Mathematical Models for Semi-Rigid Connections: Top and Seat Angle with Double Web Angles Connection

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Abstract

Two types of connection are generally considered in practice: completely rigid (moment) and simple (shear) connections. In reality rigid connections have relative flexibility and simple connections have some capacity to transfer moments. Realizing this fact, semi-rigid (partially restrained) connections are introduced in many modern design specifications. Data from physical tests are needed in order to provide better representation of the mechanical action in these connections. This is mostly achieved through experimental research, but with the advance of computational power, nonlinear 3-d finite element analyses can be performed in producing data, as well. Certain semi-rigid connection types, such as top and seat angle connection with double web angles, have few available experimental data, and thus the developed mathematical models should be validated with respect to their accuracy for this type of connection. In this study, mathematical models in terms of moment-rotation relations are considered in order to quantify the capability of these models in representing the response of this specific type of semi-rigid connection. For comparison, the polynomial model and several versions of the power model are selected. The responses obtained from these models are compared with the few available experimental data, as well as the response obtained from a nonlinear 3-d finite element model analysis of these connections. The best fit with the experimental data are seen with the finite element model and the polynomial model.

Keywords: *Semi-rigid connections; connections; steel; mathematical models; nonlinear finite element*

1 Introduction

Steel structures have to be connected through effective connections with sufficient moment capacity and sustained energy dissipation characteristics in order to resist against earthquake induced loads. Before the Northridge (1994) and Kobe (1995) earthquakes, designers and also the codes at that times were in favor of welded connections since they have great moment capacities. However, the failure of the welded connections in brittle manner in these earthquakes resulted in severe damage to buildings. In the following decade after Northridge and Kobe earthquakes, structural steel codes together with seismic codes established new rules for connections. In this perspective, partially restrained connections or semi-rigid connections have been more recently cited in the codes. Moreover, they are also acknowledged as an economical way to accomplish better earthquake performance in steel frames. The economical advantages of using semi-rigid connections in steel frames do not automatically lead to their use in practical design applications, since there is not enough analytical and experimental research on the response of these types of connections. Furthermore, complicated analysis procedure and not knowing the range of application of these types of connections impede its use in practice by the designers. As a result, designers still approach steel framing connections as either fixed or pinned. But, both of these assumptions do not reflect the actual nonlinear behavior of the connections.

Nonlinear response of the connection behavior was first recognized in the early 1930's. Then, certain attempts have been undertaken ranging from simple linear and bilinear curves to more sophisticated polynomial and exponential models for reflecting actual nonlinear behavior of the connections. These models are moment-rotation type relations, and they are simply curve fitted with available experimental data. Due to lack of experimental data and with the advance of computational power, finite element applications are now used to obtain moment-rotation curves. Among semi-rigid connections that received small attention is the top and seat angle connection with double web angles. This connection type provides increased connection restraint over the top and seat angle connections.

2 Mathematical Models for Semi-Rigid Connections

2.1 Linear and Multi-Linear Models

The simplest mathematical model that can be found in literature is the linear connection model. The model needs only one connection parameter which is the initial stiffness. The model can be described as follows:

$$M = R_{ki}\theta \quad \text{where} \quad R_{ki} = \lambda \frac{4EI}{L} \tag{1}$$

where EI and L stand for the beam rigidity and length, respectively, and λ is called as the rigidity index that varies from 0 to 1 accounting for pinned connection to fixed connection, respectively. The validity of the linear model obviously fails when deformations exceed the linear range. This model is mostly recommended and used in vibration and bifurcation analysis. The multi-linear models start with the bilinear model. More complicated models could be obtained within this approach by introducing further number of parameters through curve fitting with the experimental data.

