Painlevé classification of coupled Korteweg–de Vries systems

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In this work, we give a classification of coupled Korteweg–de Vries equations. We found new systems of equations that are completely integrable in the sense of Painlevé. © *1997 American Institute of Physics.* [S0022-2488(97)01407-2]

I. INTRODUCTION

The coupled Korteweg–de Vries (KdV) type equations have been the most important class of nonlinear evolution equations and are extensively studied by many authors.[1–8] Recently, Svinolupov[9,10] has introduced a class of integrable multicomponent KdV equations associated with Jordan algebras. We have shown that the Jordan–KdV systems have a Painlevé property.[11] Very recently,[12] Svinolupov's work was extended on KdV systems to a more general form,

$$q_t^i = b_j^i q_{xxx}^j + s_{jk}^i q^j q_x^k, \tag{1}$$

where $i,j,k=1,2,\ldots,N$, q^i depend on the variables x,t, and s_{jk}^i, b_j^i are constants. It is shown that there are infinitely many integrable subclasses of (1) having recursion operators,

$$R_j^i = b_j^i D^2 + a_{jk}^i q^k + c_{jk}^i q_x^k D^{-1} + F_{lkj}^i q^l D^{-1} q^k D^{-1}, \tag{2}$$

where a_{jk}^i, c_{jk}^i and F_{lkj}^i are constants with

$$s_{jk}^i = a_{kj}^i + c_{jk}^i, \quad F_{lkj}^i = -F_{ljk}^i. \tag{3}$$

In this work we applied the Painlevé test for PDE introduced by Weiss *et al.*[13] to find the integrable subclasses of (1) when $N=2$. We consider a system of coupled KdV equations in the form

$$\begin{aligned} u_t &= \eta_1 u_{xxx} + \eta_2 v_{xxx} + c_1 u u_x + c_2 u v_x + c_4 v u_x + c_3 v v_x, \\ v_t &= \mu_1 v_{xxx} + \mu_2 u_{xxx} + d_1 u u_x + d_2 u v_x + d_4 v u_x + d_3 v v_x, \end{aligned} \tag{4}$$

where

$$u=q^1, \quad v=q^2, \quad \eta_1=b_1^1, \quad \eta_2=b_2^1, \quad \mu_2=b_1^2, \quad \mu_1=b_2^2, \quad c_1=s_{11}^1,$$

$$c_2=s_{12}^1, \quad c_4=s_{21}^1, \quad c_3=s_{22}^1, \quad d_1=s_{11}^2, \quad d_2=s_{12}^2, \quad d_4=s_{21}^2, \quad d_3=s_{22}^2.$$

The main problem is to find the conditions satisfied by η_l, μ_l, c_n, d_n, ($l=1,2$; $n=1,2,3,4$) in order to have P type-subclasses of (4).

II. PAINLEVÉ ANALYSIS

Let $\phi=0$ be the singularity manifold of (4). By setting $u\approx u_0\phi^{\alpha_1}$, $v\approx v_0\phi^{\alpha_2}$ into the leading terms of (4), we have $\alpha_1=\alpha_2=-2$ and the equations for u_0 and v_0,

$$u_0^2 c_1 + u_0 v_0 (c_2+c_4) + v_0^2 c_3 + 12\phi_x^2 (u_0\eta_1 + v_0\eta_2) = 0, \tag{5}$$

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$$u_0^2 d_1 + u_0 v_0 (d_2 + d_4) + v_0^2 d_3 + 12\phi_x^2 (u_0 \mu_1 + v_0 \mu_2) = 0. \tag{6}$$

To determine the resonances we set $u \approx u_0 \phi^{-2} + \beta_1 \phi^{\tau-2}$, $v \approx v_0 \phi^{-2} + \beta_2 \phi^{\tau-2}$ into the leading terms of (4) and obtain a sixth-order polynomial equation in r. One root of this polynomial must be -1. Substituting $r=-1$ into the polynomial, we have the condition

$$12\phi_x^2 \{u_0(\eta_2 d_1 - \mu_1 c_1) + v_0[\eta_2(d_2+d_4) - \eta_1 d_3 + c_3\mu_2 - \mu_1(c_2+c_4)]\}$$
$$+ v_0[u_0(c_3 d_1 - c_1 d_3) - v_0 d_3(c_2+c_4) + v_0 c_3(d_2+d_4)] - 144\phi_x^4(\eta_1\mu_1 - \eta_2\mu_2) = 0. \tag{7}$$

