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Vibration Characteristics of Various Wide Flange Steel Beams and Columns

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Abstract: Vibration characteristics of steel framed structures are affected by accurate modelling of the mass and stiffness matrices of beam and column members. In this regards, wide flange sections are the most popularly used steel sections, and calculation of axial, flexural and shear responses of these sections become critical. In this study, a mixed formulation frame finite element is developed from three-fields Hu-Washizu-Barr functional. Consistent mass matrix of the element is obtained such that determination of vibration frequencies of members with varying geometry and material distribution is modelled without any need for specification of different displacement shape functions for each individual case. An accurate shear correction coefficient for wide flange I and H sections is taken into account in order to get closer match with exact solutions. Comparative study is undertaken by the use of proposed beam finite element solutions and 3 dimensional solid finite element analyses in ANSYS. Results indicate that proposed beam finite element can get fundamental and higher modes of vibration for varying aspect ratios of wide flange beams and columns.

Keywords: Steel framed structures; Wide Flange Sections; Finite element modelling; Vibration characteristics

1. Introduction

The strength of the members in a structure determines the performance of the whole system. Likely, deformations have crucial influence on the serviceability of the structures, where different types of deformations have consequential effects on each other due to the continuum phenomenon of the bodies. Shear deformations can be crucial for the determination of the lateral flexibility of steel moment resisting frames. The study by Charney et al. [1] clearly shows the effects of shear deformations for various structural steel beam and column sections. Consideration of the shear effects on the members are provided via the definition of effective shear area. The studies on the shear effects of the cross sections on linear basis [2] or nonlinear basis [3, 4] presented various ways to take into account such effects.

In this study, the members examined in [1] that are W36x135, W24x250 and W14x730 sections and European sections that are HEB180, IPE270 and IPE750x147 are modelled with ANSYS [5] finite element program to verify the performance of the proposed beam model in this paper. The beam finite element model proposed in this research study relies on Hu-Washizu-Barr variational. In order to calculate an accurate stiffness and mass matrix, force-based interpolation functions are used, and the shear correction coefficient suggested by [1] is adopted.

2. Frame Element Formulation

2.1 Kinematic Relations

Displacements on a material point on the section of a beam that deforms in *xy*-plane can be obtained through Timoshenko beam theory as follows;

$$\begin{Bmatrix} u_x(x,y) \\ u_y(x,y) \end{Bmatrix} = \begin{Bmatrix} u(x) - y\theta(x) \\ v(x) \end{Bmatrix} \quad (1)$$

where u_x(x,y) and u_y(x,y) are the displacements in x and y directions, respectively of any point in the section. $u(x)$ is the displacement of the point $(x,0)$ along x-axis. $v(x)$ is the transverse deflections of the point $(x,0)$ from x-axis in y direction. $\theta(x)$ is the small rotation of the beam cross section around z-axis.

The non-zero strain components $\boldsymbol{\varepsilon}$ include the normal strain in the x direction and shear strain with xy component, where these are calculated from section deformations as follows;

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u'(x) - y\theta'(x) \\ -\theta(x) + v'(x) \end{Bmatrix} = \begin{Bmatrix} \varepsilon_a(x) - y\kappa(x) \\ \gamma(x) \end{Bmatrix} = \mathbf{a}_s(y,z)\,\mathbf{e}(x) \tag{2}$$

where $\mathbf{e}(x)$ is the section deformation vector given as follows;

$$\mathbf{e}(x) = \begin{bmatrix} \varepsilon_a(x) & \gamma(x) & \kappa(x) \end{bmatrix}^{\mathrm{T}} \tag{3}$$

In Equation (3), $\varepsilon_a(x)$ is the axial strain of the reference axis, $\gamma(x)$ is the shear deformation along y-axis and κ is the curvature about z-axis. Section deformations can be easily obtained from section reference displacements through a one to one comparison of the terms in Equation (2). Furthermore, section compatibility matrix, $\mathbf{a}_s(y,z)$ introduced in Equation (2) is written as follows;

$$\mathbf{a}_s(y,z) = \begin{bmatrix} 1 & 0 & -y \\ 0 & 1 & 0 \end{bmatrix} \tag{4}$$

