updated further as follows:

$$P(G \mid T_2) = \frac{P(T_2 \mid G)P(G)}{P(T_2 \mid G)P(G) + P(T_2 \mid \bar{G})P(\bar{G})}$$
$$= \frac{(0.8)(0.95)}{(0.8)(0.95) + (0.1)(0.05)} = 0.993$$

This updating is performed sequentially. The updating may also be performed in a single step using the two test results together. In this latter case, we have

$$P(G \mid T_1T_2) = \frac{P(T_1T_2 \mid G)P(G)}{P(T_1T_2 \mid G)P(G) + P(T_1T_2 \mid \bar{G})P(\bar{G})}$$
$$= \frac{(0.8)(0.8)(0.7)}{(0.8)(0.8)(0.7) + (0.1)(0.1)(0.3)} = 0.993$$

which is clearly the same as the result obtained sequentially above, as it should be.

2.4. CONCLUDING REMARKS

In this chapter, we learn that a probabilistic problem involves the determination of the probability of an event within an exhaustive set of possibilities (or possibility space). Two things are paramount in the formulation and solution of such problems: (1) the definition of the possibility space and the identification of the event within this space; and (2) the evaluation of the probability of the event. The relevant mathematical bases useful for these purposes are the theory of sets and the theory of probability. In this chapter, the basic elements of both theories are developed in elementary and nonabstract terms, and are illustrated with physical problems.

Defined in the context of sets, events can be combined to obtain other events via the operational rules of sets and subsets; basically, these consist of the *union* and *intersection* of two or more events including their complements. Similarly, the operational rules of the theory of probability provide the bases for the deductive relationships among probabilities of different events within a given possibility space; specifically, these consist of the *addition rule*, the *multiplication rule*, the *theorem of total probability*, and *Bayes' theorem*.

In essence, the concepts developed in this chapter constitute the fundamentals of applied probability. In Chapters 3 and 4, additional analytical tools will be developed based on these fundamental concepts.

PROBLEMS

Sections 2.1 & 2.2

2.1 The possible settlements for the three supports of a bridge shown in Fig. P2.1 are as follows:

support A—0 in., 1 in., 2 in.
support B—0 in., 2 in.
support C—0 in., 1 in., 2 in.

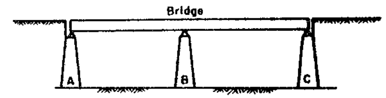


Figure P2.1

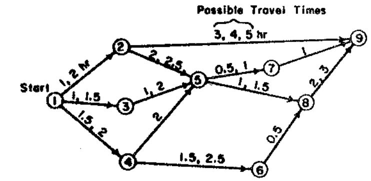


Figure P2.2

(a) Identify the sample space representing all possible settlements of the three supports; for example (1, 0, 2) means A settles 1 in., B settles 0 in., and C settles 2 in.
(b) If E is the event of 2 in. differential settlement between any adjacent supports of the bridge, determine the sample points of E.

2.2 Figure P2.2 shows a network of highways connecting the cities 1, 2, . . . , 9.
(a) Identify the sample space representing all possible routes between cities 1 and 9.
(b) The possible travel times between any two connecting nodes are as indicated in Fig. P2.2 (for example, from 2 to 9, the possible travel times are 3, 4, 5 hr). What are the possible travel times between 1 and 9 through route ① → ② → ⑨? How about through route ① → ④ → ⑥ → ⑧ → ⑨?

2.3 A 6 m × 48 m apartment building may be divided into 1-, 2-, or 3-bedroom units, or combinations thereof (Fig. P2.3). If 1-bedroom units are each 6 m × 6 m, 2-bedroom units are each 6 m × 12 m, and 3-bedroom units are each 6 m × 16 m, how may the apartment building be subdivided into one or more types of units?

2.4 A left-turn pocket of length 60 ft is planned at a street intersection. Assume that only two types of vehicles will be using it; a type-A vehicle will occupy 15 ft of the pocket, whereas a type-B vehicle will occupy 30 ft.
(a) Identify all the possible combinations of types A and B vehicles waiting for left turns from the pocket.
(b) Group these possibilities into events of 1, 2, 3, and 4 vehicles waiting for left turns.

2.5 Strong wind at a particular site may come from any direction between due east ($\theta = 0°$) and due north ($\theta = 90°$). All values of wind speed V are possible.
(a) Sketch the sample space for wind speed and direction.
(b) Let $A = \{V > 20 \text{ mph}\}$
$B = \{12 \text{ mph} < V \leq 30 \text{ mph}\}$
$C = \{\theta \leq 30°\}$

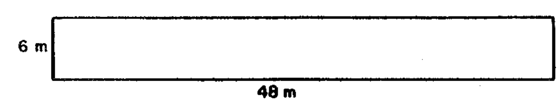


Figure P2.3

Identify the events A, B, C, and $\bar{A}$ in the sample space sketched in part (a).