Together with (5), (6), (7) the equation for resonances becomes

$$(r+1)(r-4)(r-6)\{(\eta_1\mu_1 - \eta_2\mu_2)(r^3 - 9r^2)\phi_x^2 + r[38(\eta_1\mu_1 - \eta_2\mu_2)\phi_x^2$$
$$+ u_0(\eta_1 d_2 - \eta_2 d_1 + \mu_1 c_1 - \mu_2 c_2) + v_0(\eta_1 d_3 - \eta_2 d_4 + \mu_1 c_4 - \mu_2 c_3)]$$
$$+ 2[-36(\eta_1\mu_1 - \eta_2\mu_2)\phi_x^2 - u_0[\eta_1(d_2+d_4) - 2(\eta_2 d_1 - \mu_1 c_1) - \mu_2(c_2+c_4)]$$
$$+ v_0[2(c_3\mu_2 - d_3\eta_1) + \eta_2(d_2+d_4) - \mu_1(c_2+c_4)]]\} = 0. \tag{8}$$

The three of the roots are $-1,4,6$. The others, say r_1, r_2, r_3, must be integers. This is possible if

$$(\eta_1\mu_1 - \eta_2\mu_2)\phi_x^2(r_1 r_2 r_3 - 72) - 2u_0[\eta_1(d_2+d_4) - 2(\eta_2 d_1 - \mu_1 c_1) - \mu_2(c_2+c_4)]$$
$$+ 2v_0[2(c_3\mu_2 - d_3\eta_1) + \eta_2(d_2+d_4) - \mu_1(c_2+c_4)] = 0, \tag{9}$$

$$(\eta_1\mu_1 - \eta_2\mu_2)\phi_x^2(r_1 r_2 + r_2 r_3 + r_1 r_3 - 38) - u_0(\eta_1 d_2 - \eta_2 d_1 + \mu_1 c_1 - \mu_2 c_2)$$
$$- v_0(\eta_1 d_3 - \eta_2 d_4 + \mu_1 c_4 - \mu_2 c_3) = 0, \tag{10}$$

$$(\eta_1\mu_1 - \eta_2\mu_2)(r_1 + r_2 + r_3 - 9) = 0. \tag{11}$$

At this point we have to divide the systems in (4) into two parts.[12] These are the *nondegenerate systems* where $(\eta_1\mu_1 - \eta_2\mu_2) \neq 0$ and the *degenerate systems* where $(\eta_1\mu_1 - \eta_2\mu_2) = 0$, that is, they reduce to lower-dimensional systems.

For the *nondegenerate systems*, the equation (11) implies that we have to have $r_1 + r_2 + r_3 = 9$, which leads the following.

Case (1): $r_1 = 0$, $r_2 = 0$, $r_3 = 9$. In this case u_0 and v_0 must be arbitrary. But (9) and (10) imply that this is impossible unless $(\eta_1\mu_1 - \eta_2\mu_2) = 0$. Thus, test fails.

Case (2): $r_1 = 0$, r_2 *may take one of the values* (1,2,3,4), $r_3 = 9 - r_2$. In these cases one of the functions u_0 or v_0 must be arbitrary. We assume that u_0 is arbitrary and $v_0 = \alpha\phi_x^2 + \beta$, where α and β are independent from ϕ_x. Then the equations (5), (6), (7), and (9) are satisfied if

$$c_3 = 0, \quad \eta_2 = 0, \quad \alpha = -\frac{12\mu_1}{d_3}, \quad \beta = \frac{u_0 c_1}{(c_2+c_4)}, \quad \eta_1 = \frac{(c_2+c_4)}{d_3},$$

$$\mu_2 = \mu_1[(c_2+c_4)(d_2+d_4) - c_1 d_3]/d_3(c_2+c_4), \tag{12}$$

$$d_1 = c_1[(c_2+c_4)(d_2+d_4) - c_1 d_3]/(c_2+c_4)^2, \quad \text{where } d_3 \neq 0, \quad c_2 + c_4 \neq 0, \quad \mu_1 \neq 0.$$

These are the only exceptable solutions; all others violate the condition $(\eta_1\mu_1 - \eta_2\mu_2) \neq 0$. For the above values of parameters we obtain the solutions of Eq. (10), which depends on r_2. Thus, we have four subcases with resonances $(0, r_2, 9 - r_2, -1, 4, 6)$. To discuss the arbitrariness of the functions corresponding to resonances, we substitute $u=\Sigma_{j=0}^{8} u_j\phi^{j-2}$, $v=\Sigma_{j=0}^{8} v_j\phi^{j-2}$ into (4), for each case separately, and obtain the recursion relations for u_j and v_j. By solving these relations we have the following results.

