Modeling the Nonlinear Behavior of Semi-Rigid Connectors With a Macro Element Model

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ABSTRACT:
In this paper, a macro element model based on force formulation and considering spread inelastic behavior along element length and incorporating nonlinear semi-rigid connectors will be presented. For a true representation of the hysteretic energy dissipation characteristic of a semi-rigid connector, a quadra-linear moment-rotation based model is developed incorporating pinching, damage, the possibility of residual moment capacity other than zero, and degradation of stiffness under various scenarios. The behavior is calibrated via experimental data available in the literature. The macro element model is used in conjunction with corotational formulation for the capture of nonlinear geometric effects, as well. Importance of accurate modeling of cyclic behavior of semi-rigid connectors is assessed through analysis of a low-rise steel moment resisting frame structure with the use of proposed element and connector model.

Keywords: Semi-rigid connector; nonlinear analysis; steel framed structures; finite element

1. INTRODUCTION

In the analysis of steel structures with frame finite elements, structural engineers in practice mostly consider beam to column connections either as shear type (pinned) or moment type (rigid). In reality moment connections have some flexibility and shear connections have some rigidity incorporated in their real behavior with significant nonlinearity included, as well; thus this behavior results into the categorization of some connections as partially restrained or also called as semi-rigid. In order to incorporate the nonlinear behavior of semi-rigid connections, an accurate modeling of the hysteretic behavior of these connections is necessary besides accurate representation of spread of plasticity along element lengths. Several experimental and analytical modeling of semi-rigid connections have been performed by researchers, and implementation of an accurate behavior of semi-rigid connections into structural analyses is still an ongoing research issue.

In the study of Lui and Chen (1986) beam elements were formulated with updated Lagrangian approach which takes formation of the plastic hinges in the beam and both axial force and bending moment effects into consideration. In the formulation of this beam element, it was assumed that the member was prismatic and plane section remained plane, and thus distortion of the members was assumed to be negligible. Plasticity was assumed to exist only as a lumped value at the plastic hinge location and the remaining part of the member was linear elastic. For the moment rotation curve of the connection an exponential function was used.

In the study of Sekulovic and Salatic (2001) the effect of the connection flexibility on the behavior of the structure under static loading was examined. They used a numerical model which takes both nonlinear behavior of the connection and geometric nonlinearity of the structure into consideration. The beam model has similar assumptions with the study by Lui and Chen. They ended up with the effect of the connection flexibility on the critical load. In a later study by Sekulovic et.al. (2002), the authors have examined the effects of connection flexibility on nonlinear analyses. In this case the connections were modeled as nonlinear rotational spring and a dashpot in parallel. For modeling of the moment rotation behavior of the connection under monotonic loading the three parameter power model by Kishi et.al. (1993) was used. The independent hardening model was adopted to simulate the inelastic connection behavior under cyclic loading. They have concluded that the flexibility of the connection has a great importance on the nonlinear behavior of the structure.

Castellazzi (2012) has also studied the influence of flexible joints on the response of framed structures. He has developed a second order shear deformable beam element incorporating flexibility and eccentricity of connections at the end of the beam. The element has similar assumptions with regards to lumping of plasticity at element ends and the remaining of the beam being linear elastic. The developed element uses second order Timoshenko's beam model. While developing the structural stiffness matrix Castellazi condensed out the additional degrees of freedoms so that the dimension of the matrix is 6×6.

In a recent study by Valipour and Bradford (2012), a frame element that considers spread inelasticity along element length and nonlinear semi-rigid behavior at element ends was developed. The element has been developed based on force formulation using total secant stiffness approach. Due to the use of force based formulation, a single macro element was used for capturing the response of the beam and connections together. The nonlinear response of the connection did not take into account stiffness and strength degradation and pinching effects, but they opted to use both a bilinear model with kinematic hardening and alternatively the Ramberg–Osgood model. The connection region considered both moment-rotation behavior and axial force-deformation behavior, thus contained both rotational and translational springs.