(c) Use new sketches to identify the following events:

$$D = A \cap C$$
$$E = A \cup B$$
$$F = A \cap B \cap C$$

(d) Are the events D and E mutually exclusive? How about events A and C?

2.6 The possible values of the water height H, relative to mean water level, at each of the two rivers A and B are as follows (in meters):

$$H = -3, -2, -1, 0, 1, 2, 3, 6$$

(a) Consider river A and define the following events:

$$A_1 = \{H_A > 0\}, \qquad A_2 = \{H_A = 0\}, \qquad A_3 = \{H_A \leq 0\}$$

List all pairs of mutually exclusive events among A_1, A_2, and A_3.

(b) At each river, define
Normal water, $N = \{-1 \leq H \leq 1\}$
Drought, $D = \{H < -1\}$
Flood, $F = \{H > 1\}$

Use the ordered pair (h_A, h_B) to identify sample points relating to joint water levels in A and B, respectively; thus $(3, -1)$ defines the condition $h_A = 3$ and $h_B = -1$ simultaneously. Determine the sample points for the events

(i) $N_A \cap N_B$ (ii) $(F_A \cup D_A) \cap N_B$

2.7 The sequence of main activities in the construction of two structures is shown in Fig. P2.7. The construction of the superstructures A and B can start as soon as their common foundation has been completed.

The possible times of completion for each phase of construction are indicated in Fig. P2.7; for example, the foundation phase may take 5 or 7 months.

(a) List the possible combinations of times for each phase of the project; for example, (5, 3, 6) denotes the event that it takes 5 months for foundation, 3 months for superstructure A, and 6 months for superstructure B.

(b) What are the possible *total* completion times for structure A alone? For structure B alone?

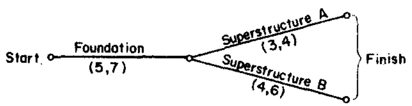


Figure P2.7

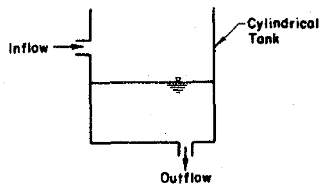


Figure P2.8

(c) What are the possible *total* completion times for the project?

(d) If the possibilities in part (a) are equally likely, what is the probability that the complete project will be finished within 10 months?

2.8 A cylindrical tank is used to store water for a town (Fig. P2.8). The available supply is not completely predictable. In any one day, the inflow is equally likely to fill 6, 7, or 8 ft of the tank. The demand for water is also variable, and may (with equal likelihood) require an amount equivalent to 5, 6, or 7 ft of water in the tank.

(a) What are the possible combinations of inflow and outflow in a day?

(b) Assuming that the water level in the tank is 7 ft at the start of a day, what are the possible water levels in the tank at the end of the day? What is the probability that there will be at least 9 ft of water remaining in the tank at the end of the day?

Section 2.3

2.9 A power plant has two generating units, numbered 1 and 2. Because of maintenance and occasional machine malfunctions, the probabilities that, in a given week, units No. 1 and 2 will be out of service (these two events are denoted by E_1 and E_2) are 0.01 and 0.02, respectively.

During a summer week there is a probability of 0.10 that the weather will be extremely hot (say average temperature > 85°F; this event is denoted by H) so that demand for power for air-conditioning will increase considerably. The performance of the power plant in terms of its potential ability to meet the demand in a given week can be classified as

(i) satisfactory S, if *both* units are functioning *and* the average temperature is below 85°F
(ii) marginal M, if one of the units is out of service *and* the average temperature is above 85°F
(iii) unsatisfactory U, otherwise.

Assume H, E_1, and E_2 are statistically independent.

(a) Define the events S, M, and U in terms of H, E_1, and E_2.

(b) What is the probability that *exactly one unit* will be out of service in any given week?

(c) Find $P(S)$, $P(M)$, and $P(U)$.

2.10 A cantilever beam has 2 hooks where weights 1 and 2 may be hung (Fig. P2.10). There can be as many as two weights or no weight at each hook. In order to design this beam, the engineer needs to know the fixed-end moment at A, that is, M_A.

(a) What are all the possible values of M_A?

(b) Let

E_1 denote the event that $M_A > 600$ ft-lb

E_2 denote the event that $200 \leq M_A < 800$ ft-lb

Are events E_1 and E_2 mutually exclusive? Why?

(c) Are events E_1 and E_3 mutually exclusive? Where $E_3 = \{0, 100, 400\}$.

(d) With the following information:

Probability that weight 1 hangs at $B = 0.2$
Probability that weight 1 hangs at $C = 0.7$
Probability that weight 2 hangs at $B = 0.3$
Probability that weight 2 hangs at $C = 0.5$

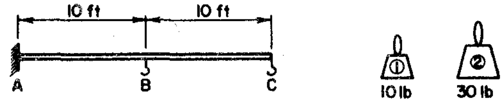


Figure P2.10

What are the probabilities associated with each sample point in part (a)? Assume that the location of weight ① does not affect the probability of the location of weight ②.

