Three-Dimensional Frame Element Formulation for Nonlinear Analysis of Semi-Rigid Steel Framed Structures

H. F. Özel[1], Afşin Sarıtaş[2] and Zafer Karakaş[3]
[1]Dept. of Civil Engineering, Middle East Technical University, Ankara, Turkey, fozel@metu.edu.tr
[2]Dept. of Civil Engineering, Middle East Technical University, Ankara, Turkey, asaritas@metu.edu.tr
[3]Dept. of Civil Engineering, Middle East Technical University, Ankara, Turkey, zafer.karakas@metu.edu.tr

Abstract

In this paper, a force-based three-dimensional frame finite element formulation with spread of inelasticity through the element and localized nonlinear semi-rigid connections is developed. The proposed model utilizes Euler-Bernoulli beam theory assumptions, and adopts fiber discretization of monitored sections along element length and section depth for the spread of inelasticity in order to capture axial force and biaxial bending moment interaction. Defining any type of semi-rigid either linear or nonlinear connection behavior along element length does not necessitate any increase in number of degrees of freedom. The element is modelled with option of capturing nonlinear geometric effects. The proposed element is designated for the biased analysis of steel structures, since considering the presence of beam to column semi-rigid connections through analyses provides true behavior. The formulated element can also consider the presence of any type of connection that may be present in a steel structure; such as beam to column, column base, brace end connections. Specific to this study, moment-rotation response of beam to column connections is implemented for both the strong axis and weak axis bending directions. Numerical examples are considered to verify the results obtained from the developed finite element models with regards to other available structural analysis programs.

Keywords: *Steel structures; Semi-rigid; Three dimensional; Finite element formulation; Nonlinear structural analysis; Beam to column connections*

1 Introduction

Phenomenologically there are 2 types of connections on the purposes of design for steel structures. Beam column connections are considered as either simple or moment type connections. However, beam column connections tend to behave with relative rotation together with moment transfer. This type of connection is called semi-rigid connection. In order to represent the actual behavior of steel structures many researchers are prone to take the effect of the semi-rigid connections on the behaviors of the structures. (Chui & Chan, 1997) and (Nader & Astaneh-Asl, 1996) applied laboratory studies on the semi-rigidly jointed steel frames accompanied with numerical analyses. Studies revealed a good match with the laboratory test results and the numerical studies when introducing the semi-rigid joints. Studies of (Saritas & Koseoglu, 2015) revealed the effects of semi-rigid connections under cycling loading and studies verified with experimental results for two dimensional steel semi-rigid framed structures. The dynamic behavior of steel structures is dependent on the definition of flexible joints on the system. The study of (Ozel & Saritas, 2015) showed the necessity of defining flexible joints for accurate modelling of vibration characteristics on steel framed structures. Again in dynamic manner, three dimensional (3D) studies of (Nguyen & Kim, 2013) revealed the significance of semi-rigid joints on steel framed structures under cyclic loading.

In this study, 3D steel frames are modelled with semi-rigid joints and the effects are discussed. In order to carry out an accurate analysis of steel framed structures, proposed element with semi-rigid connections is compared with (Frank McKenna, Fenves, & Scott, 2015) force based element, which by the way does not contain semi-rigid connections in its formulation, but require introduction of additional nodes and spring elements for capturing the presence of semi-rigid connections at the expense of increased matrix sizes, thus storage in computers. Although not discussed in current paper, there are also differences between proposed element and OpenSees's force based element with regards to mass matrix implementation. OpenSees model utilizes lumped mass approach; however, proposed model utilizes force-based consistent mass matrix based as proposed by (Soydas & Saritas, 2016. In Press).

2 Frame Element Formulation

2.1 Kinematic Relations

Displacements on a material point on the section of a beam that deforms in xyz-plane can be obtained by calculating Timoshenko beam theory as follows;

$$\begin{Bmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{Bmatrix} = \begin{Bmatrix} u(x) - y\theta_z(x) + z\theta_y(x) \\ v(x) - z\theta_x(x) \\ w(x) + y\theta_x(x) \end{Bmatrix} \tag{1}$$

where $u_x(x,y,z)$, $u_y(x,y,z)$ and $u_z(x,y,z)$ are the displacements in x, y and z directions, respectively of any point in the section. u(x) is the displacement of the point (x,0) along x-axis. v(x) and w(x) are the transverse deflections of the point (x,0) from x-axis in y and z direction. $\theta_x(x)$, $\theta_y(x)$ and $\theta_z(x)$ are the small rotation of the beam cross section around x,y and z-axis.