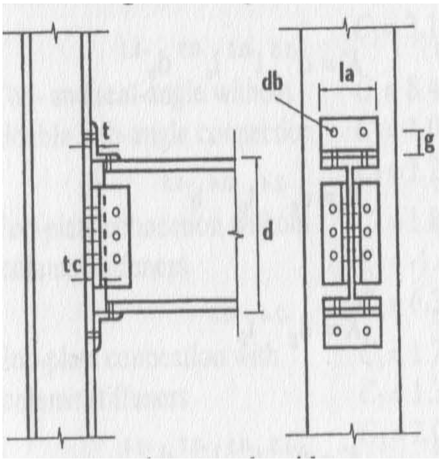

Figure 1 Size parameters for top and seat angle with double web angles connection

2.2 Polynomial Models

Polynomial model is one of the most widely known mathematical models for connections, and it is adopted by many researchers due to its simplicity. This model was extensively used by Frye and Morris (1975), and is also called as Frye and Morris polynomial model in literature. The model is expressed as follows:

$$\theta = C_1(KM) + C_2(KM)^3 + C_3(KM)^5 \tag{2}$$

where K is the standardization parameter that depends on geometrical properties and connection type. C_1 , C_2 and C_3 values are the curve fitting constants obtained from the experimental data. For top and seat angle with double web angles, $C_1 = 1.50\times10^{-3}$, $C_2 = 5.60\times10^{-3}$, and $C_3 = 4.35\times10^{-3}$, and the standardization parameter K is given as follows:

$$K = d^{-1.287} t_a^{-1.128} t_c^{-0.415} l_a^{-0.694} (g - d_b/2)^{1.350} \tag{3}$$

where all size parameters in Equation (3) are expressed in centimeters, and the description of the size parameters are presented in Figure (1).

The stiffness than can be calculated as the first derivative of the moment-rotation function in Equation (2), i.e. $R_k = \partial M / \partial \theta$.

2.3 Power Models

The first power model composed of two parameters and was first suggested by Krishnamurthy et al. (1979). The following form of the equation was suggested for the two parameter power model:

$$\theta = aM^b \tag{4}$$

in which *a* and *b* are the curve fitting parameters. This model requires great amount of sampling of experimental data from a certain type of connection. On the other hand, the three parameter power model provides a simpler description for the representation of the connection behavior. Using the initial connection stiffness R_{ki} and the ultimate moment capacity M_u of the connection, and further assuming zero slope at the point of ultimate moment, the three parameter power model can be written as follows:

$$M = \frac{R_{ki}\theta}{\left[1+\left(\theta/\theta_0\right)^n\right]^{1/n}} \quad \text{where} \quad \theta_0 = \frac{M_u}{R_{ki}} \tag{5}$$

where the shape parameter *n* can be determined by using the method of least squares for the differences between the predicted moments and the experimental data. The initial stiffness R_{ki} is calculated as a sum of the contributions from the initial stiffness of top angle, seat angle and web angles, where certain assumptions are made with respect to the material behavior and kinematics of the connection configuration. These are thoroughly discussed by Uslu (2009). The ultimate moment capacity M_u is very much dependent on the elastic-plastic collapse mechanism of the connection. In the model suggested by Kishi et al. (1993), the collapse mechanism is obtained by the summation of the plastic moment capacities contributed by each angle. In this perspective, plastic beam theory considering moment-shear interaction is used to evaluate ultimate moment capacity. This model uses Drucker's yield criterion for moment-shear interaction. In depth description of the model by Kishi et al. is available in Uslu (2009). The last parameter that defines the three parameter model in Equation (5) is the shape parameter *n*. The shape parameter is expressed in terms of θ_0 defined in Equation (5), and it is given as follows for the top and seat angle with double web angles:

$$\begin{aligned} n &= 2.003\log_{10}\theta_0 + 6.070; \quad && \text{if } \log_{10}\theta_0 > -2.880 \\ n &= 0.302; && \text{otherwise} \end{aligned} \tag{6}$$

With the definition of the three parameters, the model presented by Kishi et al. is completed. The same model is also adopted with slight differences to the Eurocode 3 (1997), and it is recently used in a model by Faella et al. (1999). The basic differences between these models result from the estimation of the ultimate moment capacity of the connection. Detailed description of the differences between the various power models are presented in Uslu (2009).

