CVE 372 HYDROMECHANICS
PIPE FLOW

FORMULA SHEET

PIPE FLOW

Reynolds Number: $\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$

Continuity Equation: $Q = V_1 A_1 = V_2 A_2 = VA = \text{constant}$

Energy Equation: $z_1 + \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} - H_P + H_T + \sum h_f + \sum h_m$

Pump Head: $H_P = \frac{\eta P_P}{\gamma Q}$ 1 watt = 1 N m/s 1 hp = 745.7 watt

Turbine Head: $H_T = \frac{P_T}{\eta \gamma Q}$

Pump Power: $P_p = \frac{\gamma Q H_p}{\eta_p}$

Parallel Pumps: $\eta_p = \frac{\gamma H_p \sum Q}{\sum P_p}$

Pumps in Series: $\eta_p = \frac{\gamma Q \sum H_p}{\sum P_p}$

Momentum Equation: $Fp_1 - Fp_2 + W \sin\theta - F_f = \rho Q\left(\beta_2 V_2 - \beta_1 V_1\right)$

Total Head Loss: $h_L = h_f + h_m$

Head Loss (Darcy-Weisbach Equation): $h_\mathrm{f} = f \frac{L}{D} \frac{V^2}{2g} = \frac{8 f L Q^2}{g \pi^2 D^5} = \frac{4 \tau_w L}{\gamma D}$

Head Loss (Hazen – Williams Equation): $h_\mathrm{f} = \frac{6.8}{C^{1.85}} \frac{L}{D^{1.165}} V^{1.85} = \frac{10.6}{C^{1.85}} \frac{L}{D^{4.87}} Q^{1.85}$

Head Loss (local/minor): $h_\mathrm{m} = K_m \frac{V^2}{2g} = K_m \frac{8Q^2}{g \pi^2 D^4}$

Pressure Change (horizontal pipe): $\Delta p = \frac{2\tau L}{r} = \frac{4 L \tau_w}{D} = \gamma h_L$

Pressure Change (sloped pipe): $\frac{\Delta p - \gamma L \sin\theta}{L} = \frac{2\tau}{r}$ Velocity: $V = \frac{(\Delta p - \gamma L \sin\theta) D^2}{32 \mu L}$

Non-circular Pipes: $R_h = \frac{A}{P}$ $D_h = 4 R_h$ $h_\mathrm{f} = f \frac{L}{D_h} \frac{V^2}{2g}$ $\mathrm{Re} = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu}$

Networks: $\Delta Q = -\dfrac{\Sigma K|Q_a|Q_a}{\Sigma 2K|Q_a|}$ (Hardy-Cross Method)

where $K = \dfrac{8fL}{g\pi^2 D^5} + \dfrac{8K_m}{g\pi^2 D^4}$ for each pipe.

Equivalent Pipe Equations:

Pipes in Parallel $\left(\dfrac{D_{eq}^5}{f_{eq}L_{eq}}\right)^{1/2} = \sum_{i=1}^{n}\left(\dfrac{D_i^5}{f_i L_i}\right)^{1/2}$

Pipes in Series $\dfrac{f_{eq}L_{eq}}{D_{eq}^5} = \sum_{i=1}^{n}\dfrac{f_i L_i}{D_i^5}$

LAMINAR FLOW IN CIRCULAR PIPES

Entrance Length: $\dfrac{l_e}{D} = 0.06\,\text{Re}$

Shear Stress: $\tau = \mu\dfrac{du}{dy} = \tau_w \dfrac{2r}{D}$

Wall Shear Stress: $\tau_w = \dfrac{\Delta pD}{4L} = \dfrac{8\mu V}{D} = \dfrac{f}{8}\rho V^2$

Velocity at the center: $V_c = \left(\dfrac{\Delta pD^2}{16\mu l}\right)$

Velocity at r: $u(r) = \left(\dfrac{\Delta pD^2}{16\mu L}\right)\left(1-\left(\dfrac{2r}{D}\right)^2\right) = V_c\left(1-\left(\dfrac{2r}{D}\right)^2\right)$

Average Velocity: $V = \dfrac{V_c}{2} = \dfrac{\Delta pD^2}{32\mu L}$

Flow Rate: $Q = VA = \dfrac{\pi D^4 \Delta p}{128\mu L}$

Friction (major) Loss: $h_f = f\dfrac{L}{D}\dfrac{V^2}{2g} = \dfrac{4L\tau_w}{\gamma D} = \dfrac{32L\mu V}{\gamma D^2}$