Development of frame finite element models could be pursued either by displacement-based or force-based formulations as stated in above studies. The use of force-based approach provides significant advantages in the modeling of semi-rigid connections at the ends of an element or along an element without the need for the introduction of additional degrees of freedom (dofs); furthermore, this approach also allows for an accurate representation of spread of plasticity along a member even with the use of single element per span. Due to its robustness and accuracy in terms of capturing nonlinear actions, a macro element model based on force formulation and considering spread inelastic behavior along element length and incorporating nonlinear semi-rigid connectors is developed in this paper. With this element, nonlinear semi-rigid behavior of a column to beam connection or a column-tree to beam connection will be easily handled in an analysis. Actually, the developed element is formulated and coded such that it can include infinite number of semi-rigid connectors at arbitrary locations along an element. For a true representation of the hysteretic energy dissipation characteristic of a semi-rigid connector, a quadra-linear moment-rotation based model is developed incorporating pinching, damage, the possibility of residual moment capacity, degradation of stiffness under various scenarios. The developed moment-rotation model is calibrated and tested via experimental cyclic data of top and seat angle with/without double web angles connections from literature. The macro element model is used in conjunction with corotational formulation for the capture of nonlinear geometric effects. An accurate capture of possible nonlinear plastification in the beam and nonlinear hinging in the connector should be strictly incorporated in the analysis of steel framed structures. Importance of accurate modeling of cyclic behavior of semi-rigid connectors is assessed through analysis of a low-rise steel moment resisting frame structure with the use of proposed element and connector model.

2. FRAME ELEMENT FORMULATION

Formulation of a nonlinear macro element model with semi-rigid connectors through the use of displacement based finite elements requires increased number of meshing and thus the use of several degrees of freedom. In a recent study, Saritas and Soydas (2012) has shown for small and large structural systems that increased element numbers and degrees of freedom as a result of the use of displacement based approach results in significant increases in computation times when compared with the use of force-based approaches. In general, if the spread of plasticity along a member is sought, at least 8 to 16 elements are necessary to accurately capture the nonlinear behavior with displacement based approach. With the introduction of semi-rigid connectors, as many more zero-length spring elements will be required. Development of a macro element model with displacement based approach in this regards would not be a truly novel contribution since it would be just the assembly of the responses of several elements with the use of finite element software and by attaching rotational springs to the ends. Despite this fact, there is still effort that should be spent in accurately capturing the cyclic behavior of semi-rigid connectors as discussed in the next chapter, and such a model should be implemented and used as part of a frame finite element analysis program nevertheless.

Formulation of the nonlinear frame element with semi-rigid connectors in this paper is based on the fact that force interpolation functions are used in predicting the element response. The element is composed of continuous element portions with discontinuities arising due to the presence of zero-length rotational springs. Kinematics of the continuous portion of the frame element follows Euler-Bernoulli Beam Theory assumptions, in which plane sections before deformation remain plane after deformation. As part of this theory, shear deformations are neglected, and thus element response is due to the presence of normal stress along element length. Nonlinear response of the continuous portion of the element is aggregated from the monitoring of the responses of several control sections along element length, and furthermore at each section, the response is assembled through fiber discretization of the section.

The formulation of the element starts with the calculation of axial force *N(x)* and bending moment *M(x)* from basic element forces q_1, q_2 and q_3 (Figure 1). In the absence of inter element loads, these internal forces can be simply calculated with statics knowledge as follows:

$$\boldsymbol{s}(x)=\begin{Bmatrix} N(x) \\ M(x) \end{Bmatrix}=\begin{Bmatrix} q_1 \\ \left(\dfrac{x}{L}-1\right)q_2+\dfrac{x}{L}q_3 \end{Bmatrix} \tag{2.1}$$