The non-zero strain components $\boldsymbol{\varepsilon}$ include the normal strain in the x direction and shear strain with xyz component, where these are calculated from section deformations as follows;

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xy} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u'(x) - y\theta_z'(x) + z\theta_y'(x) \\ -\theta_z(x) + v'(x) - z\theta_x'(x) \\ \theta_y(x) + w'(x) + y\theta_x'(x) \end{Bmatrix} = \begin{Bmatrix} \varepsilon_a(x) - y\kappa_z(x) + z\kappa_y(x) \\ \gamma_y(x) - z\varphi(x) \\ \gamma_z(x) + y\varphi(x) \end{Bmatrix} = \mathbf{a}_s(y,z)\,\mathbf{e}(x) \tag{2}$$

where $\mathbf{e}(x)$ is the section deformation vector given as follows;

$$\mathbf{e}(x) = \begin{bmatrix} \varepsilon_a(x) & \kappa_z(x) & \kappa_y(x) & \gamma_y(x) & \gamma_z(x) & \varphi(x) \end{bmatrix}^{\mathrm{T}} \tag{3}$$

In Equation (3), $\varepsilon_a(x)$ is the axial strain of the reference axis, $\varphi(x)$ is torsional strain, $\gamma_y(x)$ and $\gamma_z(x)$ are the shear deformations about *y* and *z*-axis, respectively and, $\kappa_y(x)$ and $\kappa_z(x)$ are the curvature about *y* and *z*-axis, respectively. Section deformations can be easily calculated from section reference displacements as clearly visible from a one to one comparison of the terms of Equation (2). Furthermore, section compatibility matrix, $\mathbf{a}_s(y,z)$ introduced in Equation (2) is written as follows;

$$\mathbf{a}_s(y,z)=\begin{bmatrix} 1 & -y & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -z \\ 0 & 0 & 0 & 0 & 1 & y \end{bmatrix} \tag{4}$$

Beam kinematics is presented for the case of Timohensko beam theory in above derivation; however, verification studies are only pursued for the case where only normal stress-strain behavior is nonlinear, and the shear strains due to shear forces vanish by considering curvatures are related to the double derivative of transverse displacements in Equation (2).

2.2 Basic System without Rigid Body Modes and Force Interpolation Functions

Element formulation is proposed in *xyz*-plane, where the formulation considers two end nodes and relies on a transformation from complete system to basic system. In the whole structure, the element has 6 degrees of freedom (dof) per node, resulting in 12 dofs, where the nodes are placed at element ends. The complete system is proposed such that the axis of the element is aligned with horizontal *x*-axis. The basic system is chosen as shown in Figure 1, where left hand side of the system only allows rotations in y and z direction, and the right hand side allows only displacement in x direction and rotations in x, y and z direction.. The transformation matrix, **a** for an element with length *L* is used to relate element end forces in complete system to basic element forces as follows;

$$\mathbf{p}=\mathbf{a}^{\mathrm{T}}\mathbf{q}; \quad \text{and} \quad \mathbf{a}=\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 1 & 0 & -1/L & 0 & 0 & 0 & 0 \\ 0 & 1/L & 0 & 0 & 0 & 0 & 0 & -1/L & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1/L & 0 & 1 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 \\ 0 & 0 & -1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 0 & 1 & 0 \end{bmatrix} \tag{5}$$