Friction Factor: $f = \dfrac{64}{\text{Re}} = \dfrac{8\tau_w}{\rho V^2}$

Friction Velocity: $u^* = \sqrt{\dfrac{\tau_w}{\rho}} = \sqrt{\dfrac{f}{8}}V$

TURBULENT FLOW IN CIRCULAR PIPES

Entrance Length: $\frac{l_e}{D} = 4.4(\text{Re})^{1/6}$

Shear Stress: $\tau = (\mu + \mu_t)\frac{du}{dy} = \mu\frac{du}{dy} - \rho\overline{u'v'} = \tau_{\text{lam}} + \tau_{\text{turb}}$

Wall Shear Stress: $\tau_w = \frac{\Delta pD}{4L} = \frac{8\mu V}{D} = \frac{f}{8}\rho V^2$

Friction Velocity: $u^* = \sqrt{\frac{\tau_w}{\rho}}$

Average Velocity in the Viscous Sublayer: $\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu}$

Viscous sublayer thickness: $\delta_* = \frac{5\nu}{u^*}$

Average Velocity in the Overlap Region: $\frac{\bar{u}}{u^*} = 2.5\ln\left(\frac{yu^*}{\nu}\right) + 5.0$

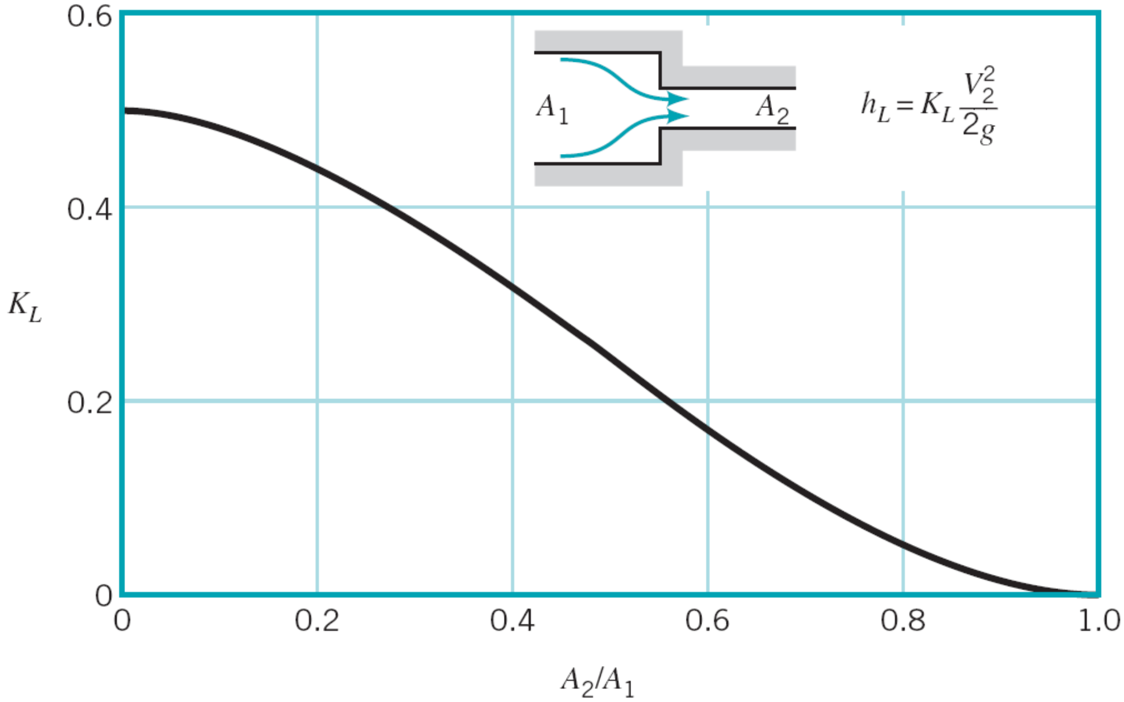
Average Velocity in the Turbuent Region: $\frac{V_c - \bar{u}}{u^*} = 2.5\ln\left(\frac{R}{y}\right)$ or $\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$

$$\frac{V}{V_c} = \frac{2n^2}{(n+1)(2n+1)}$$

The Gradient of Average Velocity: $\frac{d\bar{u}}{dr} = -\frac{V_c}{nR}\left(1 - \frac{r}{R}\right)^{\frac{1-n}{n}}$

Friction Factor: $\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$

Loss Coefficient for a Sudden Contraction:



Loss Coefficient for a Sudden Expansion: