HINTS to ASSIGNMENT #5

A&T-4.1

(a) $F : N(c\mu_W, c\sigma_W)$ where $c = \sqrt{h^2 + l^2}/h$.

(b) $W : N(20,5) \Rightarrow F : N(44.7,11.2) \Rightarrow P(F > 30) = 0.906583$

A&T-4.6 $T_{AB} : N(10,1)$ and $T_{BC} : N(15,2)$

(a) Since T_{AB} and T_{BC} are indep., $T = T_{AB} + T_{BC} : N(25,\sqrt{5})$

(b) $P(T + 2 > 30) = 1 - \phi(1.34)$, $P(T + 2 < 20) = \phi(-3.13)$

A&T-4.8

(a) (i) $\lambda_A = 32/50$ (ii) $\lambda_H = 28/4(50)$ (iii) $\lambda_H = 4/8(50)$

(b) $\lambda = 7\lambda_H + 12\lambda_{NH} = 1.1 \text{ hurricanes}/19 \text{ months}$

(c) $P(N \geq 1) = 1 - e^{-\lambda}$ where N is Poisson r.v. with λ as in (b)

(d) $E\{T\} = 10{,}000 E\{N\} = 10{,}000\lambda = 11{,}000$

(e) $P(T > 10{,}000) = P(N > 1) = 1 - e^{-\lambda}(1 + \lambda) = 0.3$

A&T-4.11 $R : N(30,3)$ and $P : N(15,4)$ are independent, so $S = R + P : N(45,5)$

(a) $P(\text{insuf.}) = .3P(S < 20) + .6P(S < 30) + .1P(S < 40) = .017$

(b) Let $D : N(28,6)$ be the demand. Then $S - D : N(17,7.8)$
$P(\text{insuf.}) = P(S - D < 0) = \phi(-2.18) = 0.015$

A&T-4.12 Given the weights of empty truck ($ET : N(15,5)$), loaded truck ($LT : N(30,7)$) and passenger car ($PC : N(2,1)$).

(a) $P(\text{-exc.}) = .1P(ET < 5) + .1P(LT < 5) + .8P(PC < 5) = .801$

(b) $P(3 \text{ vehicles exc.}) = p^3$ where $1 - p = .801$ from (a).

(c) Let $p_T = 0.5P(ET > 5) + 0.5P(LT > 5) = 0.9885$ and
$p_C = P(PC > 5) = 0.00135 \Rightarrow P(3 \text{ vehicles exc.}) = p_C{}^2 p_T$

(d) Let $T = PC_1 + PC_2 + PC_3 + ET$, then $T : N(21,5.3)$
$P(T > 30) = 1 - \phi(1.70) = 0.0446$

(e) Let X be Poisson r.v. for # of vehicles with $\lambda = 3$.
$P(X = 1) = 3e^{-3} \Rightarrow P(\text{only one truck}) = (0.2)3e^{-3} = 0.03$

A&T-4.13 Let $P_1 : N(50,5)$ and $P_2 : N(20,3)$

(a) $M_A = 30P_1 - 20P_2 \Rightarrow M_A : N(1100,161.6)$

(b) Let $M_R : N(1750,150)$ then $M_R - M_A : N(650,220.5)$
$P() = P(M_R < M_A) = P(M_R - M_A < 0) = \phi(-2.95) = p$

(c) $\text{P(survive)} = \binom{3}{1}p^2q^3 + \binom{2}{1}pq^4 + \binom{1}{1}p^0q^5$ where 3, 4 and 5 supporting poles are counted as one in 5 poles, respectively.

(d) $P(3 \text{ fail}) = 10p^3q^2$

A&T-4.14 $P(A) = .153$, $P(R) = .106$, $P(B) = .094$, $P(C) = .647$
$T_A : N(.9,.135)$, $T_R : N(4.5,.45)$, $T_B : N(4.8,.72)$, $T_C : N(4.5,.9)$

(a) $P(\text{in 4 hr.}) = .153P(T_A < 4) + .106P(T_R < 4) +$
$+ .094P(T_B < 4) + +.647P(T_C < 4)$

(b) Let $D = T_B - T_C$, then $D : N(.3,1.15)$
$P(\text{bus faster}) = P(T_B < T_C) = P(D < 0) = \phi(-0.26)$

A&T-4.18 $P : N(5,1)$ and $W : N(1,0.2)$ with $\rho_{PW} = 0.5$

(a) Given $M_a = 50W + 10P$, then $E\{M_a\} = 50\mu_W + 10\mu_P = 100$ and $V\{M_a\} = 50^2\sigma_W{}^2 + 10^2\sigma_P{}^2 + 2(50)(10)\rho_{WP}\sigma_W\sigma_P = 300$

(b) Given $M_r : N(200,50)$ then $M_r - M_a : N(100,52.9)$
$P(M_r < M_a) = \phi(-1.89) = 0.0294$

M&A-p152/70

(a) $E\{X\} = \int_0^{\sqrt{8}} x f_X(x)dx = 1.89$ and $E\{Y\} = E\{X\} + 3 = 4.89$

(b) $f_Y(y) = f_X(\psi(y))|\psi'(y)| = (y - 3)/4$ for $3 \leq y \leq 3 + \sqrt{8}$
where $\psi(y) = y - 3$

(c) $cov(X,Y) = 0 \Rightarrow \rho_{XY} = 0$

(d) $E\{Y\} = \int_3^{3+\sqrt{8}} y f_Y(y)dy = 4.89$

M&R-5.95 $W_i : N(160,30)$

(a) Let $T = W_1 + \cdots + W_{25}$, then $T : N(4000,150)$
$P(T > 4500) = 1 - \phi(3.33)$

(b) Let $T = W_1 + \cdots + W_n$, then $T : N(160n,30\sqrt{n})$
$P(T > 4500) = 0.0001 \Rightarrow (4500 - 160n)/30\sqrt{n} = 3.72 \Rightarrow n \approx 25$

M&R-S.5.2 $f_X(x) = \binom{n}{x}p^x q^{n-x}$ for $x = 0,1,2,\ldots,n$ and $Y = X^2$, then $f_Y(y) = \binom{n}{\sqrt{y}}p^{\sqrt{y}}q^{n-\sqrt{y}}$ for $y = 0,1,4,\ldots,n^2$

M&R-S.5.8 $f_X(x) = \begin{cases} 1 & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$ and $Y = e^X$, then

$$f_Y(y) = \begin{cases} 1/y & e \leq y \leq e^2 \\ 0 & \text{otherwise} \end{cases}$$

M&R-S.5.21 $M_X(t) = (1 - \frac{t}{\lambda})^{-1}$

(a) $M_Y(t) = (1 - \frac{t}{\lambda})^{-r}$

(b) Y is gamma distribution from S.5.20

M&R-S.5.22 $M_X(t) = exp(\mu t + \frac{1}{2}\sigma^2 t^2)$

(a) $M_Y(t) = exp((\mu_1 + \mu_2)t + \frac{1}{2}(\sigma_1^2 + \sigma_2^2)t^2)$

(b) $Y : N(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2})$