(e) Determine the probabilities of the following events:

$$E_1,\ E_2,\ E_1 \cap E_2,\ E_1 \cup E_2,\ \bar{E}_2$$

2.11 In a building construction project, the completion of the building requires the successive completion of a series of activities. Define

E = excavation completed on time; and $P(E) = 0.8$
F = foundation completed on time; and $P(F) = 0.7$
S = superstructure completed on time; and $P(S) = 0.9$

Assume statistical independence among these events.

(a) Define the event {project completed on time} in terms of E, F, and S. Compute the probability of on-time completion.
(b) Define, in terms of E, F, S and their complements, the following event:
G = excavation will be on time and at least one of the other two operations will not be on time
Calculate $P(G)$.
(c) Define the event
H = only one of the three operations will be on time

2.12 The waste from an industrial plant is subjected to treatment before it is ejected to a nearby stream. The treatment process consists of three stages, namely: primary, secondary, and tertiary treatments (Fig. P2.12). The primary treatment may be rated as good (G_1), incomplete (I_1) or failure (F_1). The secondary treatment may be rated as good (G_2) or failure (F_2), and the tertiary treatment may also be rated as good (G_3) or failure (F_3). Assume that the ratings in each treatment are *equally likely* (for example, the primary treatment will be equally likely to be good or incomplete or failure). Furthermore, the performances of the three stages of treatment are statistically independent of one another.

(a) What are the possible combined ratings of the three treatment stages? (for example, G_1, F_2, G_3 denotes a combination where there is a good primary and tertiary, but a failure in the secondary treatment). What is the probability of each of these combinations (or sample points)?
(b) Suppose the event of satisfactory overall treatment requires at least two stages of good treatment. What is the probability of this event?
(c) Suppose:

E_1 = good primary treatment
E_2 = good secondary treatment
E_3 = good tertiary treatment

Determine

$$P(\bar{E}_1), \qquad P(E_1 \cup E_2), \qquad P(E_2E_3)$$

(d) Express in terms of E_1, E_2, E_3 the event of satisfactory overall treatment as defined in part (b). (*Hint.* E_1E_2 is part of this event.)

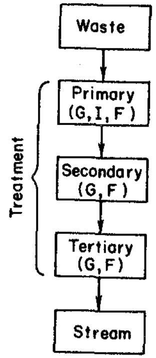


Figure P2.12

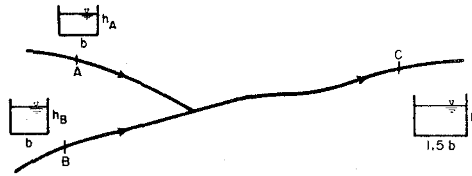


Figure P2.13

2.13 The cross-sections of the rivers at A, B, and C are shown in Fig. P2.13 and the flood levels at A and B, above mean flow level, are as follows:

Flood level at A (ft)	Probability
0	0.25
2	0.25
4	0.25
6	0.25

Flood level at B (ft)	Probability
0	0.20
2	0.20
4	0.20
6	0.20
8	0.20

Assume that the flow velocities at A, B, and C are the same. What is the probability that the flood at C will be higher than 6 ft above the mean level? Assume statistical independence between flood levels at A and B. *Ans. 0.3.*

2.14 Figure P2.14 is a plot of test results showing the degree of subgrade compac-

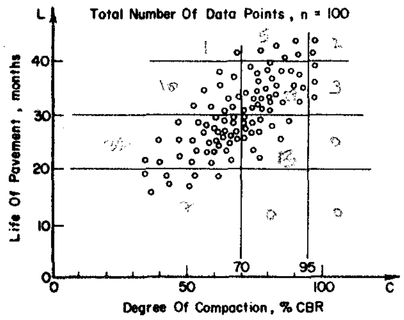


Figure P2.14

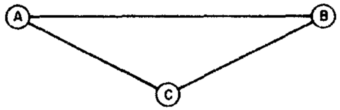


Figure P2.15

tion C versus the life of pavement L. Determine the following:

(a) $P(20 < L \leq 40 \mid C \geq 70)$
(b) $P(L > 40 \mid C \leq 95)$
(c) $P(L > 40 \mid 70 < C \leq 95)$
(d) $P(L > 30 \text{ and } C < 70)$

2.15 The highway system between cities A and B is shown in Fig. P2.15. Travel between A and B during the winter months is not always possible because some parts of the highway may not be open to traffic, because of extreme weather condition. Let E_1, E_2, E_3 denote the events that highway AB, AC, and CB are open, respectively.

