HINTS to ASSIGNMENT #4

A&T-3.61

(a) Divide each number by 665 to get joint probabilities.

(b)

x	0	1	2	3	4
$f_X(x)$	0.18	0.18	0.18	0.19	0.27

(c)

y	0	1	2	3	4
$f_{Y\|X=3}(y)$	0.35	0.25	0.20	0.12	0.08

(d) $f_{XY}(4,4) = 5/665$

(e) $cov(X,Y) = E\{XY\} - E\{X\}E\{Y\}$

$= \sum_x \sum_y xyf_{XY}(x,y) - \sum_x xf_X(x) \sum_y yf_Y(y) = -1.315$

A&T-3.62

(a) $P(\{X<1\}\cap\{Y<2\}) = \int_0^1\int_0^2 2ye^{-y(2+x)}dydx = 0.32$

(b) $f_X(x) = \int_0^\infty 2ye^{-y(2+x)}dy = \dfrac{2}{(2+x)^2}$

(c) $f_Y(y) = \int_0^\infty 2ye^{-y(2+x)}dx = 2e^{-2y}$

(d) $f_X(x)f_Y(y) \neq f_{XY}(x,y)$, so not independent

(e) $P(Y>2 \mid X=2) = \int_2^\infty f_{Y|X=2}(y)dy = \int_2^\infty \dfrac{f_{XY}(2,y)}{f_X(2)}dy = 9e^{-8}$

M&A-p181/2 Clearly $X+Y=n=4, N=7, r=3$ and they are hypergeometric r.v.'s. The range of X is $\{0,1,2,3\}$ and thus the range of (X,Y) is $\{(0,4),(1,3),(2,2),(3,1)\}$.

(a) $f_{XY}(x,y) = \binom{3}{x}\binom{4}{y}\Big/\binom{7}{4}$

(b) $f_X(x) = f_Y(y) = f_{XY}(x,y)$ for x,y in their respective ranges.

(c) They are dependent since $X+Y=4$. Note also that $f_{XY}(0,1)=0$, however $f_X(0)f_Y(1) = \frac{1}{35}\frac{4}{35} \neq 0$

M&A-p182/8

(a) $1 = \int_0^{40}\int_0^2 c(4x+2y+1)dydx = 6640c \Rightarrow c = 1/6640$

(b) $P(\{X>20\}\cap\{Y>1\}) = \int_{20}^{40}\int_1^2 f_{XY}(x,y)dydx = 31/83$

(c) $f_X(x) = \int_0^2 f_{XY}(x,y)dy = (4x+3)/3320$

$f_Y(y) = \int_0^{40} f_{XY}(x,y)dx = (2y+81)/166$

(d) $P(Y>1) = \int_1^2 f_Y(y)dy = 42/83$

(e) $P(X>20) = \int_{20}^{40} f_X(x)dx = 123/166$

(f) Not independent since $f_X(x)f_Y(y) \neq f_{XY}(x,y)$.

M&A-p184/15

(a) Note that as X increases, Y decreases, so $cov(X,Y)<0$.

(b) $cov(X,Y) = E\{XY\} - E\{X\}E\{Y\} = -600/35^2$ where $E\{X\} = nr/N = 12/7$, $E\{Y\} = n(N-r)/N = 16/7$ and $E\{XY\} = \sum_x\sum_y xyf_{XY}(x,y) = 120/35$

M&A-p185/20

(a) Physically, as X increases, Y decreases, so $cov(X,Y)<0$

(b) $cov(X,Y) = E\{XY\} - E\{X\}E\{Y\} = -3200/249^2$ where

$E\{X\} = \int_0^{40} xf_X(x)dx = 6580/249$, $E\{Y\} = \int_0^2 yf_Y(y)dy = 251/249$

$E\{XY\} = \int_0^{40}\int_0^2 xyf_{XY}(x,y)dydx = 6620/249$

M&A-p189/57 X&Y are Poisson r.v.'s with $\lambda_X = 5$ and $\lambda_Y = 3$

(a) $f_{XY}(x,y) = f_X(x)f_Y(y) = e^{-(\lambda_X+\lambda_Y)}(\lambda_X)^x(\lambda_Y)^y/x!\,y!$

(b) $P(X+Y=4) = P(T=4) = e^{-\lambda_T}(\lambda_T)^4/4!$ where $T = X+Y$ is a Poisson r.v. with $\lambda_T = \lambda_X + \lambda_Y$ since X and Y are indep.

(c) $cov(X,Y) = 0 \Rightarrow \rho_{XY} = 0$

(d) $f_{X|Y=y}(x) = f_X(x) = e^{-\lambda_X}(\lambda_X)^x/x!$

M&R-5.9 $\sum_x\sum_y f_{XY}(x,y) = 1$

M&R-5.10

(a) $P(X<0.5, Y<1.5) = f_{XY}(-1,-2) + f_{XY}(-0.5,-1) = 3/8$

(b) $P(X<0.5) = 3/8$ as in (a)

(c) $P(Y<1.5) = f_{XY}(-1,-2) + f_{XY}(-0.5,-1) + f_{XY}(0.5,1) = 7/8$

(d) $P(X>0.25, Y<4.5) = f_{XY}(0.5,1) + f_{XY}(1,2) = 5/8$

M&R-5.11 $E\{X\} = 1/8$, $E\{Y\} = 1/4$

M&R-5.46 $1 = \int_0^\infty\int_0^y ce^{-2x-3y}dxdy = c/15 \Rightarrow c = 15$

M&R-5.47

(a) $P(X<1, Y<2) = \int_0^1\int_x^2 15e^{-2x-3y}dydx = 0.9879$

(b) $P(1<X<2) = \int_1^2\int_x^\infty 15e^{-2x-3y}dydx = 0.0067$

(c) $P(Y>2) = \int_2^\infty\int_0^y 15e^{-2x-3y}dxdy = 0.0061$

(d) $P(X<2, Y<2) = \int_0^2\int_x^2 15e^{-2x-3y}dydx = 0.9939$

(e) $E\{X\} = \int_0^\infty\int_x^\infty xf_{XY}(x,y)dydx = 0.2$

(f) $E\{Y\} = \int_0^\infty\int_x^\infty yf_{XY}(x,y)dydx = 0.5333$

M&R-5.48

(a) $f_X(x) = \int_x^\infty f_{XY}(x,y)dy = 5e^{-5x}$

(b) $f_{Y|X=1}(y) = f_{XY}(1,y)/f_X(1) = 3e^{3-3y}$

(c) $E\{Y \mid X=1\} = \int_1^\infty yf_{Y|X=1}(y)dy = 1.333$

(d) $P(Y<2 \mid X=1) = \int_1^2 f_{Y|X=1}(y)dy = 1 - e^{-3}$

(e) $f_{X|Y=2}(x) = f_{XY}(x,2)/f_Y(2) = 2e^{-2x}/(1-e^{-4})$ where $f_Y(y) = \int_0^y f_{XY}(x,y)dy = \frac{15}{2}(e^{-3y} - e^{-5y})$.

M&R-5.68 $cov(X,Y) = \sigma_{XY} = E\{XY\} - E\{X\}E\{Y\} = 27/32$ and $\rho_{XY} = \dfrac{\sigma_{XY}}{\sigma_X\sigma_Y} = 1$ where $E\{XY\} = 7/8$, $E\{X\} = 1/8$, $E\{Y\} = 1/4$, $V\{X\} = \sigma_X{}^2 = 0.421875$, $V\{Y\} = \sigma_Y{}^2 = 1.6875$.