2.2 Basic System without Rigid Body Modes and Force Interpolation Functions

Element formulation is proposed in xy-plane, where the formulation considers two end nodes and relies on a transformation from complete system to basic system. In the whole structure, the element has 3 degrees of freedom (dof) per node, resulting in 6 dofs in total, where the nodes are placed at element ends. The complete system is proposed such that the axis of the element is aligned with horizontal x-axis. The basic system is prescribed for the purpose of removing rigid body modes of motion, and the basic system is chosen as the cantilever beam as shown in Figure 1, where the fixed and free ends are the left and right ends, respectively. The transformation matrix, **a** for an element with length L is used to relate element end forces in complete system to basic element forces as follows;

$$\mathbf{p} = \mathbf{a}^{\mathrm{T}}\mathbf{q}; \quad \text{where} \quad \mathbf{a} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & -L & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

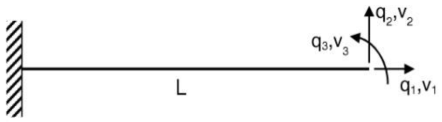

Figure 1:Cantilever basic system forces and deformations

It is also possible to relate basic element deformation vector **v** to displacements in complete system by separating 3 rigid body modes and keeping only the basic deformation modes for the element. By this way, it is feasible to derive flexibility matrix that would have been impossible to get in the complete system because of the singularity caused by rigid body modes. Basic element deformations **v** can be calculated from nodal displacements **u** in complete system as follows;

$$\mathbf{v} = \mathbf{a}\mathbf{u} \tag{6}$$

Basic element forces at free end, **q** are shown in Figure 1 and given in Equation (5). These forces can be related to internal section forces, $\mathbf{s}(x)$ by using the force interpolation matrix $\mathbf{b}(x,L)$ for the cantilever beam configuration as follows;

$$\mathbf{s}(x) = \begin{bmatrix} N(x) & V(x) & M(x) \end{bmatrix}^{\mathrm{T}} = \mathbf{b}(x,L)\,\mathbf{q} + \mathbf{s}_p(x)$$
$$\mathbf{b}(x,L) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & (L-x) & 1 \end{bmatrix} \quad \text{and} \tag{7}$$
$$\mathbf{s}_p(x) = \begin{bmatrix} L-x & 0 \\ 0 & L-x \\ 0 & (L-x)^2/2 \end{bmatrix} \begin{Bmatrix} w_x \\ w_y \end{Bmatrix}$$

By using Equation (7), it is possible to attain exact equilibrium between the forces at free end of the element and forces at any section that is x units away from the fixed end. Section forces are axial force $N(x)$, shear force in y direction $V(x)$, and moment about z-axis $M(x)$. In above equation $\mathbf{s}_p(x)$ is the particular solution for uniformly distributed loads in the axial and transverse directions, i.e. w_x and w_y, respectively. By the way, with this approach, it is easy to calculate the particular solution under arbitrary inter element loads that are concentrated or distributed.

2.3 Variational Base and Finite Element Formulation of the Element

Variational form of the element is written by considering independent element nodal displacements $\mathbf{u}$, element basic forces $\mathbf{q}$, and section deformations $\mathbf{e}$ by using three-fields Hu-Washizu functional and implemented as part of beam finite elements by Taylor et al. [6] and Saritas and Filippou[7]. Extension to dynamic case is achieved through introduction of inertial forces $\mathbf{m}\ddot{\mathbf{u}}$ acting at nodes by considering D'Alembert's principle to get the following variational form of the element

$$\begin{aligned}\delta\Pi_{\mathrm{HW}} &= \int_0^L \delta\mathbf{e}^{\mathrm{T}}\left(\hat{\mathbf{s}}(\mathbf{e}(x)) - \mathbf{b}(x,L)\,\mathbf{q} - \mathbf{s}_{\boldsymbol{p}}\left(x\right)\right) dx \\ &\quad - \delta\mathbf{q}^{\mathrm{T}}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{e}(x)\; dx + \delta\mathbf{q}^{\mathrm{T}}\mathbf{a}_g\mathbf{u} \\ &\quad + \delta\mathbf{u}^{\mathrm{T}}\mathbf{a}_g{}^{\mathrm{T}}\mathbf{q} + \delta\mathbf{u}^{\mathrm{T}}\mathbf{m}\ddot{\mathbf{u}} - \delta\mathbf{u}^{\mathrm{T}}\mathbf{p}_{app} = 0\end{aligned} \tag{8}$$

Above equation can also be obtained by considering the general Hu-Washizu variational form with extension to dynamic case by Barr [8]. Equation (8) should hold for arbitrary $\delta\mathbf{u}$, $\delta\mathbf{q}$ and $\delta\mathbf{e}$, thus the following three equations should be satisfied in order for the Hu-Washizu-Barr variational to be zero.