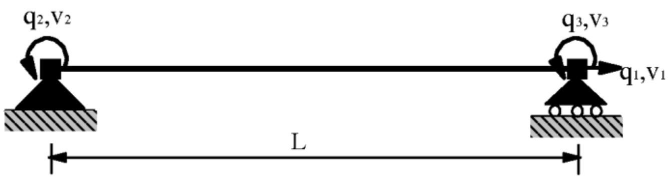

Figure 1. Basic Forces and Deformation for Beam Element

Above equation can be extended to the presence of distributed uniform element loads w_x and w_y applied in the axial and transverse directions, respectively, and can be written as:

$$\boldsymbol{s}(x)=\boldsymbol{b}(x)\boldsymbol{q}+\boldsymbol{s}_{\boldsymbol{p}}(x) \tag{2.2}$$

where $\boldsymbol{q}$ is the vector of basic element forces, $\boldsymbol{b}(x)$ represents the force-interpolation functions and can be regarded as an equilibrium transformation between section forces $\boldsymbol{s}(x)$ and basic forces $\boldsymbol{q}$, and the particular solution $\boldsymbol{s_p}(x)$ is due to uniform loads w_x and w_y, where these are all given as:

$$\boldsymbol{q}=\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix};\ \boldsymbol{b}(x)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & x/L-1 & x/L \end{bmatrix};\ \ \boldsymbol{s}_{\boldsymbol{p}}(x)=\begin{bmatrix} L(1-x/L) & 0 \\ 0 & L^2\left(\left(x/L\right)^2-x/L\right)/2 \end{bmatrix}\begin{Bmatrix} w_x \\ w_y \end{Bmatrix} \tag{2.3}$$

Principal of virtual forces necessitate the equality between virtual element end forces $\delta\boldsymbol{q}$ multiplied with real element deformations $\boldsymbol{v}$ to be calculated from the integration of virtual section forces $\delta\boldsymbol{s}$ multiplied with real section deformations $\boldsymbol{e}$. This equality is expressed in the following equation,

$$\delta\boldsymbol{q}^T\boldsymbol{v}=\int_L \delta\boldsymbol{s}^T(x)\boldsymbol{e}(x)\,dx \tag{2.4}$$

where section deformations are $\mathbf{e} = \left(\varepsilon_a \quad \kappa\right)^{\mathrm{T}}$.

Introduction of the section interpolation functions from above equations and with the fact that virtual forces are arbitrary, element end forces are calculated from the section deformations as follows:

$$\boldsymbol{v} = \int_L \boldsymbol{b}^T\left(x\right)\boldsymbol{e}\left(x\right)dx \tag{2.5}$$

Element flexibility matrix ***f*** can be calculated from above equation by considering partial differentiation with respect to element end forces:

$$\boldsymbol{f} = \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{q}} = \frac{\partial}{\partial \boldsymbol{q}}\left[\int_L \boldsymbol{b}^T\left(x\right)\boldsymbol{e}\left(x\right)dx\right] = \int_L \boldsymbol{b}^T\left(x\right)\underbrace{\frac{\partial \boldsymbol{e}(x)}{\partial \boldsymbol{s}(x)}}_{\boldsymbol{f}_s(x)}\underbrace{\frac{\partial \boldsymbol{s}(x)}{\partial \boldsymbol{q}}}_{\boldsymbol{b}(x)}dx \tag{2.6}$$

where $\boldsymbol{f}_s$ is the section flexibility matrix that can be calculated from the inversion of the section stiffness matrix $\boldsymbol{k}_s$.

At this point in the element formulation, continuous integrals written above will be discretized and furthermore the presence of semi-rigid connectors will be introduced through the following extended version of above equation for the calculation of element end deformations:

$$\begin{aligned} &\boldsymbol{v} = \boldsymbol{v}_{Elem} + \boldsymbol{v}_{Con} \\ &\boldsymbol{v}_{Elem} = \sum_{i=1}^{nIP}\boldsymbol{b}^T\left(x_i\right)\boldsymbol{e}_{\boldsymbol{i}}\, wIP_{\boldsymbol{i}} \text{ ;and } \boldsymbol{v}_{Con} = \sum_{i=1}^{nSC}\boldsymbol{b}^T\left(x_i\right)\Delta_{SC,i} \end{aligned} \tag{2.7}$$

where *nIP* is the total number of monitoring sections used for the calculation of the nonlinear response of the continuous portion of the element and *nSC* is the total number of semi-rigid connectors present along the element; *wIP* is the integration weight corresponding to that integration location, and eventually $\Delta_{SC} = [\delta \quad \theta]^T$ is the vector of semi-rigid connector deformations composed of axial deformation δ and rotation θ of a connector.