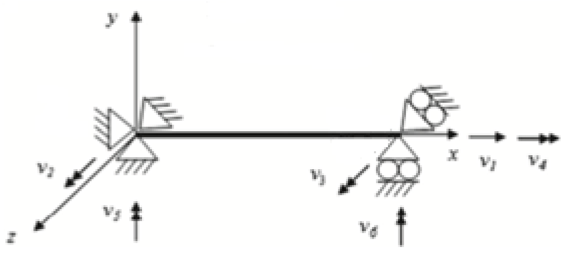

Figure 1. Basic system forces and deformations

Basic element deformations **v** can be calculated from nodal displacements **u** in complete system as follows;

$$\mathbf{v}=\mathbf{a}\mathbf{u} \tag{6}$$

Basic element forces at free end, **q** are shown in Figure 1 and given in Equation (5). These forces can be related to internal section forces, $\mathbf{s}(x)$ by using the force interpolation matrix $\mathbf{b}(x,L)$ for the cantilever beam configuration as follows;

$$\mathbf{s}(x) = \begin{bmatrix} N(x) & M_z(x) & M_y(x) & V_y(x) & V_z(x) & T(x) \end{bmatrix}^{\mathrm{T}} = \mathbf{b}(x,L)\,\mathbf{q}$$

$$\mathbf{b}(x,L) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & x/L-1 & x/L & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-x/L & -x/L & 0 \\ 0 & -1/L & -1/L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/L & 1/L \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

By using Equation (7), it is possible to attain exact equilibrium between the forces at free end of the element and forces at any section that is *x* units away from the fixed end. Section forces are axial force $N(x)$, shear force in *y and z* direction $V_y(x)$, $V_z(x)$ and moment about *y* and *z*-axis $M_y(x)$, $M_z(x)$.

2.3 Variational Base and Finite Element Formulation of the Element

Variational form of the element is written by considering independent element nodal displacements **u**, element basic forces **q**, and section deformations **e** by using three-fields Hu-Washizu functional and implemented as part of beam finite elements by (Taylor, Filippou, Saritas, & Auricchio, 2003) and (Saritas & Filippou, 2009).

$$\begin{aligned} \delta\Pi_{\mathrm{HW}} &= \int_0^L \delta\mathbf{e}^{\mathrm{T}} \left(\hat{\mathbf{s}}(\mathbf{e}(x)) - \mathbf{b}(x,L)\,\mathbf{q} - \mathbf{s}_{\boldsymbol{p}}(x)\right) dx \;\; \cdots \\ &\quad - \delta\mathbf{q}^{\mathrm{T}} \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{e}(x)\, dx - \delta\mathbf{u}^{\mathrm{T}}\mathbf{p}_{app} = 0 \end{aligned} \tag{8}$$

Above equation can also be obtained by considering the general Hu-Washizu variational form. Equation (8) should hold for arbitrary $\delta\mathbf{u}$, $\delta\mathbf{q}$ and $\delta\mathbf{e}$, thus the following three equations should be satisfied in order for the Hu-Washizu variational to be zero.

$$\mathbf{v} \equiv \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\,\mathbf{e}(x)\,dx; \quad \text{where} \quad \mathbf{v} = \mathbf{a}_{\mathrm{g}}\mathbf{u} \tag{9}$$

$$\hat{\mathbf{s}}(\mathbf{e}(x)) \equiv \mathbf{b}(x,L)\mathbf{q} \tag{10}$$

For linear elastic material response, section deformations can be calculated as $\mathbf{e}=\mathbf{k}_s^{-1}\hat{\mathbf{s}}$ to obtain the section deformations from section forces through the use of section stiffness matrix $\mathbf{k}_s$. Substitution of section deformations **e** to Equation (9) gives:

$$\mathbf{a}_{\mathrm{g}}\mathbf{u} = \mathbf{v} = \mathbf{f}\;\mathbf{q}; \quad \text{where} \quad \mathbf{f} = \int_0^L \mathbf{b}^{\mathrm{T}}(x,L)\mathbf{f}_s(x)\,\mathbf{b}(x,L)\,dx \tag{11}$$