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{p} \equiv \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{p} = \mathbf{a}_{\mathrm{g}}{}^{\mathrm{T}}\mathbf{q} \tag{9}$$

$$\mathbf{v} \equiv \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\,\mathbf{e}(x)\,dx; \quad \text{where} \quad \mathbf{v} = \mathbf{a}_{\mathrm{g}}\mathbf{u} \tag{10}$$

$$\hat{\mathbf{s}}(\mathbf{e}(x)) \equiv \mathbf{b}(x,L)\mathbf{q} + \mathbf{s}_{\boldsymbol{p}}\left(x\right) \tag{11}$$

Equation (9) is the equation of motion that holds for linear or nonlinear material response, and this equation can be collected for each element to get structure's equation of motion. A numerical time integration scheme can be employed to get a solution. Consequence of viscous damping can be simply achieved by adding $\mathbf{c}\dot{\mathbf{u}}$ to the left hand side of the equation, where $\mathbf{c}$ is the damping matrix. It is also possible to determine resisting forces $\mathbf{p}$ not only in terms of displacements $\mathbf{u}$ but also as a function of velocities $\dot{\mathbf{u}}$ through the use of a material model that considers time-dependent effects, such as visco-elastic or visco-plastic material models.

For linear elastic material response, section deformations can be calculated as $\mathbf{e}=\mathbf{k}_{\mathrm{s}}^{-1}\hat{\mathbf{s}}$ to obtain the section deformations from section forces through the use of section stiffness matrix $\mathbf{k}_{\mathrm{s}}$. Substitution of section deformations $\mathbf{e}$ to Equation (10) gives:

$$\begin{aligned}&\mathbf{a}_{\mathrm{g}}\mathbf{u} = \mathbf{v} = \mathbf{f}\;\mathbf{q}; \\ &\text{where} \quad \mathbf{f} = \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{f}_s\,(x)\,\mathbf{b}(x,L)\,dx\end{aligned} \tag{12}$$

In above equation f is the flexibility matrix of the element in the basic system. $\mathbf{f}_{\mathrm{s}}$ is the section flexibility matrix that can be calculated from the inversion of the section stiffness matrix $\mathbf{k}_{\mathrm{s}}$. Further substitution of above equation for linear elastic response in Equation (9) results in

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{k} = \mathbf{a}^{\mathrm{T}}\mathbf{f}^{-1}\mathbf{a} \tag{13}$$

where $\mathbf{k}$ is the 6×6 element stiffness matrix in the complete system.

As a remark, Equations (10) and (11) are related to the element state determination, i.e. these equations can be solved independent of Equation (9), and then the solution can be condensed out into Equation (9) such that the equations of motion can be assembled for all elements. This process was demonstrated above for the linear elastic case. In general, state determination of the element requires an iterative solution in the case of nonlinear behavior, where Equations (9) to (11) are needed to be solved. This solution requires also the calculation of element flexibility matrix $\mathbf{f}$ under nonlinear response, where taking derivative of element deformations $\mathbf{v}$ in Equation (10) with respect to element forces $\mathbf{q}$ results into the same flexibility integration expression given in Equation (12), but this time the section stiffness will be nonlinear, as well.