Element flexibility matrix is similarly discretized as follows:

$$\begin{aligned} &\boldsymbol{f} = \boldsymbol{f}_{Elem} + \boldsymbol{f}_{Con} \\ &\boldsymbol{f}_{Elem} = \sum_{i=1}^{nIP}\boldsymbol{b}^T\left(x_i\right)\boldsymbol{f}_{s\boldsymbol{i}}\boldsymbol{b}\left(x_i\right)wIP_{\boldsymbol{i}} \quad \text{and} \quad \boldsymbol{f}_{Con} = \sum_{i=1}^{nSC}\boldsymbol{b}^T\left(x_i\right)\boldsymbol{f}_{SC,i}\boldsymbol{b}\left(x_i\right) \end{aligned} \tag{2.8}$$

The use of Gauss-quadrature or Gauss-Lobatto quadrature integration points with number of integration points nIP selected as 5 results in an accurate nonlinear behavior for the continuous part [24]. With regards to the nonlinear behavior of semi-rigid connectors, the current study will focus on only the presence of flexibility and nonlinear behavior for the rotational component of semi-rigid connectors, thus axial behavior is assumed as rigid with infinite strength. With this assumption, deformation and flexibility due to the connector parts can be modified as follows:

$$\boldsymbol{f}_{Elem} = \sum_{i=1}^{nIP}\boldsymbol{b}^T\left(x_i\right)\boldsymbol{f}_{s\boldsymbol{i}}\boldsymbol{b}\left(x_i\right)wIP_{\boldsymbol{i}} \quad \text{and} \quad \boldsymbol{f}_{Con} = \sum_{i=1}^{nSC}\boldsymbol{b}^T\left(x_i\right)\boldsymbol{f}_{SC,i}\boldsymbol{b}\left(x_i\right) \tag{2.9}$$

$$\boldsymbol{f}_{Con} = \sum_{i=1}^{nSC}\begin{bmatrix} 0 \\ x_i/L-1 \\ x_i/L \end{bmatrix}\left[\frac{\partial M}{\partial \theta}\right]_{SC,i}^{-1}\left[0 \quad x_i/L-1 \quad x_i/L\right] \tag{2.10}$$

The proposed frame element with semi-rigid connectors is implemented in a standard finite element software package that assumes that the element response is composed of resisting forces and stiffness matrix, where in this regards the element formulation should appear to be displacement-based to the program.

3. HYSTERETIC SECTION MODEL FOR CONNECTORS

Experiments available in the literature show that under cyclic loading it is reasonable to characterize the behavior of steel connections with hysteretic loops incorporating strength and stiffness degradation. Traditionally multi-parameter mathematical models have been developed to achieve this task; however, especially under cyclic loading the behavior is quite complex due to some phenomena named as pinching that results from material and geometric nonlinearities in the connection. It should be emphasized that the development of a multi-parameter model alone would not be a complete work without calibrating those parameters to reproduce moment rotation curves for different connection topologies.

Several simplified mathematical models are available in literature; namely linear, bilinear, tri-linear or more general multi-linear and nonlinear, where the first two models are inadequate in reflecting the cyclic behavior in an accurate manner. In this study, a quadra-linear model is preferred. Selected model bases on the same tri-linear model present in both Fedeaslab and OpenSees finite element programs. The tri-linear backbone curve, loading-unloading and pinching behavior of the existing trilinear model is shown in Figure 2. In using such a model, yielding and ultimate moments can be taken from steel specifications and change according to type of the connection. The unloading stiffness has been taken as equal to initial stiffness which is reasonable when considering experimental tests.

