac{3x^{\frac{4}{3}}}{4} - \frac{3x^{\frac{2}{3}}}{2}\right)\Bigg|_1^8$
$= 6 - \left(-\frac{3}{4}\right) = \frac{27}{4}$

22. $\int_{-e^e}^{-1} \frac{6}{x}\,dx = 6\ln|x|\Big|_{-e^e}^{-1} = 6\ln 1 - 6\ln e^e$
$= 0 - 6e = -6e$

24. $\int_2^{e+1} \frac{1}{x-1}\,dx = \ln|x-1|\Big|_2^{e+1} = \ln e - \ln 1 = 1 - 0 = 1$

30. $\int_{-1}^1 q\sqrt{q^2+3}\,dq = \frac{1}{2}\int_{-1}^1 \left(q^2+3\right)^{\frac{1}{2}}\,[2q\;dq]$
$= \frac{\left(q^2+3\right)^{\frac{3}{2}}}{3}\Bigg|_{-1}^1 = \frac{8}{3} - \frac{8}{3} = 0$

36. $\int_{-2}^1 8|x|\,dx = 8\left(\int_{-2}^0 -x\,dx + \int_0^1 x\,dx\right)$
$= 8\left(-\frac{x^2}{2}\Bigg|_{-2}^0 + \frac{x^2}{2}\Bigg|_0^1\right) = 8\left\{[0-(-2)] + \left(\frac{1}{2} - 0\right)\right\}$

40. $\int_1^{95} \frac{x}{\ln e^x}\,dx = \int_1^{95} \frac{x}{x}\,dx = \int_1^{95} 1\,dx = x\Big|_1^{95}$
$= 95 - 1 = 94$

42. $\int_{-1}^1 \frac{2}{1+e^x}\,dx = \int_{-1}^1 \frac{2}{1+e^x}\cdot\frac{e^{-x}}{e^{-x}}\,dx$
$= 2\int_{-1}^1 \frac{e^{-x}}{e^{-x}+1}\,dx$
$= -2\int_{-1}^1 \frac{1}{e^{-x}+1}\left[-e^{-x}dx\right] = -2\ln\left(e^{-x}+1\right)\Big|_{-1}^1$
$= -2\ln\left(e^{-1}+1\right) + 2\ln(e+1) = 2\ln\frac{e+1}{\frac{1}{e}+1}$
$= 2\ln\frac{e^2+e}{1+e} = 2\ln\frac{e(e+1)}{1+e} = 2\ln e = 2$

Problem 14.10 (p. 673)
Suggested Problems: 1-34

2. Area $= \int_a^b (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx = \int_0^2 (2x - x^2)\,dx$

4. Intersection points: $x(x-3)^2 = 2x$, $x(x-3)^2 - 2x = 0$, $x[$

(from the quadratic formula)

Area $= \int_0^{3-\sqrt{2}} (y_{\text{UPPER}} - y_{\text{LOWER}})\,dx + \int_{3-\sqrt{2}}^{3+\sqrt{2}} (y_{\text{UPPER}} - y$

$= \int_0^{3-\sqrt{2}} \left[x(x-3)^2 - 2x\right]dx + \int_{3-\sqrt{2}}^{3+\sqrt{2}} \left[2x - x(x-3)^2\right]dx$

6. The graphs of $y = 2x$ and $y = -2x - 8$ intersect when $2x = -2$

use horizontal elements, where y ranges from -4 to 4. Solvi

gives $2x = -y - 8$, $x = \dfrac{-y-8}{2}$.

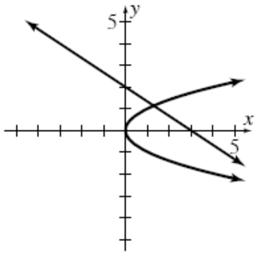
Area $= \int_{-4}^{4} (x_{\text{RIGHT}} - x_{\text{LEFT}})dy = \int_{-4}^{4} \left[\dfrac{y}{2} - \left(\dfrac{-y-8}{2}\right)\right] dy$

8. The curves $y^2 = x$ and $2y = 3 - x$ (or $x = 3 - 2y$)

intersect when $y^2 = 3 - 2y$, $y^2 + 2y - 3 = 0$,
$(y + 3)(y - 1) = 0 \Rightarrow y = -3$ or 1. We use
horizontal elements.

Area $= \int_0^1 (x_{\text{RIGHT}} - x_{\text{LEFT}})\,dy$

$= \int_0^1 [(3 - 2y) - y^2]\,dy$



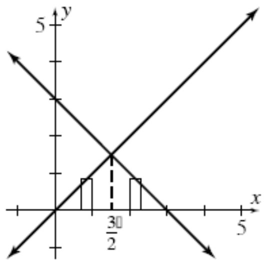
10. $y = x$, $y = -x + 3$, $y = 0$. Region appears below.

Intersection: $x = -x + 3$, $2x = 3$, $x = \dfrac{3}{2}$

Area $= \int_0^{3/2} x\,dx + \int_{3/2}^{3} (-x+3)\,dx$

$= \dfrac{x^2}{2}\Big|_0^{3/2} + \left(-\dfrac{x^2}{2} + 3x\right)\Big|_{3/2}^{3}$

$= \left[\dfrac{9}{8} - 0\right] + \left[\left(-\dfrac{9}{2} + 9\right) - \left(-\dfrac{9}{8} + \dfrac{9}{2}\right)\right] = \dfrac{9}{4}$



12. $y = x^2 + 1$, $y = x + 3$. Region appears below.