Case (2a): $r=0,1,8,-1,4,6$, $c_4=-3c_2$, $c_1=-c_2(3d_2+d_4)/(2d_3)$. This test fails, because the functions corresponding to resonances 0, 1, 6 are arbitrary without additional conditions, but u_4 or v_4 is arbitrary if $d_3=c_2(3d_2+d_4)/(d_4-d_2)$ and u_8 or v_8 is arbitrary if $d_2=d_4$, which implies $(\eta_1\mu_1-\eta_2\mu_2)=0$.

Case (2b): $r=0,2,7,-1,4,6$, $c_2=0$, $c_1=c_4d_2/d_3$. The equations pass the test if $c_4=d_3$, $d_4=0$. Thus the system,

$$u_t=\mu_1u_{xxx}+d_2u_x\,u+d_3u_x\,v,\qquad v_t=\mu_1v_{xxx}+d_2v_x\,u+d_3v_x\,v, \tag{13}$$

is of the P type, where u_0,v_2,u_4,v_6,u_7 are arbitrary functions of the solutions.

Case (2c): $r=0,3,6,-1,4,6$, $c_4=2c_2$, $c_1=3c_2(2d_2-d_4)/d_3$. The test fails, since the equations under investigation would be of the P type if $\mu_1=0$ or $d_3=0$, which violates the condition $(\eta_1\mu_1-\eta_2\mu_2)\neq 0$.

Case (2d): $r=0,4,5,-1,4,6$, $c_2=c_4$, $d_2=d_4$. In this case we obtained two subclasses of equations that are of the P type. For the first, we have $d_3=2c_4$ and $c_1=2d_4$,

$$u_t=\mu_1u_{xxx}+2\,d_4u_xu+c_4(u_xv+uv_x),\qquad v_t=\mu_1v_{xxx}+d_4(u_xv+uv_x)+2c_4v_xv, \tag{14}$$

which is the Jordan KdV system given by Svinolupov.[9,10] For the second, we have $d_3=-c_4$ and $c_1=-d_4$,

$$u_t=-2\mu_1u_{xxx}-d_4u_xu+c_4(u_xv+uv_x),\qquad v_t=-\frac{3d_4}{2c_4}\,u_{xxx}+\mu_1v_{xxx}-\frac{3d_4^2}{4c_4}\,u_xu+d_4(u_xv+uv_x)-c_4v_xv. \tag{15}$$

For both of subclasses u_0,u_4,v_4,u_5,v_6 are arbitrary functions of the solutions.

Case (3) $r_1=1$, r_2 may take one of the values (1,2,3,4), $r_3=8-r_2$.

In these and the following cases, Eqs. (9) and (10) imply that u_0 and v_0 must be in the form $u_0=\delta\phi_x^2$, $v_0=\alpha\phi_x^2$, where α and δ are constants. Using these in Eqs. (5), (6), (7), (9), (10), we find the conditions satisfied by $\eta_l,\mu_l,c_n,d_n,\alpha,\delta$ for different values of r_2 and r_3. Thus we have four subcases.

Case (3a): $r=1,1,7,-1,4,6$. Substituting $u=\Sigma_{j=0}^{7} u_j\phi^{j-2}$, $v=\Sigma_{j=0}^{7} v_j\phi^{j-2}$ into (4), we find that u_1 and v_1 are arbitrary functions if $\delta\eta_1+\alpha\eta_2=0$, $\delta\mu_2+\alpha\mu_1=0$. The solutions of these equations violate the condition $(\eta_1\mu_1-\eta_2\mu_2)\neq 0$, and the test fails.