2.4 Section Response

Section response can be obtained by the basic assumption that plane sections before deformation remain plane after deformation along the length of the beam by the use of following section compatibility matrix as given in Equation (2), where the section compatibility matrix now contains the shear correction factor κ_{s} as follows

$$\mathbf{a}_s = \mathbf{a}_s(y) = \begin{bmatrix} 1 & 0 & -y \\ 0 & \kappa_s & 0 \end{bmatrix} \tag{14}$$

Shear correction factor κ_s is taken as the inverse of the form factor suggested by Charney et al. [1] for I-sections:

$$\kappa_s = 1/\kappa; \quad \text{where} \quad \kappa = 0.85 + 2.32\frac{b_f t_f}{d\, t_w} \tag{15}$$

In above equation, b_f and t_f stand for the width and thickness of flange, respectively; d is the depth of the section and finally t_w is the thickness of the web.

The section forces are obtained by integration of the stresses that satisfy the material constitutive relations $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon})$ according to

$$\mathbf{s} = \int_A \mathbf{a}_s{}^{\mathrm{T}} \boldsymbol{\sigma}\, dA; \quad \text{where} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix} \tag{16}$$

The derivative of section forces from (16) with respect to the section deformations results in the section tangent stiffness matrix

$$\mathbf{k}_s = \frac{\partial \mathbf{s}}{\partial \mathbf{e}} = \int_A \mathbf{a}_s{}^{\mathrm{T}} \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon})}{\partial \mathbf{e}}\, dA = \int_A \mathbf{a}_s{}^{\mathrm{T}} \mathbf{k}_m \mathbf{a}_s\, dA \tag{17}$$

The material tangent modulus $\mathbf{k}_m$ is obtained from the stress-strain relation according to $\mathbf{k}_m = \partial\boldsymbol{\sigma}(\boldsymbol{\varepsilon})/\partial\boldsymbol{\varepsilon}$. Gauss-quadrature, the midpoint or the trapezoidal rule can be used for the numerical evaluation of the integrals in (16) and (17). While Gauss-quadrature gives better results for smooth strain distributions and stress-strain relations, the midpoint rule is preferable for strain distributions and stress-strain relations with discontinuous slope.

2.5 Force-Based Consistent Mass Matrix

The derivation of the consistent mass matrix requires the determination of the section mass matrix, where the mass is considered like a distributed load along the length of the beam in cantilever basic system for this derivation. The section mass matrix is easily computed by the following equation through the use of section compatibility matrix that is given in Equation (4) without the presence of shear correction:

$$\mathbf{m}_s(x) = \int_A \mathbf{a}_s^{\mathrm{T}} \rho(x,y)\, \mathbf{a}_s\, dA; \tag{18}$$

Mass matrix of the force-based element, which will be used in Equation (9), is written in 6×6 dimension by the method provided by [9], i.e. in the complete system with 3 degrees of freedom per node, as follows:

$$\mathbf{m} = \begin{bmatrix} \mathbf{m}_{00} & \mathbf{m}_{0L} \\ \mathbf{m}_{L0} & \mathbf{m}_{LL} \end{bmatrix} \tag{19}$$

where the components of element mass matrix are calculated from following sub-matrices

$$\begin{aligned}
\mathbf{m}_{LL} &= \mathbf{f}^{-1}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{k}_s^{-1}(x) \\
&\quad \cdot \left(\int_x^L \mathbf{b}^{\mathrm{T}}(x,\xi)\mathbf{m}_s(\xi)\mathbf{f}_p(\xi)\mathbf{f}^{-1} d\xi\right) dx \\
\mathbf{m}_{L0} &= \mathbf{f}^{-1}\int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{k}_s^{-1}(x) \\
&\quad \cdot \left(\int_x^L \mathbf{b}^{\mathrm{T}}(x,\xi)\mathbf{m}_s(\xi)\left(\mathbf{b}^{\mathrm{T}}(0,\xi) - \mathbf{f}_p(\xi)\mathbf{f}^{-1}\mathbf{b}^{\mathrm{T}}(0,L)\right) d\xi\right) dx \\
\mathbf{m}_{0L} &= \mathbf{m}_{L0} = -\mathbf{b}(0,L)\mathbf{m}_{LL} \\
&\quad + \int_0^L \mathbf{b}(0,x)\mathbf{m}_s(x)\mathbf{f}_p(x)\mathbf{f}^{-1} dx \\
\mathbf{m}_{00} &= -\mathbf{b}(0,L)\mathbf{m}_{L0} \\
&\quad + \int_0^L \mathbf{b}(0,x)\mathbf{m}_s(x)\left(\mathbf{b}^{\mathrm{T}}(0,x) - \mathbf{f}_p(x)\mathbf{f}^{-1}\mathbf{b}^{\mathrm{T}}(0,L)\right) dx
\end{aligned} \tag{20}$$

