HINTS to ASSIGNMENT #3

A&T-3.9 Y and X denote the water level and increase in water level at B, respectively. $\{\text{O}:\text{Overflow at B}\} = \{X+Y>40\}$

(a) Area $=1 \Rightarrow a = 2/25$

(b) $P(\text{O}) = 0.7P(X>15) + 0.3P(X>5) = 0.227$

(c) $p = 0.7P(X<15)/(1-P(\text{O})) = 0.845$

A&T-3.19 $X : \text{N}(30,\sigma)$ and $P(X<40) = 0.9$

(a) $P(X<40) = 0.9 \Rightarrow \sigma = 7.78$ and thus $P(X<50) = 0.994915$

(b) $P(X<0) = \phi(-3.86) = 0.00006$

(c) $X : \text{L-N}(\lambda = 30, \zeta = 7.78) \Rightarrow \ln X : \text{N}(\mu = 3.37, \sigma = 0.255)$

$P(X<50) = P(\ln X < \ln 50) = \phi(2.13) = 0.983414$

A&T-3.20 Let T be the time btw breakdowns and $T : \text{L-N}(6,1.5)$ thus $\ln T : \text{N}(1.76, 0.25)$

(a) T_M : time until maintenance. $P(T \ge T_M) = .9 \Rightarrow T_M = 4.22$

(b) $P(T > T_M + 1 | T > T_M) = P(T > T_M + 1)/P(T > T_M) = 0.74$

A&T-3.22 $H : L-N(30, 0.2(30)) \Rightarrow \ln H : N(3.38, 0.198)$

(a) $P(H>39) = 0.078$

(b) $P(H<44 | H>39) = P(39<H<44)/P(H>39) = 0.72$

A&T-3.46 Let X be # of hurricanes : Poisson($\lambda = .8$) and let $W : L-N(100,20) \Rightarrow \ln H : N(4.59, 0.198)$ be wind speed.

(a) $\text{P}(\text{X} \ge 1) = 1 - e^{-2\lambda} = 0.798$

(b) $\text{P}(\text{W} > 130) = 1 - \phi(1.40) = 1 - 0.919243 = 0.081 = 1-p$

(c) $\text{P}(\text{no damage}) = P(X=0) + P(X=1)p + P(X=2)p^2 = 0.72$

A&T-3.50 Given consumption $C : N(500,150)$ and supply S with $P(S=600) = 0.7$ and $P(S=750) = 0.3$

(a) $P(\text{shortage}) = 0.7P(C>600) + 0.3P(C>750) = 0.19 = p$

(b) $P(\text{shortage in a week}) = 1 - p^0q^7 = 0.77$

(c) How often $= 1/p = 5$ days $\Rightarrow \lambda = 1/5$

$\Rightarrow P(\text{shortage in a week}) = 1 - e^{-7\lambda} = 0.75$

(d) $P(C>S) = 0.01 \Rightarrow \phi(S - 500/150) = 0.99 \Rightarrow S \approx 850$

A&T-3.53

(a) $E\{X\} = 1/c = 2 \Rightarrow c = 0.5$

(b) $\text{P}(\text{X} > 6) = \int_6^\infty ce^{-cx}dx = e^{-6c} = p$

(c) Return Period $= 1/p = 20$ days

(d) $P(\text{at most once}) = p^0q^3 + 3pq^2 = 0.993$

(e) For $X : N(2,2)$: $P(X>6) = 0.023$

A&T-3.56 Let T be time until breakdown : Exp. with $E\{T\} = 24$

(a) $\text{P}(\text{T} < 5) = 1 - exp(-5/24) = p$

(b) $P(T>10 | T>5) = P(T>5) = 1-p$

(c) $P(\text{at most one}) = p^0q^5 + 5pq^4 = 0.76$

(d) $P(\text{repair}) = 1 - p^0q^5 = .1 \Rightarrow q = exp(-T/24) = .979 \Rightarrow T = 0.5$

A&T-3.57 Let T be interarrival time: Exp. with $E\{T\} = 0.5$

(a) T_O :time of operation $\Rightarrow \text{P}(\text{T} > T_O) = .8 = e^{-2T_O} \Rightarrow T_O = .11$

(b) Let $p = P(T > T_O) = 0.8 \Rightarrow P(\text{none wait}) = p^4 = 0.41$

(c) $n = 3$ boats/24 hours

$\Rightarrow P(\text{at least 1 wait}) = 1 - P(\text{none wait}) = 1 - p^3 = .488$

M&A- p146/37 T : time btw quakes : Exp. with $E\{T\} = 12$

(a) $\text{P}(\text{T} > 3) = exp(-3/12)$

(b) $P(T>7 | T>4) = P(T>3) = exp(-3/12)$

M&A- p154/86 T : time btw calls : Exp. with $E\{T\} = 480/50$

$P(T < 9:15 a.m.) = 1 - exp(-15(50/480)) = 0.79$

$P(T > 3 p.m.) = exp(-360(50/480)) \approx 0$

$P(9:30 a.m. < T < 10 a.m.) = \int_{30}^{60} f_T(t)dt = 0.042$

M&A- p155/90 X_B : # of failures : Binomial($n = 100, p = .08$)

$E\{X_B\} = np = 8$. Since $np > 5$

$P(X_B \ge 10) \approx P(X_N \ge 10) = 0.23$

$P(X_B \le 5) \approx P(X_N \le 5) = 0.1335$ where $X_N : N(np, \sqrt{npq})$

M&R-4.44 Let $X : N(10,2)$

(a) $P(X > x) = 0.5 \Rightarrow x = 10$

(b) $P(X > x) = 0.95 \Rightarrow x = 6.71$

(c) $P(x < X < 10) = 0.2 \Rightarrow x = 8.95$

(d) $P(-x < X - 10 < x) = 0.95 \Rightarrow x = 3.92$

(e) $P(-x < X - 10 < x) = 0.99 \Rightarrow x = 5.16$

M&R-4.66 Let X_P be Poisson r.v. with $\lambda = 1000$. Since $\lambda > 5$

$P(X_P > 10000) \approx P(X_N > 10000) = .5$ with $X_N : N(10\lambda, \sqrt{10\lambda})$

M&R-4.83 Let X_E be distance btw cracks: Exp. with $E\{X_E\} = 5$ and X_P be the corresponding Poisson r.v.

(a) $P(X_E > 10) = e^{-2}$ **(b)** $P(X_P = 2) = 2e^{-2}$ **(c)** $\sqrt{V\{X_E\}} = 5$

M&R-4.84

(a) $P(12 < X_E < 15) = \int_{12}^{15} f_{X_E}(x)dx = 0.041$

(b) $P(\text{no cracks}) = [P(X_E > 5)]^2 = e^{-2}$

(c) $P(X_E > 15 | X_E > 5) = P(X_E > 10) = e^{-2}$

M&R-4.146 Let X_B be the binomial r.v. with $n = 200$ and $p = .9$

(a) $P(X_B \le 185) \approx P(X_N \le 185) = \phi(1.18) = 0.881$

(b) $P(X_B \le 184) \approx P(X_N \le 184) = \phi(0.94) = 0.826391$

(c) Find n so that $P(X_B \le 185) = 0.95$ with $p = .9$

$\Rightarrow \phi((185 - np)/\sqrt{npq}) = 0.95$

$\Rightarrow (185 - np)/\sqrt{npq} = 1.65$

$\Rightarrow n \approx 198$