On any given day, assume

$$P(E_1) = 2/5 \qquad P(E_3 \mid E_2) = 4/5$$
$$P(E_2) = 3/4 \qquad P(E_1 \mid E_2E_3) = 1/2$$
$$P(E_3) = 2/3$$

(a) What is the probability that a traveler will be able to make a trip from A to B if he has to pass through city C? *Ans. 0.6.*
(b) What is the probability that he will be able to get to city B? *Ans. 0.7.*
(c) Which route should he try first in order to maximize his chance of getting to B?

2.16 A contractor is submitting bids to two jobs A and B. The probability that he will win job A is $P(A) = \frac{1}{4}$ and that for job B is $P(B) = \frac{1}{3}$.

(a) Assuming that winning job A and winning job B are independent events, what is the probability that the contractor will get at least a job?
(b) What is the probability that the contractor got job A if he has won at least one job?
(c) If he is also submitting a bid for job C with probability of winning it $P(C) = 1/4$, what is the probability that he will get at least one job? Again assume statistical independence among A, B, and C. What is the probability that the contractor will not get any job at all?

2.17 Cities 1 and 2 are connected by route A, and route B connects cities 2 and 3

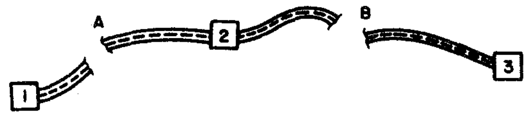


Figure P2.17

(Fig. P2.17). Denote the eastbound lanes as A_1 and B_1, and the westbound lanes as A_2 and B_2, respectively.

Suppose that the probability is 90% that a lane in route A will not require major repair work for at least 2 years; the corresponding probability for a lane in route B is only 80%.

(a) Determine the probability that route A will require major repair work in the next two years. Do the same for route B.
Assume that if one lane of a route needs repair, the chances that the other lane will also need repair is 3 times its original probability. *Ans. 0.17; 0.28.*
(b) Assuming that the need for repair works in routes A and B are independent of each other, what is the probability that the road between cities 1 and 3 will require major repair in two years? *Ans. 0.40.*

2.18 The water supply system for a city consists of a storage tank and a pipe line supplying water from a reservoir some distance away (Fig. P2.18). The amount of water available from the reservoir is variable depending on the precipitation in the watershed (among other things). Consequently, the amount of water stored in the tank would be also variable. The consumption of water also fluctuates considerably.

To simplify the problem, denote

A = available water supply from the reservoir is low
B = water stored in the tank is low
C = level of consumption is low

and assume that

$$P(A) = 20\%$$
$$P(B) = 15\%$$
$$P(C) = 50\%$$

The reservoir supply is regulated to a certain extent to meet the demand, so

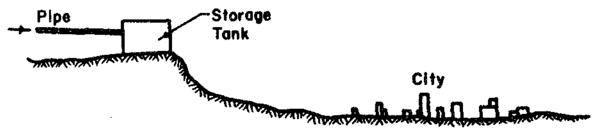


Figure P2.18

that

$$P(\bar{A} \mid \bar{C}) \equiv P(\text{reservoir supply is high} \mid \text{consumption is high})$$
$$= 75\%$$

Also, $P(B \mid A) = 50\%$, whereas the amount of water stored is independent of the demand.

Suppose that a water shortage will occur when there is high demand (or consumption) for water but either the *reservoir supply is low* or the *stored water is low*. What is then the probability of a water shortage? Assume that $P(AB \mid \bar{C}) = 0.5\, P(AB)$.

2.19 The time T (in minutes) that it takes to load crushed rocks from a quarry onto a truck varies considerably. From a record of 48 loadings, the following were observed.

Loading time T (minutes)	No. of observations
1	0
2	5
3	12
4	15
5	10
6	6
≥ 6	0
	Total = 48

(a) Sketch the histogram for the above data.
(b) Based on these data, what is the probability that the loading time T for a truck will be at least 4 minutes?
(c) What is the probability that the total time for loading 2 consecutive trucks will be less than 6 minutes? Assume the loading times for any two trucks to be statistically independent.
(d) In order to make a conservative estimate of the loading time, it is assumed that loading a truck will require at least 3 minutes; on this assumption, what will be the probability that the loading time for a truck will be less than 4 minutes?

2.20 A gravity retaining wall may fail either by sliding (A) or overturning (B) or both (Fig. P2.20). Assume:
(i) Probability of failure by sliding is twice as likely as that by overturning; that is, $P(A) = 2P(B)$.
(ii) Probability that the wall also fails by sliding, given that it has failed by overturning, $P(A \mid B) = 0.8$
(iii) Probability of failure of wall $= 10^{-3}$
(a) Determine the probability that sliding will occur. *Ans. 0.00091.*
(b) If the wall fails, what is the probability that *only* sliding has occurred? *Ans. 0.546.*

2.21 Two cables are used to lift a load W (Fig. P2.21). However, normally only cable A will be carrying the load; cable B is slightly longer than A, so nor-