Case (3b): $r=1,2,6,-1,4,6$. Substituting $u=\Sigma_{j=0}^{6} u_j\phi^{j-2}$, $v=\Sigma_{j=0}^{6} v_j\phi^{j-2}$ into (4) and requiring that Eqs. (5), (6), (7), (9), and (10) have to satisfy, we observe that two subclasses of (4) pass the Painlevé test. The first subclass is

$$u_t=\eta_1u_{xxx}-\frac{12\eta_1}{\delta}\,u_xu+2c_2u_xv+c_2v_xu-\frac{\delta c_2^2}{6\eta_1}\,v_xv,\qquad v_t=\eta_1v_{xxx}-\frac{6\eta_1}{\delta}\,v_xu+c_2v_xv, \tag{16}$$

and the second subclass is

$$u_t=\eta_1 u_{xxx}-\frac{\delta c_4}{4}v_{xxx}-\frac{12\eta_1}{\delta}u_x u+c_4 u_x v+2c_4 v_x u+c_3 v_x v,$$
$$v_t=-2\eta_1 v_{xxx}+\frac{12\eta_1}{\delta}v_x u-c_4 v_x v, \tag{17}$$

where, in both cases, $\alpha=0$, $\delta\neq 0$ and v_1, v_2, u_4, u_6, v_6 are arbitrary functions. We observe that the second subclass reduces to the equations given by Hirota–Satsuma[1,2,14] if $c_4=0$, $\delta=-2$,

$$u_t=\eta_1(u_{xxx}+6u_x u)+c_3 v_x v,$$
$$v_t=-2\eta_1(v_{xxx}+3v_x u),\quad \text{where}\quad \eta_1=a=\tfrac{1}{2},\quad c_3=2b. \tag{18}$$

Case (3c): $r=1,3,5,-1,4,6$. In this case we obtain the system of equations passing the P test,

$$u_t=-\frac{\delta c_1}{12}u_{xxx}+\frac{3\delta c_1^2}{4d_1}v_{xxx}+c_1 u_x u-\frac{3c_1^2}{d_1}u_x v-\frac{6c_1^2}{d_1}v_x u,$$
$$v_t=-\frac{\delta d_1}{12}u_{xxx}-\frac{7\delta c_1}{12}v_{xxx}+d_1 u_x u-c_1 u_x v-2c_1 v_x u-\frac{6c_1^2}{d_1}v_x v, \tag{19}$$

where $\alpha=0$, $\delta\neq 0$ and v_1, v_3, u_4, u_5, u_6 are arbitrary functions of the solutions.

Case (3d): $r=1,4,4,-1,4,6$. This test fails since the number of arbitrary functions is less than the number of resonances.

Case (4): $r_1=2$, r_2 may take one of the values (2,3), $r_3=7-r_2$. For these values of resonances we have two subcases.

Case (4a): $r=2,2,5,-1,4,6$. In order to have arbitrary functions at $r=2$, which are u_2 and v_2, the conditions $\delta\eta_1+\alpha\eta_2=0$, $\delta\mu_2+\alpha\mu_1=0$ must hold. But the solutions of these violate the condition $(\eta_1\mu_1-\eta_2\mu_2)\neq 0$. The test fails.

Case (4b): $r=2,3,4,-1,4,6$. In this case we have two subclasses of (4) passing the P test: The first subclass is

$$u_t=\eta_1 u_{xxx}+\eta_2 v_{xxx}-\frac{1}{\delta^2}[12(\delta\eta_1+\alpha\eta_2)+\alpha(2\delta c_2+\alpha c_3)]u_x u+c_2(u_x v+u v_x)+c_3 v_x v,$$
$$v_t=\frac{\Gamma}{\Delta}\eta_2 u_{xxx}+\mu_1 v_{xxx}+\frac{\Gamma}{\Delta}[c_2 u_x u+c_3(u_x v+v_x u)]$$
$$+\left\{c_2+\frac{2\delta c_3}{\Delta}[6(\delta\mu_1+\alpha\eta_2)+\alpha(\delta c_2+\alpha c_3)]\right\}v_x v, \tag{20}$$

where

$$\Gamma=-\alpha[\alpha(\delta c_2+\alpha c_3)+12\delta\mu_1],\quad \Delta=\delta^2(\delta c_2+\alpha c_3+12\eta_2)$$

and

$$(\delta c_2+\alpha c_3)[\delta(\eta_1-\mu_1)+2\eta_2\alpha]+12\eta_2(\delta\eta_1+\alpha\eta_2)=0.$$