In above equation **f** is the flexibility matrix of the element in the basic system. $\mathbf{f}_s$ is the section flexibility matrix that can be calculated from the inversion of the section stiffness matrix $\mathbf{k}_s$. Further substitution of above equation for linear elastic response results in

$$\mathbf{ku} = \mathbf{p}_{app}; \quad \text{where} \quad \mathbf{k} = \mathbf{a}^{\mathrm{T}}\mathbf{f}^{-1}\mathbf{a} \tag{12}$$

where **k** is the 12×12 element stiffness matrix in the complete system. At this point in the element formulation, presence of semi-rigid connections will be introduced through the following extended version of above equation for the calculation of element end deformations:

$$\mathbf{v}=\mathbf{v}_{\text{Frame}}+\mathbf{v}_{\text{Con}};\quad \text{where}\quad \mathbf{v}_{\text{Frame}}=\int_L \mathbf{b}^{\mathrm{T}}(x)\,\mathbf{e}(x)\,dx;\quad \mathbf{v}_{\text{Con}}=\sum_{i=1}^{nSC}\mathbf{b}^{\mathrm{T}}(x_i)\,\boldsymbol{\Delta}_{SC,i}$$
$$\text{and}\quad \boldsymbol{\Delta}_{SC}=\begin{bmatrix}\delta_{SC}^{axial} & \theta_{Z,SC} & \theta_{Y,SC} & \delta_{Y,SC}^{shear} & \delta_{Z,SC}^{shear} & \varphi_{SC}\end{bmatrix}^{\mathrm{T}} \tag{13}$$

The first integral along the length of the frame element can be numerically calculated by using a quadrature rule to capture spread of inelastic behavior and *nSC* is the total number of semi-rigid connections discreetly located along element length; Δ_{SC} is the vector of semi-rigid connection deformations. Introduction of semi-rigid connections along element length in Figure 1 does not alter the force field under small deformations. Element flexibility matrix is similarly discretized as follows:

$$\mathbf{f}=\mathbf{f}_{\text{Frame}}+\mathbf{f}_{\text{Con}};\quad \text{where}\quad \mathbf{f}_{\text{Frame}}=\int_L \mathbf{b}^{\mathrm{T}}(x)\,\mathbf{f}_s(x)\,\mathbf{b}(x)\,dx;$$
$$\text{and}\quad \mathbf{f}_{\text{Con}}=\sum_{i=1}^{nSC}\mathbf{b}^{\mathrm{T}}(x_i)\,\mathbf{f}_{SC,i}\,\mathbf{b}(x_i) \tag{14}$$

As a remark, Equations (9) and (10) are related to the element state determination, i.e. these equations can be solved independent of Equation (8), and then the solution can be condensed out into Equation (8) such that the equations of motion can be assembled for all elements. This process was demonstrated above for the linear elastic case. In general, state determination of the element requires an iterative solution in the case of nonlinear behavior, where Equations (8) to (10) are needed to be solved. This solution requires also the calculation of element flexibility matrix **f** under nonlinear response, where taking derivative of element deformations **v** in Equation (9) with respect to element forces **q** results into the same flexibility integration expression given in Equation (11), but this time the section stiffness will be nonlinear, as well.

2.4 Section Response

Section response can be obtained by the basic assumption that plane sections before deformation remain plane after deformation along the length of the beam by the use of following section compatibility matrix $\mathbf{a}_s$ as given in Equation (2)

$$\mathbf{a}_s=\mathbf{a}_s(y)=\begin{bmatrix}1 & -y & z & 0 & 0 & 0\\ 0 & 0 & 0 & \kappa_{sy} & 0 & -z\\ 0 & 0 & 0 & 0 & \kappa_{sz} & y\end{bmatrix} \tag{15}$$

Shear correction factor κ_s is taken as the inverse of the form factor suggested by (Charney, Iyer, & Spears, 2005) for I-section about the major bending axis:

$$\kappa_s=1/\kappa;\quad \text{where}\quad \kappa=0.85+2.32\frac{b_f t_f}{d\,t_w} \tag{16}$$

The section forces are obtained by integration of the stresses that satisfy the material constitutive relations $\boldsymbol{\sigma}=\boldsymbol{\sigma}(\boldsymbol{\varepsilon})$ according to

$$\mathbf{s} = \int_A \mathbf{a}_\mathrm{s}{}^\mathrm{T} \boldsymbol{\sigma}\, dA; \quad \text{where} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \end{pmatrix} \tag{17}$$

