HINTS to ASSIGNMENT #6

A&T-5.3

$$L(\alpha)=\frac{h_1\cdots h_n}{\alpha^{2n}}exp\left\{-\frac{1}{2}\left[\left(\frac{h_1}{\alpha}\right)^2+\cdots+\left(\frac{h_n}{\alpha}\right)^2\right]\right\}$$

$$ln\,L(\alpha)=ln(h_1\cdots h_n)-2n\,ln\,\alpha-\frac{1}{2}\left[\left(\frac{h_1}{\alpha}\right)^2+\cdots+\left(\frac{h_n}{\alpha}\right)^2\right]$$

$$\frac{\partial}{\partial\alpha}ln\,L(\alpha)=0\Rightarrow\alpha^2=\frac{1}{2n}\left[h_1^2+\cdots+h_n^2\right]\Rightarrow\alpha\approx 2$$

A&T-5.6 Assume a normal population

(a) $\bar{x}=85$ and $s_{n-1}=6.76$

(b) $P(|\bar{x}-\mu|<L)=.98\Rightarrow L=z^*\sigma/\sqrt{n}$ where $\phi(z^*)=\frac{1+\gamma}{2}=.99$

$\Rightarrow z^*=2.33\Rightarrow L=5.25$

(c) $P(|\bar{x}-\mu|<L)=.98\Rightarrow L=t^*s_{n-1}/\sqrt{n}$ where $t^*=t_{\alpha,\nu}=2.9$ with $\alpha=0.01$ and $\nu=9-1=8\Rightarrow L=6.52$

A&T-5.7

(a) $\bar{x}=2.03$ and $s_{n-1}=0.485$

(b) $P(|\bar{x}-\mu|<L)=.95\Rightarrow L=t^*s_{n-1}/\sqrt{n}$ where $t^*=t_{\alpha,\nu}=1.8$ with $\alpha=0.025$ and $\nu=10-1=9\Rightarrow L=0.35$

(c) $P(\bar{x}-L<\mu)=.95\Rightarrow L=t^*s_{n-1}/\sqrt{n}$ where $t^*=t_{\alpha,\nu}=1.8$ with $\alpha=1-\gamma=0.05$ and $\nu=10-1=9\Rightarrow L=0.28$

A&T-5.10 Assume a normal population

(a) $P(|\bar{x}-\mu|<L)=.90\Rightarrow L=z^*\sigma/\sqrt{n}$ where $\phi(z^*)=\frac{1+\gamma}{2}=.95$

$\Rightarrow z^*=1.65\Rightarrow L=1.65$

(b) $n=\left(\frac{z^*\sigma}{L}\right)^2$ where $L=1.65$ and $\phi(z^*)=.975\Rightarrow z^*=1.96$

$\Rightarrow n=12.7\Rightarrow$ 4 additional

(c) $s_{n-1}=\sqrt{84.5/8}=3.25$ and $P(|\bar{x}-\mu|<L)=.90$

$\Rightarrow L=t^*s_{n-1}/\sqrt{n}$ where $t^*=t_{\alpha,\nu}=1.86$ with $\alpha=.05$ and $\nu=8\Rightarrow L=2.015$

A&T-6.8 H_o : Poisson model is good

i	x_i	o_i	e_i
1	0	6	6
2	1	8	7
3	≥ 2	3, 2, 1 $\}=6$	4, 2, 1 $\}=7$

where $e_i=f_X(x_i)\times 20$ with $f_X(x)=e^{-\lambda}\lambda^x/x!$ and $\lambda\approx\hat{\lambda}=\bar{x}=1.2$

$$\chi^2=\sum_{i=1}^{3}\frac{(o_i-e_i)^2}{e_i}=0.29$$

Since $\chi^2<\chi^2_{\alpha,\nu}=6.63$ with $\alpha=0.01$ and $\nu=3-1-1=1$. H_o cannot be rejected.

M&A-p298/30 $H_o:\mu=20$ and $H_1:\mu>20\ (=28)$

(a) $\bar{X}:N(20,5/\sqrt{9})$ if H_o is true.

(c) $\alpha=P(\bar{X}>25)=1-\phi\left(\frac{25-20}{5/\sqrt{9}}\right)=1-0.998650=0.00135$

(d) $\bar{X}:N(28,5/\sqrt{9})$ if H_1 is true.