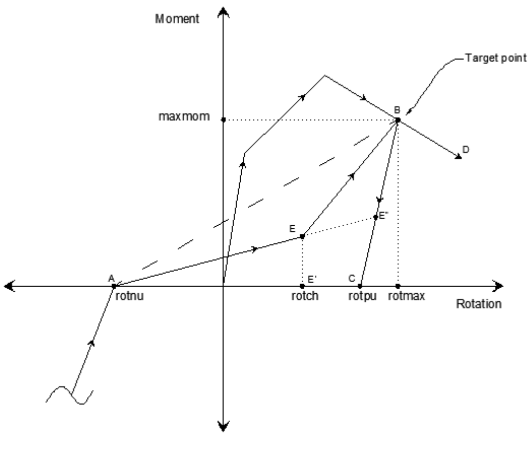

Figure 2. Behavior of the existing trilinear model

Extension to quadra-linear behavior with added parameters was needed in order to better capture pinching and residual moment capacities in the connection. In steel bolted connections pinching results from permanent deformations of bolt holes that change the shape of the holes from circle to ellipse. In the experiments in which pinching governs clearly it is also seen that the stiffness degradation due to pinching is observed at a point away from zero-moment crossing. This means that the stiffness of the unloading branch remains same up to a moment value in the reloading region. This moment value can be called as residual moment and used to determine the starting point of pinching. With this modification the user is allowed to define the residual moment value. Then this residual moment value is transformed to a rotation value. In other words, since the program algorithm is based on rotation values, the rotation value of the point at which the residual moment is reached (point A’ in Figure 3) is accepted as the starting point of pinching. In the modified model, this residual moment value is also used as a limiting value for the value of *maxmom* shown in Figure 3. In other words, if the stiffness of the any branch of the positive envelope curve is smaller than zero, the moment can decrease up to this residual moment value and then remains constant. This limitation is applied by calculating the rotation value of point D, *rotlim*. In the original model this limiting moment value is zero instead of residual moment and it is the point D' instead of D that is used to determine the value of *rotlim*.

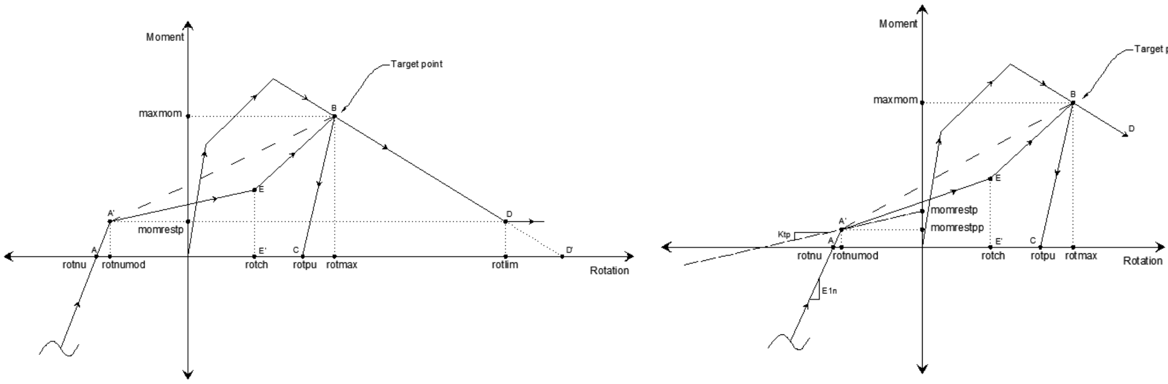

Figure 3. Behavior of the proposed quadralinear model

Another modification introduced to the model is based on an observation that in some cases the residual moment gets smaller as load cycles get larger. This modification is arranged by introducing a slope value, *Ktp*, which is the slope of the line passing through a point on the moment axis. This point is selected as the point of residual moment on the moment axis (see Figure 3). This slope value Ktp is controlled by the user since it varies according to experimental results. In this case a new variable called as *momrestpp* is introduced which is changing according to parameters *momrestp* and *Ktp*. It can also be noticed that as cycles get larger the residual moment can take negative values which is reasonable when considering experimental results.