The derivative of section forces with respect to the section deformations results in the section tangent stiffness matrix

$$\mathbf{k}_\mathrm{s} = \frac{\partial \mathbf{s}}{\partial \mathbf{e}} = \int_A \mathbf{a}_\mathrm{s}{}^\mathrm{T} \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\varepsilon})}{\partial \mathbf{e}}\, dA = \int_A \mathbf{a}_\mathrm{s}{}^\mathrm{T} \mathbf{k}_\mathrm{m} \mathbf{a}_\mathrm{s}\, dA \tag{18}$$

The material tangent modulus $\mathbf{k}_\mathrm{m}$ is obtained from the stress-strain relation according to $\mathbf{k}_m=\partial\boldsymbol{\sigma}(\boldsymbol{\varepsilon})/\partial\boldsymbol{\varepsilon}$. Gauss-quadrature, the midpoint or the trapezoidal rule can be used for the numerical evaluation of the integrals in (17) and (18). While Gauss-quadrature gives better results for smooth strain distributions and stress-strain relations, the midpoint rule is preferable for strain distributions and stress-strain relations with discontinuous slope.

3 Numerical Examples

A three dimensional portal frame model is built on OpenSees software and in Matlab by the use of proposed model (Figure 2). Model height is 3 m and bays in each direction are 6 m. Each node has six degrees of freedom and all beam and column elements have linear coordinate transformation. In other words, second order P-Delta effects are ignored.

HEB180 steel section for all columns and IPE240 steel section for all beams are defined with and without the presence of semi-rigid connections. Elasticity modulus, Poisson's ratio are taken as 210 GPa and 0.3, respectively. For semi-rigid case, connection stiffness ratio is taken as λ=2, 11 and 20, where λ is the ratio of connection stiffness to flexural rigidity EI/L of beam. It is worth to mention that λ=2 is close to pin/shear connection case, while λ=20 is close to rigid/moment connection case. Bilinear steel material with strain-hardening ratio of $1 * 10^{-6}$ is used to define material for both beams and columns and they are modelled as a nonlinear force beam-column element. The frame is loaded by 200 mm displacement in the strong axes of the columns and results are presented for verification of the response of proposed beam element.

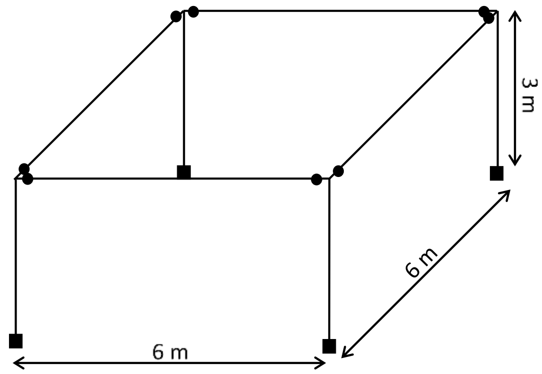

Figure 2. 3D Portal Frame Model

To model a semi-rigid steel beam-column connection in OpenSees model, the node that connects beam to column is duplicated and a zero-length rotational spring is added between the nodes. This results in an addition of 50% more nodes for this simple structure, where this number will be larger and thus be a more significant issue as the structure size grows. All the degrees of freedom except the rotational degree of freedom about which the zero-length rotational spring is presumed to rotate are constrained in OpenSees model, i.e. master and slave nodes are considered through a penalty formulation to enforce the equivalence of all dofs except than the dof at which semi-rigid response is introduced. It is worth to mention that the proposed model doesn't necessitate this kind of action while defining the semi-rigidity at the ends of the beams and there won't be any dof increase in the system.