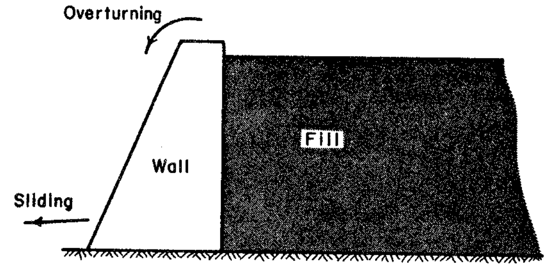


Figure P2.20

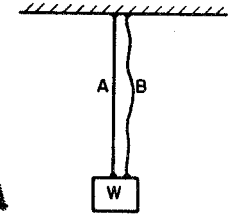


Figure P2.21

mally it does not participate in carrying the load. But if cable A breaks, then B will have to carry the full load, until A is replaced.

The probability that A will break is 0.02; also, the probability that B will fail if it has to carry the load by itself is 0.30.
(a) What is the probability that both cables will fail?
(b) If the load remains lifted, what is the probability that none of the cables have failed?

• 2.22 The preliminary design of a bridge spanning a river consists of four girders and three piers as shown in Fig. P2.22. From consideration of the loading and resisting capacities of each structural element the failure probability for each girder is 10^{-5} and each pier is 10^{-6}. Assume that failures of the girders and piers are statistically independent. Determine:

(a) The probability of failure in the girder(s).
(b) The probability of failure in the pier(s).
(c) The probability of failure of the bridge system.

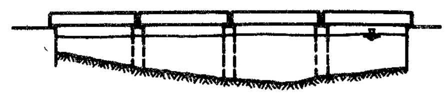


Figure P2.22

• 2.23 The town shown in Fig. P2.23 is protected from floods by a reservoir dam that is designed for a 50-year flood; that is, the probability that the reservoir will overflow in a year is 1/50 or 0.02. The town and reservoir are located in an active seismic region; annually, the probability of occurrence of a destructive earthquake is 5%. During such an earthquake, it is 20% probable that the dam will be damaged, thus causing the reservoir water to flood the town. Assume that the occurrences of natural floods and earthquakes are statistically independent.
(a) What is the probability of an earthquake-induced flood in a year?
(b) What is the probability that the town is free from flooding in any one year?
(c) If the occurrence of an earthquake is assumed in a given year, what is the probability that the town will be flooded that year?

Figure P2.23

2.24 From a survey of 1000 water-pipe systems in the United States, 15 of them are reported to be contaminated by bacteria alone whereas 5 are reported to have an excessive level of lead concentration and among these 5, there are 2 that are found to contain bacteria also.
(a) What is the probability that a pipe system selected at random will contain bacteria? *Ans. 0.017.*
(b) What is the probability that a pipe system selected at random is contaminated? *Ans. 0.02.*
(c) Suppose that a pipe system is found to contain bacteria. What is the probability that its lead concentration is also excessive? *Ans. 2/17.*
(d) Assume that the present probability of contamination as computed in part (b) is not satisfactory, and it is proposed that it should not exceed 0.01. Suppose that it is difficult to control the lead contamination, but it is possible to reduce the likelihood of bacteria contamination. What should be the permissible probability of bacteria contamination? Assume that the value of the conditional probability computed in part (c) still applies. *Ans. 0.00567.*

2.25 The structural component shown in Fig. P2.25 has welds to be inspected for flaws. From experience, the likelihood of detecting flaws in a foot of weld provided by the manufacturer is 0.1; and the probability of detecting flaws in a weld of length L ft is given by

$$P(F_L) = 0.1L \qquad \text{for} \quad 0 \leq L \leq 2\text{ ft}$$

In general, the quality between sections of welds in a structural component is

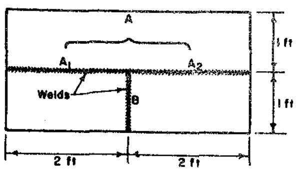


Figure P2.25

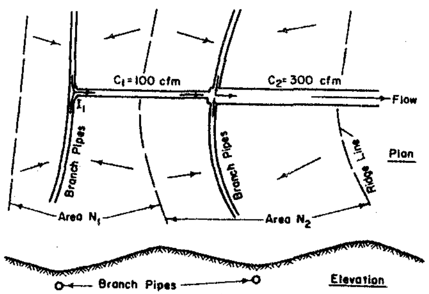


Figure P2.26

correlated. Assume the following:
(i) If flaws are detected in section A_1, the probability of flaws being detected in A_2 will be three times its original probability.
(ii) If flaws are detected in section A, the probability of detecting flaws in section B will be doubled.