In above equations, element flexibility matrix $\mathbf{f}$ is obtained as given in Equation (12). The partial flexibility matrix $\mathbf{f}_p$ is calculated as follows:

$$\mathbf{f}_p(x) = \int_0^x \mathbf{b}^{\mathrm{T}}(\xi,x)\mathbf{k}_s^{-1}(x)\mathbf{b}(\xi,x)\, d\xi \tag{21}$$

3. Numerical Examples

In this paper, for the numerical example, the proposed model is verified with the 3D model created in ANSYS environment. For the verification of the proposed model, the section used in the study given in the literature [1], i.e. W36x135, W24x250 and W14x730 sections and in addition European sections HEB180, IPE270 and IPE750x147 are considered. For the verification of the proposed model different types of sections are utilized. ANSYS model could be accepted as control model of this study.

To have a fair comparison with the model data, the use of solid finite elements is a suitable modelling approach for simulation. For this purpose, the ANSYS Workbench Design Modeller is selected to perform 3-D finite element analyses after implementing bodies geometries and utilizing certain mesh conditions. In such a numerical model, modelling approximations would have great influence on the finite element analysis, like the considered element type, meshing elements number and size, boundary conditions and environment representations.

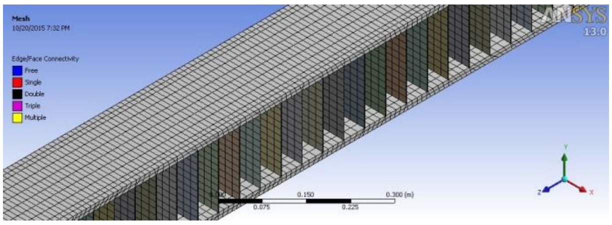

Figure 2: Representative ANSYS model sketch for one section

During the geometry implementation of the wide flange beams, 1 mm thick stiffeners, with very low mass density 1E-10 kg/m^3, are supplemented along the length of the elements to constraint the flanges and reduce their inordinate behaviour relatively; adopting such flange constraining approach converges to a more realistic behaviour of wide flange beams in structures. This way mode stabilization is achieved, where capturing clear axial, bending and shearing modes become possible for ANSYS simulations. The beam elements and the stiffeners in ANSYS model are both considered of solid type. After the geometry employment, quadrilateral mesh is generated using Solid187. This method uses linear elements to just obtain the correct results using the enhanced strain formulation. In the beams models, the number of mesh elements used changes from a test to another, but as a common value, the element size is set to 0.03 for all the beams. Yet, to set up conformal meshing among the body parts of the beams, the available Shared Topology tool is used to share faces and edges creating an analogous topology.

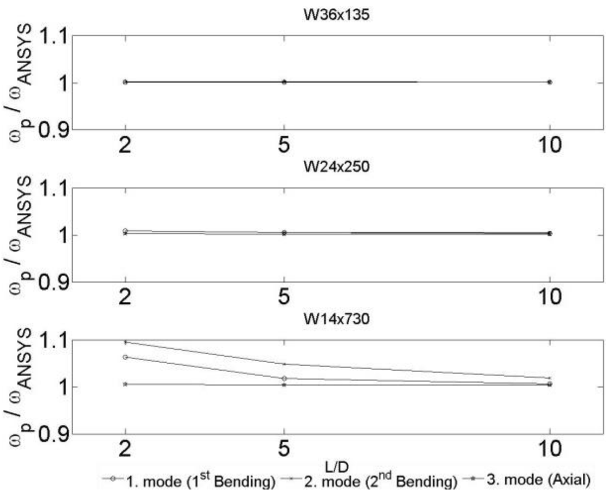

Figure 3: Natural frequency proposed over ANSYS result ratios for W-Section profiles