(f) $\beta=P(\bar{X}<25)=\phi\left(\frac{25-28}{5/\sqrt{9}}\right)=1-0.964070=0.036$

(g) Power $=1-\beta=0.96$

(h) Thinner and taller for larger n

(i) Smaller α & β for larger n.

M&A-p301/42 $H_o:\mu=0$ and $H_1:\mu\neq 0$

$t_{\nu,\alpha/2}=\frac{\bar{X}-\mu}{s/\sqrt{n}}$ is student's r.v. with $\nu=n-1=24$

$t^*=\frac{0.03-0}{0.1/\sqrt{25}}=1.5\Rightarrow .05<\alpha/2<.1\Rightarrow .1<\alpha<.2$

Since $P(\text{Type-I error})=\alpha$ is too high for practical purposes, H_o should be rejected.

M&A-p309/62 $H_o:\mu=5$ and $H_1:\mu>5$

$t^*=\frac{5.5-5}{2/\sqrt{25}}=1.25\Rightarrow .1<\alpha<.25$ for $\nu=n-1=24$

The P-value is btw 10% and 25%.

M&A-p676/10

o_{ij}				
	1	3	24	**28**
	12	28	76	**116**
	12	14	30	**56**
	25	**45**	**130**	**200**

e_{ij}			
	4	6	18
	15	26	75
	7	13	36

$\chi^2_{\alpha,\nu}=11.17$ for $\nu=(3-1)(3-1)=4\Rightarrow\alpha=\text{P-value}\approx .025$

Thus, H_o would be rejected at 5% significance level and accepted at 1% significance level.

M&R-7.2

(a) $E\{\hat{\theta}_1\}=\frac{7\mu}{7}=\mu$, $E\{\hat{\theta}_2\}=\frac{2\mu-\mu+\mu}{2}=\mu$. Both are unbiased.

(b) $V\{\hat{\theta}_1\}=\frac{7\sigma^2}{7^2}=\frac{\sigma^2}{7}$, $V\{\hat{\theta}_2\}=\sigma^2+\frac{1}{4}\sigma^2+\frac{1}{4}\sigma^2=\frac{3}{2}\sigma^2$

Since $V\{\hat{\theta}_1\}<V\{\hat{\theta}_2\}$, $V\{\hat{\theta}_1\}$ is better.

M&R-7.21 $f_X(x)=pq^{x-1}\Rightarrow L(p)=p^nq^{k-n}$ where $k=\sum x_i$

$$\frac{d}{dp}ln\,L(p)=\frac{n}{p}+\frac{n-k}{1-p}=0\Rightarrow p=\frac{n}{k}=\frac{1}{\bar{x}}$$

M&R-7.34 $\bar{X}:N(\mu_{\bar{X}}=\mu,\sigma_{\bar{X}}=\sigma/\sqrt{n})$

$P(\mu_{\bar{X}}-1.8\sigma_{\bar{X}}<\bar{X}<\mu_{\bar{X}}+1.0\sigma_{\bar{X}})=\phi(1.0)-\phi(-1.8)=0.81$

M&R-7.43 $\bar{X}_1:N(75,2)$ and $\bar{X}_2:N(70,4)$

(a) Let $Y=\bar{X}_1-\bar{X}_2:N(5,\sqrt{20})\Rightarrow P(Y>4)=\phi(.22)=0.59$

(b) $P(3.5<Y<5.5)=\phi(.11)-\phi(-.34)=0.18$

M&R-9.62 H_o : Binomial model is good

i	x_i	o_i	e_i
1	0	4	9
2	1	21	18
3	2	10	15
4	≥ 3	13, 2 $\}=15$	7, 1 $\}=8$

where $e_i=\binom{6}{x_i}(0.25)^{x_i}(0.75)^{6-x_i}\times 50$.

$$\chi^2=\sum_{i=1}^{4}\frac{(o_i-e_i)^2}{e_i}=11.07$$

(a) Since $\chi^2_{\alpha,\nu}=7.81<\chi^2$, reject H_o where $\alpha=.05$ & $\nu=4-1=3$

(b) $\chi^2_{\alpha,\nu}=11.07\Rightarrow\alpha=\text{P-value}\approx 0.01$