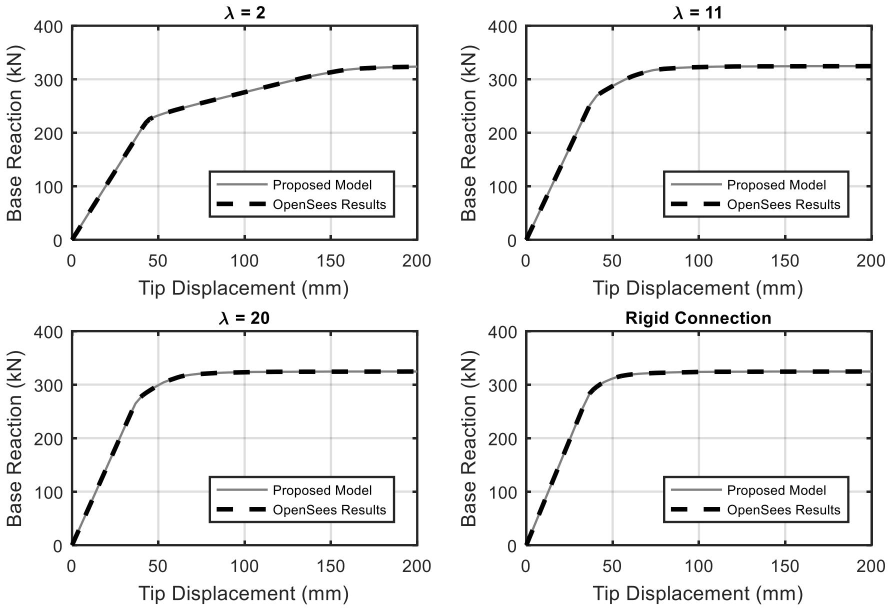

Figure 3. Nonlinear material and linear elastic connection response

First, the 3D portal frame is modelled with rigid connections to control the frame element formulation with OpenSees model and proposed model. From Figure 3, both models show a perfect match since they both utilize force method formulation. Semi-rigid connections are added to the system with elastic and bilinear behavior in different cases. In Figure 3, the comparison between models with elastic semi-rigid connections is represented and models present perfect match. In Figure 4, semi-rigid connections with bilinear moment-rotation behavior are considered, and perfect match with OpenSees is observed in all simulations.

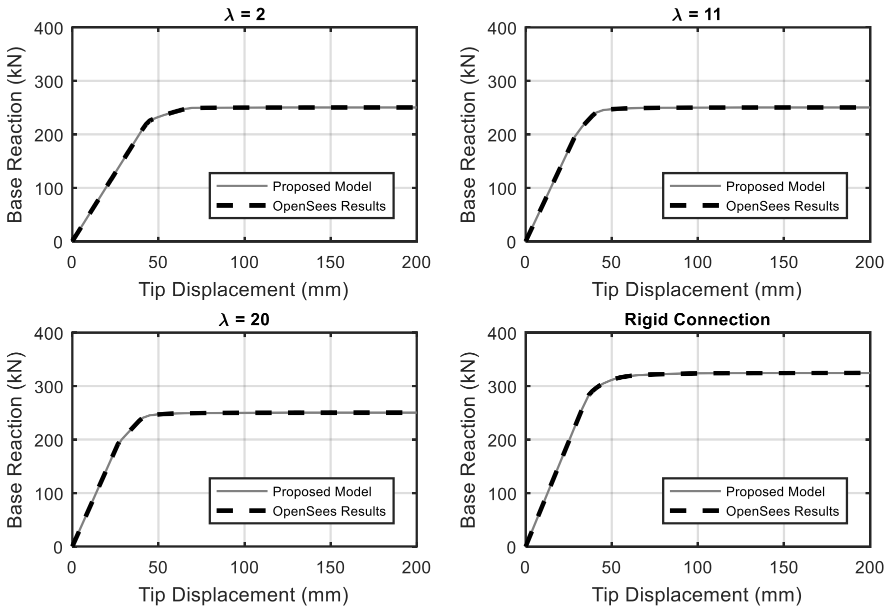

Figure 4. Nonlinear material and nonlinear connection response

The effects of the semi-rigid connections with changing connection stiffness ratio (λ) presented the difference for the 3D systems in the Figure 5, clearly. The yield of the connections decreased the dissipation of the energy of the system.