The second subclass is

$$u_t=\eta_1 u_{xxx}+\eta_2 v_{xxx}-\frac{1}{\delta^2}[12(\delta\eta_1+\alpha\eta_2)+\delta c_2\alpha]u_x u+c_2(u_x v+uv_x)-\frac{\delta c_2}{\alpha}v_x v,$$

$$v_t=\frac{\alpha}{\delta}[\delta(\eta_1-\mu_1)+\eta_2\alpha]u_{xxx}+\mu_1 v_{xxx}-\frac{\alpha}{\delta^3\eta_2}[12\eta_2(\delta\eta_1+\alpha\eta_2)+\delta^2 c_2\mu_1]u_x u \quad (21)$$

$$+\frac{c_2\mu_1}{\eta_2}(u_x v+v_x u)-\frac{\delta}{\eta_2\alpha}c_2 v_x v,$$

where, in both cases u_2, u_3, u_4, v_4, u_6 are arbitrary functions. If we substitute $c_2=a_1$, $c_3=-a_0$, $\eta_2=1$, $\mu_1=\eta_1=0$, $\delta=(a_0\alpha-6)/a_1$, $\alpha=6/(a_0\pm ia_1)$ the first subclass reduces to the system given in Ref. 12,

$$u_t=v_{xxx}+(a_0u+a_1v)u_x+(a_1u-a_0v)v_x,$$
$$v_t=u_{xxx}+(a_0u+a_1v)v_x+(a_0v-a_1u)u_x. \quad (22)$$

Case (5): $r_1=3$, $r_2=3$, $r_3=3$. In this case test fails, since the number of resonances at $r=3$ is higher than the number of arbitrary functions, which are u_3 and v_3.

In order to discuss the *degenerate systems*, let us assume that $\mu_2=\mu_1=0$; then from (8) we have the relation $v_0=\lambda u_0$.

We know that the roots of (8) must be integers and three of the roots are -1, 4, 6. Let the fourth root be σ. When $\sigma\neq 0$, we can choose $u_0=\gamma\phi_x^2$. Substituting u_0 and v_0 into Eqs. (5) and (6), we have

$$d_1=-(d_2+d_4+d_3\lambda)\lambda,$$
$$\eta_1=-[12\eta_2\lambda+c_1\gamma+(c_2+c_4)\gamma\lambda+c_3\gamma\lambda^2]/12. \quad (23)$$

Together with these equations, the fourth root of (8) is

$$\sigma=2(d_2+d_4+2d_3\lambda)/(d_2+d_3\lambda), \quad (24)$$

which can be solved for λ,

$$\lambda=[(2-\sigma)d_2+2d_4]/(\sigma-4)d_3, \quad (25)$$

where $d_3\neq 0$, $\sigma\neq 4$. In this work we discuss the cases when $\sigma=1,2,3,4,5,6$ and obtained the following.

Case (d1): $r=1$, $-1,4,6$.

$$u_t=-c_1\gamma u_{xxx}+12\eta_2 v_{xxx}+12(c_1u+c_4v)u_x+12(c_2u+c_3v)v_x,$$
$$v_t=d_4u_xv-(2d_4u-d_3v)v_x, \quad (26)$$

where $\eta_2=-\gamma(c_2+2c_4)/36$, $c_1=(c_2+2c_4)d_4/(c_2-c_4)$, $c_3=(c_2+2c_4-3d_3)(c_2-c_4)/9d_4$, $d_2=-2d_4$, and v_1, u_4, u_6 are arbitrary.

Case (d2): $r=2$, $-1,4,6$.

$$u_t=-\frac{c_1\gamma}{12}u_{xxx}+\eta_2 v_{xxx}+(c_1u+c_4v)u_x+(c_2u+c_3v)v_x,$$
$$v_t=(d_2u+d_3v)v_x, \quad (27)$$

where $d_4=0$ and u_2, u_4, u_6 are the arbitrary functions.

