MAT 101
MATHEMATICS FOR SOCIAL SCIENCES

FUNCTIONS AND GRAPHS
Solutions of Selected Problems from Sections 2.1-2.4

Problem 2.1 (page 81)
Suggested Problems: 1-42

2. The functions are different because they have different domains. The domain of $G(x)$ is $[-1, \infty)$ (all real numbers ≥ -1) because you can only take the square root of a non-negative number, while the domain of $H(x)$ is all real numbers.

4. The functions are equal. For $x = 3$ we have $f(3) = 2$ and $g(3) = 3 - 1 = 2$, hence $f(3) = g(3)$. For $x \neq 3$, we have

$$f(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-3)(x-1)}{x-3} = x - 1.$$

Note that we can cancel the $x - 3$ because we are assuming $x \neq 3$ and so $x - 3 \neq 0$. Thus for $x \neq 3$ $f(x) = x - 1 = g(x)$.
$f(x) = g(x)$ for all real numbers and they have the same domains, thus the functions are equal.

6. Any real number can be used for x.
Answer: all real numbers

8. For $\sqrt{z-1}$ to be real, $z - 1 \geq 0$, so $z \geq 1$. We exclude values of z for which $\sqrt{z-1} = 0$, so $z - 1 = 0$, thus $z = 1$.
Answer: all real numbers > 1

10. We exclude values of x for which
$x + 8 = 0$
$x = -8$
Answer: all real numbers except -8

14. We exclude values of x for which

$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3,\ 2$$

Answer: all real numbers except -3 and 2

16. r^2+1 is never 0.
Answer: all real numbers

18. $H(s)=5s^2-3$

$H(4)=5(4)^2-3=80-3=77$

$H(\sqrt{2})=5(\sqrt{2})^2-3=10-3=7$

$H\left(\frac{2}{3}\right)=5\left(\frac{2}{3}\right)^2-3=\frac{20}{9}-3=-\frac{7}{9}$

22. $h(v)=\frac{1}{\sqrt{v}}$

$h(16)=\frac{1}{\sqrt{16}}=\frac{1}{4}$

$h\left(\frac{1}{4}\right)=\frac{1}{\sqrt{\frac{1}{4}}}=\frac{1}{\frac{1}{2}}=2$

$h(1-x)=\frac{1}{\sqrt{1-x}}$

24. $H(x)=(x+4)^2$

$H(0)=(0+4)^2=16$

$H(2)=(2+4)^2=6^2=36$

$H(t-4)=[(t-4)+4]^2=t^2$

26. $k(x)=\sqrt{x-3}$

$k(4)=\sqrt{4-3}=\sqrt{1}=1$

$k(3)=\sqrt{3-3}=\sqrt{0}=0$

$k(x+1)-k(x)=\sqrt{(x+1)-3}-\sqrt{x-3}$
$=\sqrt{x-2}-\sqrt{x-3}$

34. $f(x)=x^3$

a. $f(x+h)=(x+h)^3=x^3+3x^2h+3xh^2+h^3$

b. $\frac{f(x+h)-f(x)}{h}=\frac{(x^3+3x^2h+3xh^2+h^3)-x^3}{h}=\frac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$

36. $f(x)=\dfrac{x+8}{x}$

a. $f(x+h)=\dfrac{(x+h)+8}{x+h}=\dfrac{x+h+8}{x+h}$

b. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{\frac{x+h+8}{x+h}-\frac{x+8}{x}}{h}=\dfrac{x(x+h)\left(\frac{x+h+8}{x+h}-\frac{x+8}{x}\right)}{x(x+h)h}=\dfrac{x(x+h+8)-(x+h)(x+8)}{x(x+h)h}$

$=\dfrac{x^2+xh+8x-x^2-hx-8x-8h}{x(x+h)h}=\dfrac{-8h}{x(x+h)h}=-\dfrac{8}{x(x+h)}$

40. $x^2+y=0$

The form $y=-x^2$ shows that for each input x there is exactly one output, $-x^2$. Thus y is a function of x. Solving for x gives $x=\pm\sqrt{-y}$. If, for example, $y=-1$, then $x=\pm 1$, so x is not a function of y.

42. $x^2+y^2=1$

Solving for y we have $y=\pm\sqrt{1-x^2}$. If $x=0$, then $y=\pm 1$, so y is not a function of x. Solving for x gives $x=\pm\sqrt{1-y^2}$. If $y=0$, then $x=\pm 1$, so x is not a function of y.

