HINTS to ASSIGNMENT #2

A&T-3.6 Let D be the event of detection.

(a) $P(D' \mid n=2) = (0.2)^2$

(b) $P(D') = \sum_{n=0}^{2} P(D' \mid n) P_N(n) = 0.424$

(c) $E\{N\} = \sum_{n=0}^{2} n P_N(n) = 0.8, \ldots$

(d) $P(\text{Flawless} \mid D') = P(D' \mid n=0) P_N(n=0) / P(D') = 0.71$

A&T-3.14 Let $Y = g(X)$

(a) $E\{Y\} = \sum_{x_i} g(x_i) p_X(x_i)$

(b) $\sqrt{V\{Y\}} = \sqrt{\sum_{x_i} [g(x_i) - E\{Y\}]^2 p_X(x_i)}$

A&T-3.26 Let X be the binomial r.v. for given n and p.

(a) $p = 3/20$ **(b)** $P(X=3)$ for given $n=10$

(c) $P(X \geq 1) = 1 - P(X=0)$ for given $n=3$

(d) Use $p = 1/20$ in (c).

A&T-3.27 Given $p = 1/10$

(a) Let X_G be the geometric r.v. $\Rightarrow \mathrm{P}(\mathrm{X_G} = 3)$

(b) Let X_B be the binomial r.v. with $n=3$ $\Rightarrow \mathrm{P}(\mathrm{X_B} \geq 1)$

(c) Let X_B be the binomial r.v. with $n=5$ $\Rightarrow \mathrm{P}(\mathrm{X_B} = 3)$

(d) Let X_B be the binomial r.v. as in (b) $\Rightarrow \mathrm{P}(\mathrm{X_B} = 1)$

A&T-3.29 Given $p = 0.02$

(a) Let X_B be the binomial r.v. with $n=4$ $\Rightarrow \mathrm{P}(\mathrm{X_B} = 0)$

(b) Let X_B be the binomial r.v. as in (a) $\Rightarrow \mathrm{P}(\mathrm{X_B} \leq 1)$

(c) Let X_G be the geometric r.v. $\Rightarrow \mathrm{P}(\mathrm{X_G} = 3)$

(d) Let Y_B be the binomial r.v. with $n=4$ and $p = \mathrm{P}(\mathrm{X_B} = 0)$ from (a) $\Rightarrow \mathrm{P}(\mathrm{Y_B} = 4)$

A&T-3.34 Let I, II, III represent the events that dikes I, II, III will be exceeded, respectively. Given $P(I) = 1/20$, $P(II) = 1/10$, $P(III) = 1/25$.

(a) $P(I \cup II) = 0.145$

(b) $P(I \cup II \cup III) = 1 - P(I' \cap II' \cap III') = 0.179$

(c) Let Y_B be the binomial r.v. with $n=4$ and $p = P(I \cup II \cup III)$ from (b) $\Rightarrow \mathrm{P}(\mathrm{Y_B} = 0)$

A&T-3.38 Given 2 eartquakes / 5 years.

(a) Let X_P be the poisson r.v. with $\lambda = 3(2/5)$ $\Rightarrow \mathrm{P}(\mathrm{X_P} = 1)$

(b) Let X_P be the poisson r.v. as in (a) $\Rightarrow \mathrm{P}(\mathrm{X_P} = 0)$

(c) Let X_P be the poisson r.v. with $\lambda = 2/5$ $\Rightarrow \mathrm{P}(\mathrm{X_P} \leq 2)$

(d) Let X_P be the poisson r.v. with $\lambda = 2$ $\Rightarrow \mathrm{P}(\mathrm{X_P} \geq 1)$

A&T-3.42 Let N denote the event of no gasoline. Given $P(N) = 0.2$ and 1 service station / 10 miles.

(a) Let X_P be the poisson r.v. with $\lambda = 15(1/10)$ $\Rightarrow \mathrm{P}(\mathrm{X_P} \leq 1)$

(b) $p = P(N) * P(N) * P(N) = 0.2^3$

(c) Let X_P be the poisson r.v. as in (a). Then

$$P(\text{stranded}) = \sum_{n=0}^{\infty} (0.2)^n P(X_P = n) \quad = e^{-0.8\lambda}$$

A&T-3.44 Let D and T denote the events of storm damage to the dish and the truss, respectively. Given $P(D) = 0.2$ and $P(T) = 0.05$ with 10 storms / 50 years.