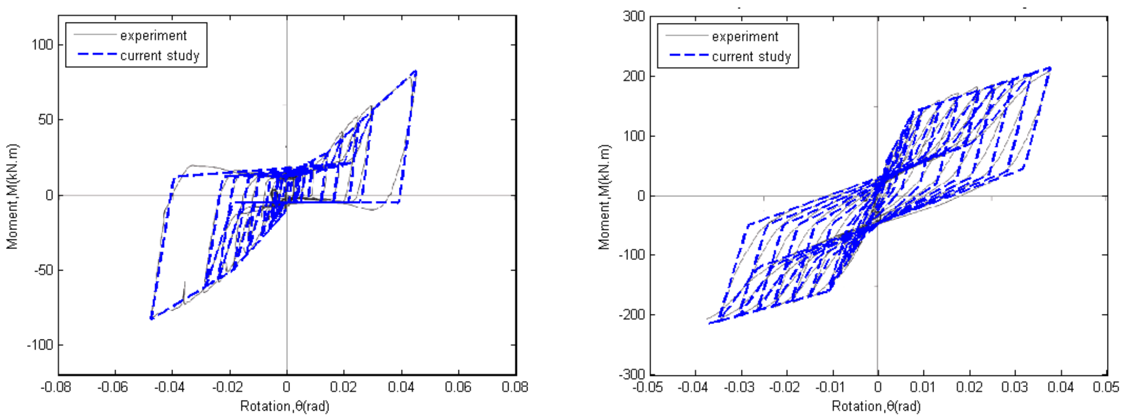

Figure 4. Comparison of Experimental and Analytical Results of Specimens by Abolmaali et.al.(2003)
Left Specimen: DW-BB-127-13-16-114-5-610; Right Specimen: DW-WB-152-19-19-191-5-610

The accuracy of the model presented above can be validated with the test results of semi-rigid connections existing in the literature. The results obtained by Abolmaali et.al. (2003) were selected for this purpose. In their study two types of connections were tested namely bolted-bolted and welded-bolted double web angle connection. Bolted-bolted means angles are bolted to both beam web and column flange. Welded bolted means angles are welded to beam web and bolted to column flange. The connection between the angle and beam web is important. Because pinching mostly occurs due to enforcing of the bolts to the beam web. If the bolts have necessary strength, the circular bolt holes in the web will be oval shaped and if the angles have necessary strength the failure mode will be most probably web bearing. On the other hand when angles are welded to the beam web, due to high strength of weld, angle yielding governs the failure mode. In that study 5 bolted-bolted and 5 welded-bolted connections were used for validation. All of the specimens were successfully simulated with the proposed connector model. For sake of presentation in this paper, only the comparison graphs of two of these specimens are shown in Figure 4.

4. ANALYSIS OF A MOMENT RESISTING STEEL FRAME WITH PROPOSED MODELS

In order to verify the response of the proposed macro element including semi-rigid connectors and to see the effect of the semi-rigid beam to column connections on the cyclic behavior of structures, a low-rise three stories with single bay structure as shown in Figure 5 is considered. All columns have 3.6 m length and IPE 330 section and all beams have 6 m length and IPE 300 sections; thus, ensuring strong column – weak beam design criteria. For all columns and beams bi-linear hysteretic steel material with f_y = 355 MPa, E = 200 GPa and E_h= 0.01E is used along element length, where each element is monitored at 5 Lobatto integration points, and at each section 4 and 8 layers are used in each flange and web, respectively. For semi-rigid connections of beams, the parameters used in the specimen DW-WB-152-19-19-191-5-610 of Figure 4 is considered in order to incorporate pinching behavior in connector's cyclic response. Initial rotational stiffness of the selected connection is 30670 kN.m/rad; thus, the ratio between the initial stiffness of the connection with respect to the beam flexural rigidity of IPE300 section satisfies the partially restrained condition $\left(2 < a = R_{ki} \times (L/EI)_{beam} \cong 11 < 20\right)$. In addition to this analysis case, an alternative analysis is also conducted, where bilinear response is present in the semi-rigid connection instead of the quadra-linear model with pinching. This comparison will demonstrate the effect of the pinching behavior on the energy absorption during cyclic loading.