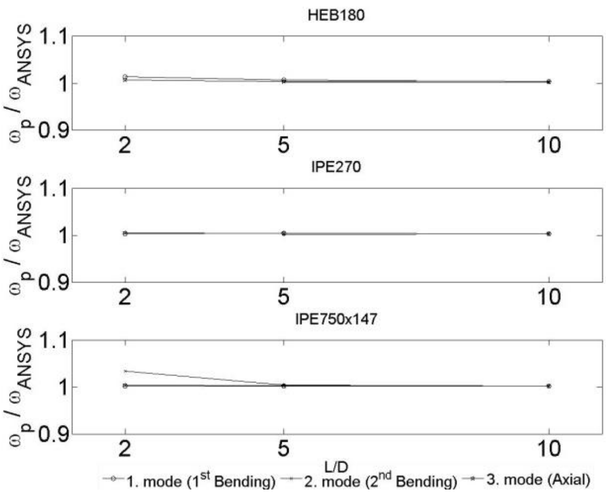

Figure 4: Natural frequency proposed over ANSYS result ratios for European section profiles

Figures (3) and (4) represent the ratio of the frequency results obtained by proposed model to ANSYS model. Here, in this study, the ANSYS FEM model is considered as control model, i.e. numerically converged exact solution for comparison. Thus, the ratio will represent the error between these control models and proposed model presented in this paper.

The results of the cantilever beams are represented separately for its first, second bending and axial modes of beams for different length over depth ratios. For the two groups of cross-sections, that are taken from study [1] and European sections, chubby and moderate sections represent significantly low errors, however, for the deep sections, approximately 10% error is obtained for smaller length over depth ratios. Yet, the effect of the shear can be better observed for the short beams, the effect on the deep sections should be further investigated.

4. Conclusions

The aim of this study is to gather and verify the modal behaviour of various I-sections under different length over depth ratios with the proposed beam finite element model.

The result of the study shows that the proposed model has satisfactory accuracy in capturing the real behaviour according to the verification carried on control models in ANSYS. The results prove that the error between the ANSYS model and proposed model is low and biased. However, the deep sections for both groups of sections, represents relatively larger errors, around 10%, which needs to be assessed in detail. Introducing stiffeners to the ANSYS model prevents the local modal deformations along the sections such as flange deformations independent from the bending deformations. Such deformations resulted into higher errors with the proposed model, since the proposed model can only show 2D frame deformation except from 3D local deformations like the results of the ANSYS FEM. For a fair comparison between a beam finite element model and 3D finite element models, it is necessary to define both top and bottom flanges and the web act together.

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References

[1]. Charney, F.A., Iyer, H., and Spears, P.W.,"Computation of major axis shear deformations in wide flange steel girders and columns", Journal of Constructional Steel Research, Vol. 61, pp. 1525–1558, 2005.

[2]. Gruttmann, F., Wagner, W., "Shear correction factors in Timoshenko's beam theory for arbitrary shaped cross-sections", Computatonal Mechanics, Vol. 27, pp. 199-207, 2001.

[3]. Chróscielewski, J., Pietraszkiewicz, W., Witkowski, W., "On shear correction factors in the non-linear theory of elastic shells", International Journal of Solids and Structures, Vol. 47, pp. 3537-3545, 2010.

[4]. Pai, P.F., Schulz, M. J., "Shear correction factors and an energy-consistent beamtheory",International Journal of Solids and Structures, Vol. 36, pp. 1523-1540, 1999.

[5]. ANSYS, Workbench 2.0 Framework, 2011.

[6]. Taylor, R.L., Filippou, F.C., Saritas, A., Auricchio, F.,"Mixed finite element method for beam and frame problems", Computational Mechanics, Vol. 31, No. 1-2, pp. 192-203, 2003.

[7]. Saritas, A., Filippou, F.C.,"Inelastic axial-flexure–shear coupling in a mixed formulation beam finite element", International Journal of Non-Linear Mechanics, Vol. 44, No. 8, pp. 913-922, 2009.

[8]. Barr, A.D.S.,"An Extension of the Hu-Washizu variational principle in linear elasticity for dynamic problems", Journal of Applied Mechanics, Vol. June, pp. 465, 1966.

[9]. Molins, C., Roca, P., Barbat, A.H.,"Flexibility-based linear dynamic analysis of complex structures with curved-"3D members", Earthquake Engineering & Structural Dynamics, Vol. 27, No. 7, pp. 731-747, 1998.