Let F_{A_1}, F_{A_2}, F_A, and F_B be the events of flaws detected in weld sections A_1, A_2, A, and B, respectively.
(a) What is the probability of detecting flaws in A? *Ans. 0.28.*
(b) What is the probability of detecting flaws in the structural component? *Ans. 0.324.*
(c) If flaws are detected in the structural component, what is the probability that they are found *only* in A? *Ans. 0.692.*

• **2.26** The storm drainage in a residential subdivision can be divided into watershed areas N_1 and N_2 as shown in Fig. P2.26. The drainage system consists of the main sewers with capacities C_1 = 100 cfm (cubic feet per minute) and C_2 = 300 cfm, respectively. The amounts of drainage from N_1 and N_2 are variable, depending on the rainfall intensities in the subdivision (assume that whenever it rains the entire subdivision is covered); in any given year, the maximum flow, I_1 and I_2, and their corresponding probabilities are as follows.

I_1 (cfm)	Probability	I_2 (cfm)	Probability
80	0.60	100	0.50
120	0.40	210	0.30
		250	0.20

Neglect the possibility of flooding in N_1 caused by the overflow of pipe C_2.
(a) What is the probability of flooding in area N_1? Flooding occurs only when the drainage exceeds the capacity of the main sewer.
(b) What is the probability of flooding in area N_2?
(c) What is the probability of flooding in the subdivision?

2.27 In order to study the parking problem of a college campus, an average worker in office building D, say Mr. X, is selected and his chance of getting a parking space each day is studied. (Assume that Mr. X will check the parking lots A, B, C in that sequence and will park his car as soon as a space is found.) Assume that there are only three parking lots available, of which A and B are free, whereas C is metered (Fig. P2.27). No other parking facilities (say street parking) are allowed. From statistical data, the probabilities of getting a parking space each week day morning in lots A, B, C are 0.2, 0.1, 0.5, respectively. However, if lot A is full, the probability that Mr. X will find a space in B is only 0.04. Also, if lots A and B are full, Mr. X will only have a probability of 40% of getting a parking space in C. Determine the following:
(a) The probability that Mr. X will not be able to secure a free space on a weekday morning. *Ans. 0.768.*
(b) The probability that Mr. X will be able to park his car on a weekday morning. *Ans. 0.539.*
(c) If Mr. X has successfully parked his car one morning, what is the probability that it will be free of charge? *Ans. 0.43.*

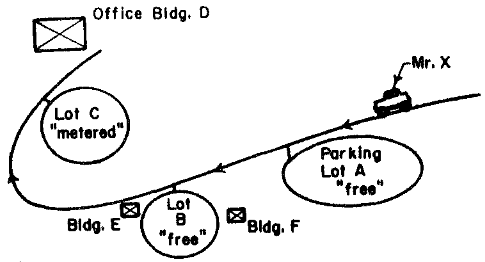


Figure P2.27

2.28 Pollution is becoming a problem in cities I and II. City I is affected by both air and water pollution, whereas city II is subjected to air pollution only. A three-year plan has been put into action to control these sources of pollution in both cities. It is estimated that the air pollution in city I will be successfully controlled is 4 times as likely as that in city II. However, if air pollution in city II is controlled, then air pollution in city I will be controlled with 90% probability.

The control of water pollution in city I may be assumed to be independent of the control of air pollution in both cities. In city I, the probability that pollution will be completely controlled (that is, both sources are controlled) is 0.32, whereas it is also estimated that water pollution is only half as likely to be controlled as the air pollution in that city. Let

A_{I} be the event "air pollution in city I is controlled"

A_{II} be the event "air pollution in city II is controlled"

W_{I} be the event "water pollution in city I is controlled"

Determine:

(a) Probability that air pollution will be controlled in both cities. *Ans. 0.18.*
(b) Probability that pollution in both cities will be completely controlled. *Ans.* 0.072.
(c) Probability that at least one city will be free of pollution. *Ans. 0.448.*

• **2.29** A form of transportation is to be provided between two cities that are 200 miles apart. The alternatives are highway (H), railway (R), or air transport

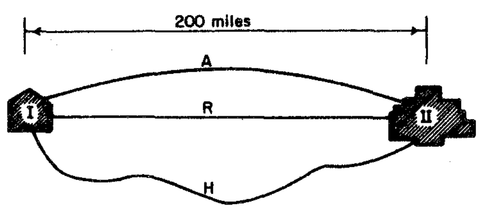


Figure P2.29

(A); the last one meaning the construction of airports in both cities. (See Fig. P2.29.) Because of the relative merits and costs, the odds that a Committee of Planners will decide on R, H, or A are 1 to 2 to 3. Only one of these three means can be constructed.

However, if the committee decides on building a railroad (R), the probability that it will be completed in one year is 50%; if it decides on a highway (H), the corresponding probability is 75%; and if it decides on air travel, there is a probability of 90% that the airports will be completed in one year.

(a) What is the probability that the two cities will have a means of transportation in one year?
(b) If some transportation facility between the two cities is completed in one year, what is the probability that it will be air transport (A)?
(c) If the committee decides in favor of land facilities, what is the probability that the final decision will be for a highway (H)?