3 Modeling of the Semi-Rigid Connections with Finite Elements

Three dimensional solid finite elements are used for the description of the inelastic behavior of a semi-rigid connection. In this study, ANSYS finite element software package is used, and the simulation of the connection elements are constructed from SOLID187 type 3-d higher order 10 nodes elements. In the connection models considered in the next section, the number of elements used changes from one test to another, but approximately 14500 elements and 33500 nodes are used for each of the top and seat angle with double web angle connections.

The contact regions in a beam to column connection should be taken into account. These contact regions include the interaction between beam element and flange angles, bolt shank and bolt hole, bolt head and the part bolt head is attached, and furthermore the column flange and the angles of the connection that are attached to the column flange. CONTA174 and TARGE170 contact and target elements are used in order to take into account the forces due to friction and the deformation pattern due to slip. The pretension in the bolts is obtained by the use of PRETS179 element type for the considered action, where this element considers only one directional pretension.

The friction and the pretension values are the two important factors that affect both slip and moment-rotation response of a connection. The pretension forces actually affect the frictional forces along the main surfaces of the bodies in contact. This interaction is highly nonlinear, and due to the complexity of the selection of these parameters, the suggested values in the current code of practice are adopted in this study. The friction coefficient is taken as 0.35 from AISC manual (2005). The suggested pretension values for various size bolts are provided in AISC manual, and these suggested values are used in this study.

A simple 3-d material model with J_2 plasticity is considered for the description of the stress-strain relation of steel. In this simple model, modulus of elasticity, yield stress and ultimate stress of steel are the input values provided to ANSYS. In depth description of the considered 3-d finite element model is presented in detail by Uslu (2009). A sample finite element model for top and seat angle with double web angles connection is presented in Figure (2).

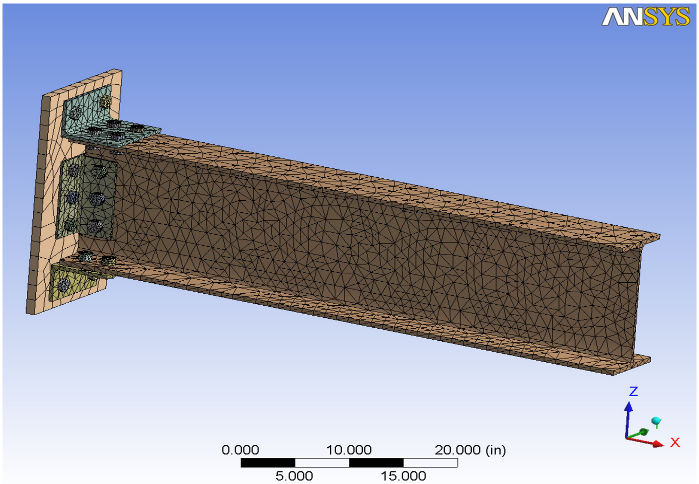

Figure 2. A sample finite element mesh for the top and seat angle with double web angles connection

4 Comparisons of the Models with Experiments

Few experiments are performed for the description of the moment-rotation relation of top and seat angle with double web angles connection in literature. The first tests in literature were conducted by Rathbun in 1936 (only 2 specimens were tested). Several decades later, Azizinamini (1985) tested several specimens. Both of these studies were limited to monotonic response. Besides these studies, the authors of this paper were not able to find any further experimental work on this type of connection. Despite the lack of experimental data, there appears to be a renewed interest in this type of semi-rigid connection at least in terms of finite element analysis (Citipitioglu et al. (2001), Danesh et al. (2007)).