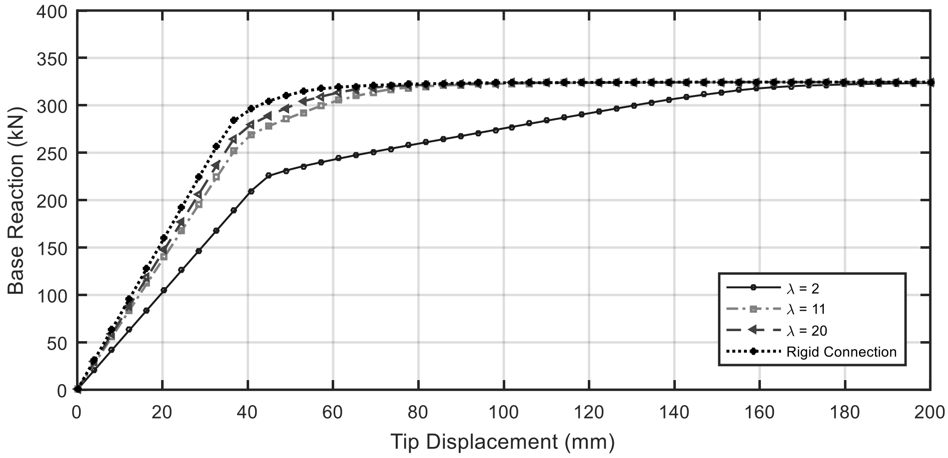

Figure 5. Influence of connection stiffness on nonlinear response of 3D portal frame structure

4 Conclusion

The proposed frame element model is compared with model created in OpenSees Software in this study. 3D portal frame is assessed with flexible connections introduced at the end of the beams. Both the proposed model and OpenSees model are created with force based element formulation. Differences between the two models are due to easier implementation of semi-rigid connections in element response for the proposed model, while OpenSees model needs extra nodes, dofs, introduction of spring elements, as well as master and slave node constraints with penalty formulation. Proposed model provides accurate and robust solutions as can be seen from the results provided in this study. The proposed model currently utilizes Euler Bernoulli beam element formulation; however, for the future research, Timoshenko Beam will be implemented with accurate shear correction factors and torsional responses. Future work will also include nonlinear static and dynamic analyses of 3D frames with semi-rigid connections.

References

Charney, F. A., Iyer, H., & Spears, P. W. (2005). Computation of major axis shear deformations in wide flange steel girders and columns. *Journal of Constructional Steel Research 61*, 1525–1558.

Chui, P. P. T., & Chan, S. L. (1997). Vibration and deflection characteristics of semi-rigid jointed frames. *Engineering Structures, 19*(12), 1001-1010.

McKenna, F., Fenves, G. L., & Scott, M. H. (2015). OpenSees (Version 2015): University of California, Berkeley.

Nader, M. N., & Astaneh-Asl, A. (1996). Shaking table tests of rigid, semirigid, and flexible steel frames. *Journal of Structural Engineering, 122*, 589-596.

Nguyen, P. C., & Kim, S. E. (2013). Nonlinear elastic dynamic analysis of space steel frames with semi-rigid connections. *Journal of Constructional Steel Research, 84*, 72-81.

Ozel, H. F., & Saritas, A. (2015). *An accurate modeling approach for calculating the vibration characteristics of steel framed structures with semi-rigid connections* Paper presented at the 8th International Conference on Behavior of Steel Structures in Seismic Areas July 1-3, 2015, Shanghai, China.

Saritas, A., & Filippou, F. C. (2009). Inelastic axial-flexure–shear coupling in a mixed formulation beam finite element. *International Journal of Non-Linear Mechanics, 44*(8), 913-922.

Saritas, A., & Koseoglu, A. (2015). Distributed Inelasticity Planar Frame Element with Localized Semi-Rigid Connections for Nonlinear Analysis of Steel Structures. *International Journal of Mechanical Sciences, 96-97*, 216-231.

Soydas, O., & Saritas, A. (2016. In Press). Free vibration characteristics of a 3d mixed formulation beam element with force-based consistent mass matrix. *Journal of Vibration and Control*.

Taylor, R. L., Filippou, F. C., Saritas, A., & Auricchio, F. (2003). Mixed finite element method for beam and frame problems. *Computational Mechanics, 31*(1-2), 192-203.