2.30 "Liquefaction of sand" denotes a phenomenon in foundation engineering, in which a mass of saturated sand suddenly loses its bearing capacity because of rapid changes in loading conditions—for example, resulting from earthquake vibrations. When this happens, disastrous effects on structures built on the site may follow.

For simplicity, rate earthquake intensities into low (L), medium (M), and high (H). The likelihoods of liquefaction associated with earthquakes of these intensities are, respectively, 0.05, 0.20, and 0.90.

Assume that the relative frequencies of occurrence of earthquakes of these intensities are, respectively, 1, 0.1, and 0.01 per year.

(a) What is the probability that the next earthquake is of low intensity? *Ans. 0.9.*
(b) What is the probability of liquefaction of sand at the site during the next earthquake? *Ans. 0.07.*
(c) What is the probability that the sand will survive the next three earthquakes (that is, no liquefaction)? Assume the conditions between earthquakes are statistically independent. *Ans. 0.80.*

2.31 There are three modes of transporting material from New York to Florida, namely, by land, sea, or air. Also land transportation may be by rail or highway. About half of the materials are transported by land, 30% by sea, and the rest by air.

Also, 40% of all land transportation is by highway and the rest by rail shipments. The percentages of damaged cargo are, respectively, 10% by highway, 5% by rail, 6% by sea, and 2% by air.

(a) What percentage of all cargoes may be expected to be damaged?
(b) If a damaged cargo is received, what is the probability that it was shipped by land? By sea? By air?

2.32 The amount of stored water in a reservoir (Fig. P2.32*a*) may be idealized into three states: full (F), half-full (H), and empty (E). Because of the probabilistic nature of the inflowing water into the reservoir, as well as the outflow from the reservoir to meet uncertain demand for water, the amount of water stored may shift from one state to another during each season. Suppose that these transitional probabilities from one state to another are as indicated in Fig. P2.32*b*. For example, in the beginning of a season, if the water storage is empty, the probability that it will become half-full at the end of the season is 0.5 and the probability that it will remain empty is 0.4, and so on. Assume that the water level is full at the start of the season.

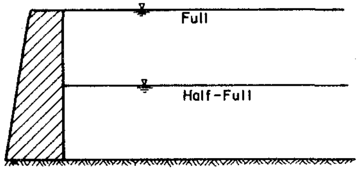


Figure P2.32a

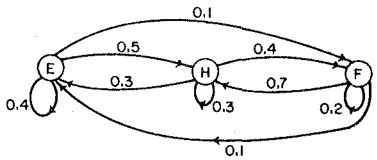


Figure P2.32b

(a) What is the probability that the reservoir will be full at the end of one season? What is the probability that the reservoir will contain water at the end of one season? *Ans. 0.2; 0.9.*

(b) What is the probability that the reservoir will be full at the end of the second season? *Ans. 0.33.*

(c) What is the probability that the reservoir will contain water at the end of the second season? *Ans. 0.73.*

2.33 At a quarry, the time required to load crushed rocks onto a truck is equally likely to be either 2 or 3 minutes (Fig. P2.33). Also the number of trucks in a queue waiting to be loaded at any time varies considerably, as reflected in the following set of 30 observations taken at random. The time required to

No. of trucks in queue	No. of observations	Relative frequency
0	6	0.2
1	3	0.1
2	9	0.3
3	9	0.3
4	3	0.1
5	0	0.0
	Total = 30	

load a truck is statistically independent of the queue size.

(a) If there are two trucks in the queue when a truck arrives at the quarry, what is the probability that its "waiting time" will be less than 5 minutes? *Ans. 0.25.*

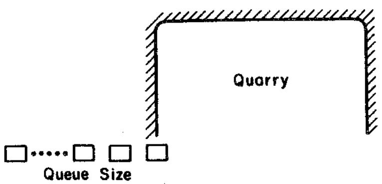


Figure P2.33

(b) Before arriving at the quarry (and thus not knowing the size of the queue), what is the probability that the waiting time of a particular truck will be less than 5 minutes? *Ans. 0.375.*

2.34 A chemical plant produces a variety of products using four different processes; the available labor is sufficient only to run one process at a time. The plant manager knows that the discharge of dangerous pollution into the plant waste water system and thence into a nearby stream is dependent on which process equipment is in operation. The probability that a particular process will be producing dangerous pollution products is as shown below:

process *A*	40%
process *B*	5%
process *C*	30%
process *D*	10%

All other processes in the plant are considered harmless.

In a typical month the relative likelihoods of processes *A*, *B*, *C*, and *D* operating through the month are 2:4:3:1, respectively.

(a) What is the probability that there will be *no* dangerous pollution discharged in a given month?

(b) If dangerous pollution is detected in the plant discharge, what is the probability that process *A* was operating?

(c) The pollution products that are discharged by the various processes have different probabilities of producing a fish kill in the stream that the plant uses for disposal, as follows.