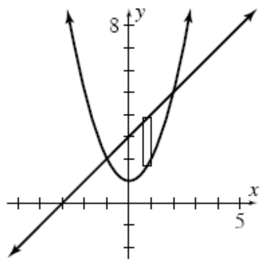
Intersection: $x^2 + 1 = x + 3$, $x^2 - x - 2 = 0$,
$(x + 1)(x - 2) = 0$, so $x = -1, 2$

Area $= \int_{-1}^{2} \left[(x+3) - (x^2+1)\right] dx$

$= \int_{-1}^{2} (x + 2 - x^2)\,dx$

$= \left(\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3}\right)\Big|_{-1}^{2}$

$= \left(2 + 4 - \dfrac{8}{3}\right) - \left(\dfrac{1}{2} - 2 + \dfrac{1}{3}\right) = \dfrac{9}{2}$

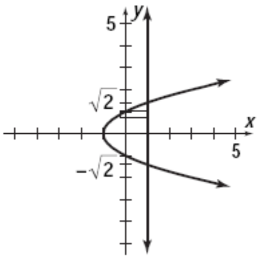


14. $y^2 = x+1$, $x = 1$. Region appears below.

Intersection: $y^2 = 2,\ y = \pm\sqrt{2}$

$$\text{Area } = \int_{-\sqrt{2}}^{\sqrt{2}} \left[1-\left(y^2-1\right)\right] dy = \left(2y - \frac{y^3}{3}\right)\Bigg|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right) - \left(-2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) = \frac{8\sqrt{2}}{3}$$



20. $y = x^3,\ y = x + 6,\ x = 0$

Region appears below.

Intersection: $x^3 = x+6,\quad x^3 - x - 6 = 0,$

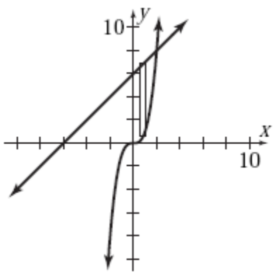
$(x-2)(x^2+2x+3) = 0 \Rightarrow x = 2$

$x^3 = 0 \Rightarrow x = 0$

$$\text{Area} = \int_0^2 [(x+6) - x^3]\, dx$$

$$= \left(\frac{x^2}{2} + 6x - \frac{x^4}{4}\right)\Bigg|_0^2$$

$$= (2+12-4) - (0) = 10$$



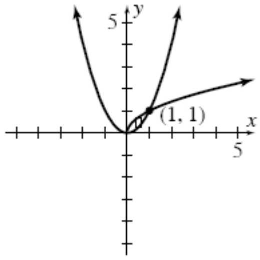
22. $y = \sqrt{x},\ y = x^2$. Region appears below.

Intersection: $x^2 = \sqrt{x},\ x^4 = x,\ x^4 - x = 0$,

$x\left(x^3-1\right) = 0$, so $x = 0,\ 1$.

$$\text{Area } = \int_0^1 \left(\sqrt{x} - x^2\right) dx = \left(\frac{2x^{\frac{3}{2}}}{3} - \frac{x^3}{3}\right)\Bigg|_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - 0 = \frac{1}{3}$$



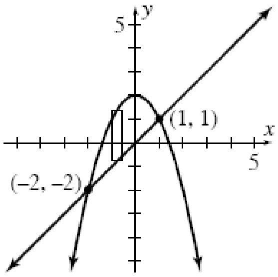
24. $y = 2 - x^2$, $y = x$. Region appears below.

Intersection: $x = 2 - x^2$, $x^2 + x - 2 = 0$,

$(x+2)(x-1) = 0 \Rightarrow x = -2$ or 1.

$$\text{Area } \int_{-2}^{1} \left[\left(2-x^2\right) - x\right] dx = \left(2x - \frac{x^3}{3} - \frac{x^2}{2}\right)\Bigg|_{-2}^{1}$$

$$= \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right) = \frac{9}{2}$$



28. $y = x^3 + x$, $y = 0$ (x-axis), $x = -1$, $x = 2$

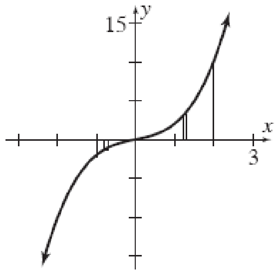
Region appears below.

$$\text{Area} = \int_{-1}^{0} -(x^3 + x)\,dx + \int_{0}^{2} (x^3 + x)\,dx$$

$$= \left(-\frac{x^4}{4} - \frac{x^2}{2}\right)\Bigg|_{-1}^{0} + \left(\frac{x^4}{4} + \frac{x^2}{2}\right)\Bigg|_{0}^{2}$$

$$= \left[0 - \left(-\frac{1}{4} - \frac{1}{2}\right)\right] + [(4+2) - 0]$$

$$= \frac{27}{4}$$



30. $y = x^3$, $y = \sqrt{x}$. Region appears below. Intersection: $x^3 = \sqrt{x}$, $x^6 = x$, $x^6 - x = 0$, $x\left(x^5 - 1\right) = 0$, $x = 0, 1$

$$\text{Area} = \int_{0}^{1} \left(\sqrt{x} - x^3\right) dx = \left(\frac{2x^{\frac{3}{2}}}{3} - \frac{x^4}{4}\right)\Bigg|_{0}^{1}$$

$$= \left(\frac{2}{3} - \frac{1}{4}\right) - 0 = \frac{5}{12}$$