SUPPLEMENTARY PROBLEMS

1. The discharge factor for flow through an open channel of parabolic cross-section is

$$K = \frac{1.2}{x}\left[\sqrt{16x^2+1} + \frac{1}{4x}\ln\left(\sqrt{16x^2+1}+4x\right)\right]^{-2/3}$$

where x is the ratio of the maximum water depth to the breadth of the channel at the top of the water. Determine K when $x = 0.45$. [Answer: $K = 1.3394$.]

2. The following Matlab code is supposed to evaluate the function

$$f(x) = \frac{x^2\cos(\pi x)}{(x^3+1)(x+2)}$$

for $x \in [0,1]$ at 200 points. Correct the code and check by evaluating the function at $x = 1$ (the 200th point) which should be $-1/6$:

```
x = linspace(0,1);
clear all
g = x^3+1;
H = x+2;
z = x.^2;
y = cos xpi;
f = y*z/g*h
```

3. Correct the following Matlab code which is supposed to plot the polynomial $f(x) = x^4 - 1$ between $x = -2$ and $x = 2$ using 20 points:

```
x = -2:0.1:2;
c = [1 0 0 -1];
y = polyval(c,x);
plot(y,x)
```

4. Correct the following Matlab code which is supposed to set up the function $f(x) = x^3\cos(x+1)$ on the grid $x = 0$ to 3 in steps of 0.1 and give the value of the function at $x = 2$ and $x = 3$:

```
x = linspace(0,3);
f = x^3.*cos x+1;
% x = 2
f(2)
% x = 3
f(End)
```

5. Create two vectors, one whose elements are $a_n = 2n-1$ and the other whose elements are $b_n = 2n+1$, $n = 0,1,...,7$. Determine the following: (a) $a+b$, (b) $a-b$, (c) $a'b$, (d) determinant of $a'b$, and (e) ab'.

6. Given the vector $x = \begin{bmatrix} 17 & -3 & -47 & 5 & 29 & -37 & 51 & -7 & 19 \end{bmatrix}$. Create a script that rearranges x into the following vector: $y = \begin{bmatrix} -3 & -7 & -37 & -47 & 51 & 29 & 19 & 17 & 5 \end{bmatrix}$. The script should be written to work on a vector of arbitrary length. Place the value 0 (for the general vector) with the negative quantities, that is, when 0 is the value of an element of the vector, it will be the first element of *y*.

7. Let $z = $ magic(5).

   a. Perform the following operations to z in the order given:

   i. Divide column 2 by $\sqrt{3}$.

   ii. Add the elements of the third row to the elements in the fifth row (the third row remains unchanged).

   iii. Multiply the elements of the first column by the corresponding elements of the fourth column and place the result in the first column.

   iv. Set the diagonal elements to 2.

   b. If the result obtained in (a) (iv) is denoted q, then obtain the diagonal of qq' (Answer: $\begin{bmatrix} 486 & 104189 & 7300 & 44522 & 111024 \end{bmatrix}$).

   c. Determine the maximum and minimum values of q.

8. Let $x = $ magic(3).

   a. Create a new matrix in which each row of has been moved up by one row and the first row becomes the last row.

   b. Create a new matrix in which each column of has been moved to the right and the last column becomes the first column.

9. Manipulate the output of magic(5) to produce the following altered matrix:

$$\text{magic(5)} = \begin{bmatrix} 17 & 24 & 1 & 8 & 15 \\ 23 & 5 & 7 & 14 & 16 \\ 4 & 6 & 13 & 20 & 22 \\ 10 & 12 & 19 & 21 & 3 \\ 11 & 18 & 25 & 2 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 17 & 24 & 1 & 8 & 0 \\ 23 & 5 & 7 & 0 & 16 \\ 4 & 6 & 0 & 20 & 22 \\ 10 & 0 & 19 & 21 & 3 \\ 0 & 18 & 25 & 2 & 9 \end{bmatrix}$$

10. The transformation from spherical to Cartesian coordinates is given by

$$x = b \sin \phi \cos \theta$$
$$y = b \sin \phi \sin \theta$$
$$z = b \cos \phi$$

Take ten equally spaced values of in the range $0 \le \phi \le 90°$ and twenty-four equally spaced values of in the range $0 \le \theta \le 360°$ and plot the hemisphere using $mesh\,(x, y, z)$ when $b = 2$.

11. If

$$A = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

then show that P is an orthogonal matrix, i.e. $P'P = I$ where I is the identity matrix and that $P^{-1}AP = eig(\,A\,)$.

12. If

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Show that $A^2 - 4A - 5I = 0$, i.e. every matrix satisfies its own characteristic polynomial (Cayley–Hamilton theorem). How can you deduce $A^{-1} = \frac{1}{5}(A - 4I)$?

13. Plot the Fourier series given below for 250 values of t over its indicated range using $plot\,(t, f(t))$. Obtain the solutions by using the vector multiplication procedures.
   a. Square wave

$$f(t) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{401} \frac{1}{n} \sin(2n\pi t) \qquad -\frac{1}{2} \le t \le \frac{1}{2}$$
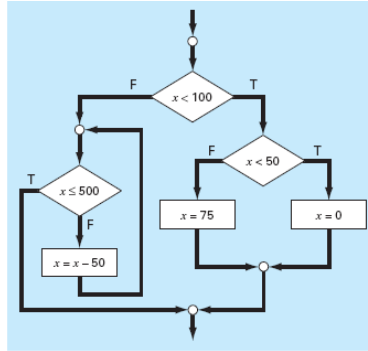
   b. Sawtooth

$$f(t) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{200} \frac{1}{n} \sin(2n\pi t) \qquad -1 \le t \le 1$$

   c. Trapezoidal

$$f(t) = \frac{4}{\alpha^2} \sum_{n=1,3,5,\ldots}^{105} \frac{\sin(n\pi\alpha)}{(\pi n)^2} \sin(n\pi t) \qquad -2 \le t \le 2$$

   Let $\alpha = 0.25$.

14. Write a Matlab pseudocode to implement the flowchart below:

15. Two quantitites are required to locate a point in 2D space, namely,

   (a) (x, y) in Cartesian coordinates

   (b) (r, θ) in Polar coordinates

   where they are related by

$$x = r\cos(\theta), \quad y = r\sin(\theta) \quad \text{and} \quad r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x).$$

   Some difficulties arises in the conversion procedure as summarized in the Table:

x	< 0	< 0	< 0	$= 0$	$= 0$	$= 0$
y	> 0	< 0	$= 0$	> 0	< 0	$= 0$
θ	$\tan^{-1}(y/x) + \pi$	$\tan^{-1}(y/x) - \pi$	π	$\pi/2$	$-\pi/2$	0

   Write a Matlab function procedure to calculate (r, θ) as a function of (x, y). Run your program to fill out the following Table:

x	1	1	0	-1	-1	-1	0	1	0
y	0	1	1	1	0	-1	-1	-1	0
r									
θ									

16. Consider the following piecewise defined function:

$$f(t) = \begin{cases} 11t^2 - 5t & \text{for} \quad 0 \le t \le 10, \\ 1100 - 5t & \text{for} \quad 10 \le t \le 20, \\ 50t + 2(t - 20)^2 & \text{for} \quad 20 \le t \le 30, \\ 1520\exp(-0.2(t - 30)) & \text{for} \quad t > 30, \\ 0 & \text{for} \quad \text{otherwise.} \end{cases}$$

   Write a Matlab function to generate a table of $f(t)$ values for $t = -5$ to 50 at increments of 0.5 and plot t versus $f(t)$.