Problem 2.2 (p. 85)
Suggested Problems: 1-28

2. $f(x)=\dfrac{x^3+7x-3}{3}=\dfrac{1}{3}x^3+\dfrac{7}{3}x-1$, which is a polynomial function.

4. yes

8. $g(x)=4x^{-4}=\dfrac{4}{x^4}$, which is a rational function.

10. all real numbers

12. all x such that $1\le x\le 3$

18. $g(x) = |x-3|$

$g(10) = |10-3| = |7| = 7$

$g(3) = |3-3| = |0| = 0$

$g(-3) = |-3-3| = |-6| = 6$

22. $F(3) = 3^2 - 3(3) + 1 = 1$

$F(-3) = 2(-3) - 5 = -11$

$F(2)$ is not defined.

28. $\frac{8!}{5!(8-5)!} = \frac{8!}{5!\cdot 3!}$

$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(5\cdot 4\cdot 3\cdot 2\cdot 1)(3\cdot 2\cdot 1)}$

$= \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1}$

$= 8\cdot 7$

$= 56$

Problem 2.3 (p. 90)
Suggested Problems: 1-16

2. $f(x) = 2x$, $g(x) = 6 + x$

a. $(f+g)(x) = f(x) + g(x)$
$= (2x) + (6+x)$
$= 3x + 6$

b. $(f-g)(x) = f(x) - g(x)$
$= (2x) - (6+x)$
$= x - 6$

c. $(f-g)(4) = (4) - 6 = -2$

d. $(fg)(x) = f(x)g(x) = 2x(6+x) = 12x + 2x^2$

e. $\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{2x}{6+x}$

f. $\frac{f}{g}(2) = \frac{2(2)}{6+2} = \frac{4}{8} = \frac{1}{2}$

g. $(f\circ g)(x) = f(g(x))$
$= f(6+x)$
$= 2(6+x)$
$= 12 + 2x$

h. $(g\circ f)(x) = g(f(x)) = g(2x) = 6 + 2x$

i. $(g\circ f)(2) = 6 + 2(2) = 6 + 4 = 10$

6. $(f \circ g)(p) = f(g(p))$
$$= f\left(\frac{p-2}{3}\right)$$
$$= \frac{4}{\frac{p-2}{3}}$$
$$= \frac{12}{p-2}$$

$$(g \circ f)(p) = g(f(p)) = g\left(\frac{4}{p}\right) = \frac{\frac{4}{p}-2}{3} = \frac{4-2p}{3p}$$

8. $(F \circ G)(t) = F(G(t))$
$$= F(3t^2+4t+2)$$
$$= \sqrt{3t^2+4t+2}$$
$(G \circ F)(t) = G(F(t))$
$$= G\left(\sqrt{t}\right)$$
$$= 3\left(\sqrt{t}\right)^2 + 4\left(\sqrt{t}\right) + 2$$
$$= 3t + 4\sqrt{t} + 2$$

12. Let $g(x) = x^2 - 2$ and $f(x) = \sqrt{x}$. Then
$$h(x) = \sqrt{x^2-2} = \sqrt{g(x)} = f(g(x))$$

14. Let $g(x) = 9x^3 - 5x$ and $f(x) = x^3 - x^2 + 11$.
Then $h(x) = [g(x)]^3 - [g(x)]^2 + 11 = f(g(x))$

16. Let $g(x) = 3x - 5$ and $f(x) = \dfrac{2-x}{x^2+2}$. Then
$$h(x) = \frac{2-(3x-5)}{(3x-5)^2+2} = f(g(x)).$$

Problem 2.4 (p. 93)
Suggested Problems: 1-12

2. $g^{-1}(x) = \dfrac{x}{2} - \dfrac{1}{2}$

4. $f^{-1}(x) = \dfrac{\sqrt{x}}{4} + \dfrac{5}{4}$

6. $r(V) = \sqrt[3]{\dfrac{3V}{4\pi}}$

8. $g(x) = (5x+12)^2$ is not one-to-one, because $g(x_1) = g(x_2)$ does not imply $x_1 = x_2$. For example, $g\left(-\frac{11}{5}\right) = g\left(-\frac{13}{5}\right) = 1$.

10. $F(x) = |x-9|$ is not one-to-one, because $F(x_1) = F(x_2)$ does not imply $x_1 = x_2$. For example, $F(8) = F(10) = 1$.

12. The inverse of $V(r) = \frac{4}{3}\pi r^3$ is $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$, so the solution is $r(100) = \sqrt[3]{\frac{3(100)}{4\pi}}$.