(a) Let X_P be the poisson r.v. with $\lambda = 10(10/50)$ $\Rightarrow \mathrm{P}(\mathrm{X_P} > 2) = 1 - \mathrm{P}(\mathrm{X_P} \leq 2)$

(b) $p_D = P(\textbf{no damage to dish}) = \sum_{n=0}^{2} [P(D')]^n P(X_P = n) = .525$

$p_T = P(\textbf{no damage to truss}) = \sum_{n=0}^{2} [P(T')]^n P(X_P = n) = .637$

$P(\text{damage to system}) = 1 - p_D p_T$

(c) Let $p_D = \sum_{n=0}^{\infty} [P(D')]^n P(X_P = n) = e^{-0.2\lambda}$ and $p_T = \sum_{n=0}^{\infty} [P(T')]^n P(X_P = n) = e^{-0.05\lambda}$ in (b).

A&T-3.59 Let X be the hypergeometric r.v. with $N = 200$, $r = 20$ and $n = 10$.

(a) $P(X=0) = \binom{20}{0}\binom{180}{10} / \binom{200}{10} \approx P(X_B = 0) = 0.9^{10}$ where X_B is the binomial r.v. with $n = 10$ and $p = 10\%$ approximating X since $n/N = 0.05 \leq 0.1$.

(b) $P(X \geq 1) = 1 - P(X=0)$ since $0 \leq X \leq 10$.

M&A- p91/41 Given $p = 0.8$

(a) Let X be the binomial r.v. with $n=10$ $\Rightarrow \mathrm{P}(\mathrm{X} \geq 9) = 0.38$

(b) Let X_B be the binomial r.v. with $n=5$ and $p = P(X \geq 9)$ as in (a) $\Rightarrow \mathrm{P}(\mathrm{X_B} = 5)$

M&A- p97/78 Given $p = 1/4$. Let X_{NB} be the pascal r.v. with $r = 2 \Rightarrow \mathrm{P}(\mathrm{X_{NB}} < 7) = 0.4661$, or alternatively let Let X_B be the binomial r.v. with $n = 6 \Rightarrow \mathrm{P}(\mathrm{X_B} \geq 2) = 0.4661$.

M&R-3.52 $f_Y(y) = 1/10$ for $y = 0, 5, 10, 15, \ldots, 45$

$E\{Y\} = \sum y f_Y(y) = 22.5$, $V\{Y\} = \sum (y - E\{Y\})^2 f_Y(y) = 206.25$

Similarly, $f_X(x) = 1/10$ for $x = 0, 1, 2, 3, \ldots, 9$, $E\{X\} = 4.5$, $V\{X\} = 8.25$. Note that $E\{Y\} = 5E\{X\}$ and $V\{Y\} = 5^2 V\{X\}$

M&R-3.67 Let X be a binomial r.v. with $n = 125$ and $p = 0.90$

(a) $P(X \leq 120)$ **(b)** $P(X \leq 119)$

M&R-3.79 Given $p = 0.2$.

(a) Let X_G be the geometric r.v. $\Rightarrow \mathrm{E}\{\mathrm{X_G}\} = 1/p = 5$

(b) Let X_{NB} be the pascal r.v. with r $\Rightarrow \mathrm{E}\{\mathrm{X_G}\} = r(1/p)$. Then $\Rightarrow \mathrm{E}\{\mathrm{X_G}$ with $r = 9\} - \mathrm{E}\{\mathrm{X_G}$ with $r = 8\} = 1/p = 5$ which also follows from lack of memory property of the geometric dist.

M&R-3.81 Let X_G be the geometric r.v. with $p = 0.6$ $\Rightarrow \mathrm{P}(\mathrm{X_G} \leq 3) = 0.936$

M&R-3.94 Let X be the hypergeometric r.v. with $N = 40$, $r = 6$ and $n = 6$.

(a) $P(X=6) = \binom{6}{6}\binom{34}{0} / \binom{40}{6} = \dfrac{6!}{40\,39\,38\,37\,36\,35} = 2.6 \times 10^{-7}$

(b) $P(X=5) = 5.3 \times 10^{-5}$ **(c)** $P(X=4) = 2.2 \times 10^{-3}$

(c) Let X_G be the geometric r.v. with $p = P(X=6)$ as in (a) $\Rightarrow \mathrm{E}\{\mathrm{X_G}\} = 1/p = 3846154$ weeks.

M&R-3.100 Given 10 calls / hour.

(a) Let X_P be the poisson r.v. with $\lambda = 10$ $\Rightarrow \mathrm{P}(\mathrm{X_P} = 5)$

(b) Let X_P be the poisson r.v. as in (a) $\Rightarrow \mathrm{P}(\mathrm{X_P} \leq 3)$

(c) Let X_P be the poisson r.v. with $\lambda = 20$ $\Rightarrow \mathrm{P}(\mathrm{X_P} = 15)$

(d) Let X_P be the poisson r.v. with $\lambda = 5$ $\Rightarrow \mathrm{P}(\mathrm{X_P} = 5)$