In the current study, 5 specimens from Azizinami (1985) are considered for comparison, and these are specimens 14S1 to 14S5. These specimens consisted of W12x96 (305x144 in mm) stub column and W14x38 (360x57 in mm) beam sections with varying bolt sizes and angle geometries. We refer to Azizinami's work or Uslu (2009) for detailed presentation of the geometrical properties.

In this section, the experimental data from Azizinami's tests are compared with the moment-rotation relations obtained from the polynomial model and the three different power models described above. The power models consist of Kishi, Faella and the Eurocode 3 models. The linear and multi-linear models are skipped, since there is not enough information and experimental data on the top and seat angle with double web angle connections.

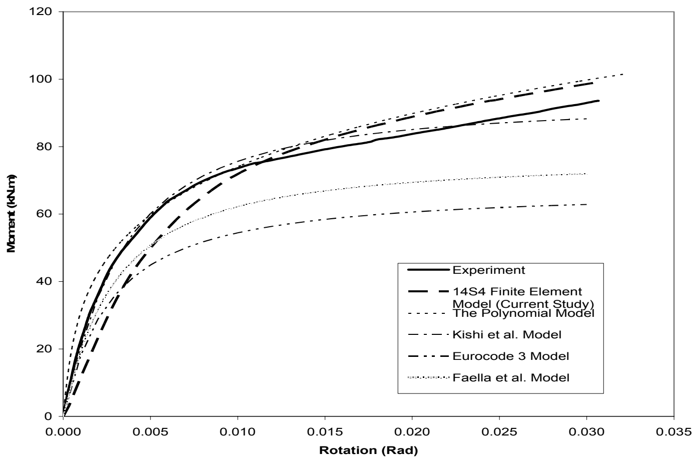

Figure 3. Comparison of moment-rotation relations for 14S3 Specimen

Table 1. Comparison of the initial and plastic stiffness values between experimental data and finite element model

Specimen	$\mathbf{(R_{ki})_{exp}/(R_{ki})_{FEM}}$	$\mathbf{(R_{kp})_{exp}/(R_{kp})_{FEM}}$
14S1	0.880	1.001
14S2	0.546	1.009
14S3	1.004	0.907
14S4	0.597	1.003
14S5	0.633	1.076
Mean	0.73	1.00
COV(%)	24.34	5.37

A sample comparison of the moment-rotation curves is presented in Figure (3). This figure actually tells the overall story about the variations in the responses. The initial stiffness value (R_{ki}) obtained from the finite element model (FEM) underestimates the experimental one, but the stiffness in the plastic range (R_{kp}) is closely approximated by FEM. The variations in the initial elastic stiffness and the plastic stiffness values between FEM and experiment are tabulated in Table 1, where the mean and the coefficient of variation (COV) are also provided. The ultimate moments obtained from the finite element model and the simplified mathematical models (moment-rotation models) are compared with the experimental one in Table 2. The finite element model and the polynomial model in general estimate the ultimate moment capacity best.

Table 2. Comparison of ultimate moments (M_u) between models and experiment

Specimen	FEM/Exp	Poly/Exp	Kishi/Exp	Faella/Exp	EC3/Exp
14S1	1.00	1.03	0.84	0.64	0.53
14S2	1.05	1.03	0.94	0.57	0.49
14S3	1.08	1.07	0.69	0.49	0.37
14S4	1.05	1.08	0.94	0.77	0.67
14S5	0.97	0.93	0.77	0.59	0.46
Mean	1.03	1.03	0.84	0.61	0.50
COV(%)	3.72	5.25	11.51	15.05	19.40

5 Conclusions

A three dimensional finite element simulation can predict the moment-rotation response of a semi-rigid connection within acceptable accuracy. Among the simplified mathematical models, the best fit with the experimental data is observed in the polynomial model. Among the power models, the best fit is obtained by Kishi et.al. model. On the other hand, the most conservative results are obtained from Eurocode 3 model.

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