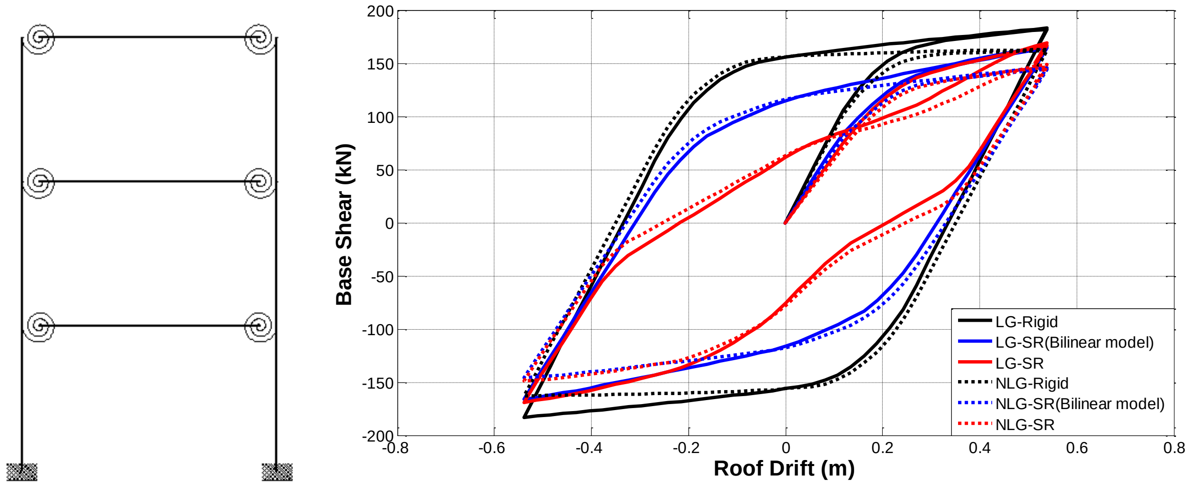

Figure 5. Effects of semi-rigid connection, pinching and nonlinear geometry on story drift

A cyclic displacement reversal with a magnitude of 0.5 m is applied at the roof level. Four different cases are examined to show the semi-rigid, nonlinear geometry and pinching effects: rigid connector model with linear geometry (LG-Rigid); rigid connector model with nonlinear geometry (NLG-Rigid); proposed semi-rigid connector model with linear geometry (LG-SR); proposed semi-rigid connector model with nonlinear geometry (NLG-SR); bi-linear semi-rigid connector model with linear geometry (LG-SR(Bi-Linear model)); bi-linear semi-rigid connector model with nonlinear geometry (NLG-SR(Linear model)). In all analyses, spread of inelasticity along element length is taken into account. The responses obtained from the rigid and semi-rigid cases in Figure 5 clearly demonstrate the importance of the linear and nonlinear behavior of semi-rigid connections. The difference in the energy absorption capacities between the bilinear model and quadra-linear model with pinching and damage for semi-rigid connections furthermore presents the necessity of development and use of advanced hysteretic models for the description of the nonlinear response of structures under earthquake excitations.

5. CONCLUSION

In this paper, an accurate and robust beam element with semi-rigid connections is developed, where the element derivation bases on force formulation. With this element it is possible to incorporate semi-rigid connectors' behavior into analysis with less degree of freedom with respect to displacement based element. With this element it is also possible to capture the behavior of the spread of plasticity along element length in the beam span, as well. Also developed macro element can include any number of connectors located at anywhere along the beam; thus column tree connections can be easily analyzed by using single element per span by using the proposed beam element.

As part of this study, a hysteretic section model is developed for capturing the cyclic behavior of semi-rigid connections. With some modifications such as pinching and residual moment behavior, moment rotation curves very close to experimental ones are obtained. The developed section model can easily be incorporated into the developed macro element. Thus a more realistic structural analysis can be performed with this element.

Effect of semi-rigid connections on the initial lateral stiffness of the building is observed by comparing the rigid connection case. A lateral stiffness reduction is observed due to semi-rigid behavior of the connections. Effect of pinching on the energy absorption of the building is observed by comparing the results with the bilinear modeling approach of the semi-rigid connection, i.e. pinching in the connection is ignored with this analysis case. It is seen that pinching of the connections reduces the energy absorption of the building significantly, and should be incorporated into the modeling when earthquake excitations will be considered through a static cyclic or nonlinear time history analyses. Effect of nonlinear geometry is observed in the structural system response, and it is seen that for each case nonlinear geometry effect leads to softening in the building.

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