HINTS to ASSIGNMENT #1

A&T-2.13 Since the flow velocities are the same $h_c = 2/3(h_a + h_b)$. This implies that $\{h_c > 6\} \equiv \{h_a + h_b > 9\}$ which gives $P(\{h_c > 6\}) = 0.3$.

A&T-2.15

(a) $P(\{\text{able to go from A to B through C}\}) = P(E_2 \cap E_3) = 3/5$.

(b) $P(\{\text{able to go to } B\}) = P((E_2 \cap E_3) \cup E_1) =$
$= P(E_1) + P(E_2 \cap E_3) - P(E_1 \cap E_2 \cap E_3) = 7/10$.

A&T-2.21 Let A and B be the events that cables A and B breaks, respectively. Given $P(A) = 0.02$ and $P(B|A) = 0.3$.

(a) $P(\{\text{both cables fail}\}) = P(A \cap B) = 0.006$.

(b) $E = \{\text{the load remains lifted}\} = (A' \cap B) \cup (A \cap B') \cup (A' \cap B')$

This implies $P(E) = 1 - P(A \cap B)$. Thus

$$P(\text{none of the cables failed} \mid E) = P(A' \cap B' \mid E) = \frac{P(A')}{P(E)} = 0.99$$

Since $A' = (A' \cap B') \cup (A' \cap B)$ and $P(A' \cap B) = 0$ (impossible event).

A&T-2.27 Let A, B, C be the events of getting a parking space in lots A, B, C, respectively. Given $P(A) = 0.2$, $P(B) = 0.1$, $P(C) = 0.5$, $P(B \mid A') = 0.04$, $P(C \mid A' \cap B') = 0.4$.

(a) $\text{P(not able to park)} = P(A' \cap B' \cap C') =$
$= P(C' \mid A' \cap B')P(B' \mid A')P(A') = 0.4608$.

(b) $P(\text{able to park}) = P(A \cup B \cup C) = 1 - P(A' \cap B' \cap C')$.

(c) $P(\text{free of charge} \mid \text{able to park}) = P(A \cup B \mid A \cup B \cup C) =$

$$= \frac{1 - P(A' \cap B')}{P(A \cup B \cup C)} = \frac{1 - P(B' \mid A')P(A')}{P(A \cup B \cup C)} = 0.43.$$

A&T-2.28 Given $P(A_I) = 4P(A_{II})$, $P(A_I|A_{II}) = 0.9$, $P(A_I \cap W_I) = 0.32$, $P(W_I) = 1/2\, P(A_I)$ and further $P(A_I \cap W_I) = P(A_I)P(W_I)$, $P(A_{II} \cap W_I) = P(A_{II})P(W_I)$, $P(W_I \mid A_I \cap A_{II}) = P(W_I)$. This implies that $P(A_I) = \sqrt{2(0.32)}$, $P(A_{II}) = 0.2$ and $P(W_I) = 0.4$.

(a) $P(A_I \cap A_{II}) = 0.18$

(b) $P(A_I \cap W_I \cap A_{II}) = 0.072$

(c) $P((A_I \cap W_I) \cup A_{II}) =$
$= P(A_I \cap W_I) + P(A_{II}) - P(A_I \cap W_I \cap A_{II}) = 0.448$.

A&T-2.31 Let L, S, A, R and H denote the events of transporting by land, sea, air, rail and highway, respectively. And let D denote damaged cargo. Given $P(L) = 0.5$, $P(S) = 0.3$, $P(A) = 0.2$ with $P(H) = 0.5(0.4) = 0.2$ and $P(R) = 0.5(0.6) = 0.3$. Further $P(D \mid H) = 0.1$, $P(D \mid R) = 0.05$, $P(D \mid S) = 0.06$, $P(D \mid A) = 0.02$

(a) $P(D) = P(D \mid H)P(H) + \cdots + P(D \mid A)P(A)$

(b) $P(L \mid D) = P(H \cup R \mid D) = P(H \mid D) + P(R \mid D) =$
$= \frac{P(D \mid H)P(H) + P(D \mid R)P(R)}{P(D)}$. Similarly for the others $P(S \mid D)$ and $P(A \mid D)$.

A&T-2.38 Let S be the event that the city will have satisfactory water supply. Given $P(B) = 0.8$, $P(A \cap B) = 0.6$, $P(A' \mid B') = 0.7$. Further $P(S \mid (A \cap B') \cup (A' \cap B)) = 0.7$, $P(S \mid A \cap B) = 0.9$ and $P(S \mid A' \cap B') = 0.9$. Total probability theorem gives
$P(S) = P(S \mid (A \cap B') \cup (A' \cap B))P((A \cap B') \cup (A' \cap B)) +$
$+ P(S \mid A \cap B)P(A \cap B) + P(S \mid A' \cap B')P(A' \cap B') = 0.764$.

A&T-2.41 Let R and $F\ (\equiv R')$ be the reliability and failure of the proposed design, respectively. Given $P(H) = 0.1$, $P(M) = 0.1$, $P(L) = 0.8$ and further $P(R \mid L) = 0.999$, $P(F \mid M) = 2(1 - 0.999)$, $P(F \mid H) = 10(1 - 0.999)$

(a) $P(R) = P(R \mid L)P(L) + \cdots + P(R \mid H)P(H) = 0.998$

(b) Now $P(M) = 1/9$, $P(L) = 8/9 \Rightarrow P(R) = 0.9989$

(c) $P(M \mid F) = \frac{P(F \mid M)P(M)}{P(F)} = 0.2$ and $P(L \mid F) = 0.8$.

M&A- p41/30 $P(A' \cap B') = 1 - P(A \cup B) =$
$= 1 - [P(A) + P(B) - P(A \cap B)] = \cdots = P(A')P(B')$.

M&A- p41/34 $P(B \mid TA) = \frac{P(TA \mid B)P(B)}{P(TA)}$

M&A- p43/42 Let E and O denote shutdowns due to equipment failure and operator error, respectively. Given $P(E \cap O') = 0.1$, $P(O) = 0.4$, $P(E \cap O) = 0.05 \Rightarrow P(E) = 0.15$.

(a) $P(E \cup O) = 0.5$

(b) $P(E' \cap O) = P(O) - P(E \cap O) = 0.35$

(c) $P(O' \cap E') = 1 - P(O \cup E) = 0.5$

(d) $P(O \mid E) = 0.33$

(e) $P(O \mid E') = 0.412$

M&R-2.117 Let F, S and T denote the events that the first, the second and the third selected washer is thicker, respectively.

(a) $P(FST) = \frac{30}{50}\frac{29}{49}\frac{28}{48}$ **(b)** $P(T \mid F'S') = \frac{30}{48}$ **(c)** $P(T) = \frac{30}{50}$

M&R-2.121 Let A be the event that the device in line 1 fails. Let B and C be the events, respectively, that the first and the second devices on line 2 fail. Similarly D and E for line 3. And F for line 4.

Given $P(B) = P(C) = P(D) = P(E) = .01$ $P(A) = P(F) = .02$
$P(\text{OPERATIONAL}) = 1 - P(\text{NOT OPERATIONAL}) =$
$= 1 - P(A \cap (B \cup C) \cap (D \cup E) \cap F) =$
$= 1 - (.02)^2(1 - (0.99)^2)^2$