Case (d3): $r=3$, $-1,4,6$.

$$u_t=-\frac{c_1\gamma}{12}u_{xxx}+\eta_2 v_{xxx}+(c_1u+c_4v)u_x+(c_2u+c_3v)v_x,$$
$$v_t=d_4u_x v+(2d_4u+d_3v)v_x, \tag{28}$$

where $d_2=2d_4$, $12\eta_2c_1+\gamma[c_1(2c_2-c_4)-3d_4(c_2-2c_4)]=0$, $d_4[-12\eta_2(c_2-2c_4)+\gamma(c_1c_3-c_2d_3+2c_4d_3)]=0$, and v_3, u_4, u_6 are arbitrary.

Case (d4): $r=4$, $-1,4,6,d_4=d_2$.

$$u_t=\eta_1u_{xxx}+\eta_2v_{xxx}+c_1u_xu+c_2(u_xv+v_xu)+c_3v_xv,$$
$$v_t=d_1u_x u+d_2(u_xv+v_xu)+d_3v_x v, \tag{29}$$

where

$c_4=c_2$, $d_1=-(2d_2+d_3\lambda)\lambda$, $\eta_1=-[12\eta_2\lambda+c_1\gamma+2c_2\gamma\lambda+c_3\gamma\lambda^2]/12$, and $\gamma=12\eta_2\{d_2(d_2-c_1)+d_2\lambda(2d_3-3c_2)-\lambda^2[d_3(c_2-d_3)+2c_3d_2]-c_3d_3\lambda^3\}/\{c_1c_2d_2+2d_2\lambda(c_1c_3+c_2^2)+c_3\lambda^2(c_1d_3+5c_2d_2)+2c_3\lambda^3(c_2d_3+c_3d_2)+c_3^2d_3\lambda^4\}$, u_4, v_4, u_6 are arbitrary.

As a special case, if $\lambda=0$, $c_2=d_3=0$, $d_2=c_3=2$, $c_1=6$, $\eta_2=0$, $\gamma=-2$, the set of equations (30) reduces to the one given by Ito,[3]

$$u_t=u_{xxx}+6u_xu+2v_xv,$$
$$v_t=2(uv)_x. \tag{30}$$

Case $(d5)$: $r=5$, $-1,4,6$. In this case we have two subclasses passing the P test: The first one is

$$u_t=-\frac{c_1\gamma}{12}u_{xxx}+\eta_2v_{xxx}+(c_1u+c_4v)u_x+(c_2u+c_3v)v_x,$$
$$v_t=d_4u_x v+(d_2u+d_3v)v_x, \tag{31}$$

where $d_4=3d_2/2$, $\lambda=0$, and the second one is

$$u_t=\frac{1}{12d_3^2}[(12\eta_2+c_4\gamma)(3d_2-2d_4)d_3-(3d_2-2d_4)^2c_3\gamma-3d_3^2\gamma(d_2-d_4)]u_{xxx}+\eta_2v_{xxx}$$
$$+\frac{1}{d_3}\{[(3d_2-2d_4)c_2+3d_3(d_2-d_4)]u+c_4d_3v\}u_x+(c_2u+c_3v)v_x, \tag{32}$$
$$v_t=-\frac{1}{d_3}[(3d_2-2d_4)(2d_2-3d_4)u-d_4d_3v]u_x+(d_2u+d_3v)v_x,$$

where, in both cases, u_4, u_5, u_6 are arbitrary.

Case (d6): $r=6$, $-1,4,6$.

$$u_t=-\frac{\gamma}{12d_3}[c_1d_3+(c_2-2c_4)(2d_2-d_4)]u_{xxx}+\frac{\gamma}{12d_3}[(c_4-2c_2)d_3+(2d_2-d_4)c_3]v_{xxx}$$
$$+(c_1u+c_4v)u_x+(c_2u+c_3v)v_x, \tag{33}$$

$$v_t = -\frac{1}{d_3}\left[(2d_2-d_4)(d_2-2d_4)u-d_4\, d_3 v\right]u_x+(d_2u+d_3v)v_x,$$

where v_4, u_6, v_6 are arbitrary.

III. SUMMARY

We found new coupled system of equations having the Painlevé property. Some of them reduce to the known equations by special choice of parameters. Some of these systems may be related by simple transformations. Furthermore, the problem studied in this work may be considered in the framework of the perturbative Painlevé approach given in Ref. 15. In most cases the recursion relations and the expressions for u_j and v_j are very extensive, and therefore are not given in this work.

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