Process	Probability of fish kill
A	0.9
B	0.1
C	0.8
D	0.3

Based on these assumptions what is the probability that fish will be killed by pollution in the stream in a given month?

(d) Of the four processes, which is the most fruitful one (in terms of minimizing the likelihood of fish kill) to select for clean-up if only one can be improved?

2.35 The probability of occurrence of fire in a subdivision has been estimated to be 30% for one occurrence and 10% for two occurrences in a year. Assume that the chance for three or more occurrences is negligible. In a fire, the probability that it will cause structural damage is 0.2. Assume that structural damages between fires are statistically independent.

(a) What is the probability that there will be no structural damage caused by fire in a year? *Ans. 0.904.*

(b) If a small town consists of two such subdivisions, what is the probability that there will be some structural damage caused by fire in the town in a year? Assume that the events of fire-induced structural damage in the two subdivisions are statistically independent. *Ans. 0.183.*

• **2.36** At a construction project, the amount of material (say lumber for falsework) available for any day is variable, and can be described with the frequency diagram of Fig. P2.36. The amount of material used in a day's construction is either 150 units or 250 units, with corresponding probabilities 0.70 and 0.30.

(a) What is the probability of shortage of material in any day? Shortage occurs whenever the available material is less than the amount needed for that day's construction.
(b) If there is a shortage of material, what is the probability that there were fewer than 200 units available?

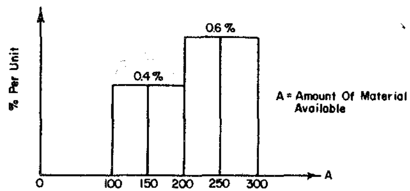


Figure P2.36 Frequency diagram of A

2.37 The completion time of a construction project depends on whether the carpenters and plumbers working on the project will go on strike. The probabilities of delay (D) are 100%, 80%, 40%, and 5% if *both go on strike*, *carpenters alone go on strike*, *plumbers alone go on strike*, and *neither of them strikes*, respectively. Also, there is 60% chance that plumbers will strike if carpenters strike, and if plumbers go on strike there is 30% chance that carpenters would follow. It is known that the chance for the plumbers' strike is 10%. Let

C = event that carpenters went on strike
P = event that plumbers went on strike
D = delay in project completion

(a) Determine probability of delay in completion. *Ans. 0.118.*
(b) If there is a delay in completion, determine the following:
 (i) Probability that both carpenters and plumbers strike. *Ans. 0.254.*
 (ii) Probability that carpenters strike and plumbers do not. *Ans. 0.136.*
 (iii) Probability of carpenters' strike. *Ans. 0.390.*

2.38 The water supply for a city comes from two reservoirs, a and b (Fig. P2.38). Because of variable rainfall conditions each year, the amount of water in each reservoir may *exceed* or *not exceed* the normal capacity. Let A denote the event that the water in reservoir a exceeds its normal capacity, and let B denote that for reservoir b. The following probabilities are given: $P(B) = 0.8$, $P(AB) = 0.6$, $P(\bar{A} \mid \bar{B}) = 0.7$. In addition, the probabilities that the city will have satisfactory supply of water *if only one reservoir exceeds*, *both reservoirs exceed*, and *none of the reservoir exceeds* the normal capacities are

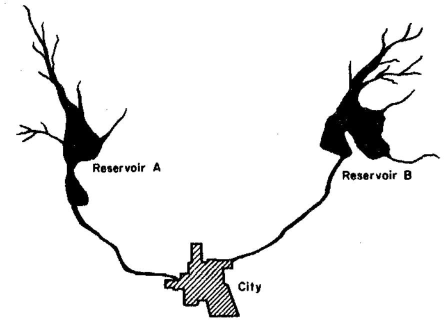


Figure P2.38

0.7, 0.9, and 0.3, respectively. What is the probability that the city will have satisfactory water supply? *Ans. 0.764.*

• **2.39** A water tower is located in an active earthquake region. When an earthquake occurs, the probability that the tower will fail depends on the magnitude of the earthquake *and* also on the amount of storage in the tank at the time of shaking of the ground. For simplicity, assume that the tank is either full (F) or half-full (H) with relative likelihoods of 1 to 3. The earthquake magnitude may be assumed to be either strong (S) or weak (W) with relative frequencies 1 to 9.

When a strong earthquake occurs, the tower will definitely collapse regardless of the storage level. However, the tower will certainly survive a weak earthquake if the tank is only half-full. If the tank is full during a weak earthquake, it will have a 50-50 chance of survival.

If the tower collapsed during a recent earthquake, what is the probability that the tank was full at the time of the earthquake?

2.40 For a county in Texas, the probabilities that it will be hit by one or two hurricanes each year are 0.3 and 0.05, respectively. The event that it will be hit by three or more hurricanes in a year may be assumed to have negligible probability.

This county may be subjected to floods each year from the melting of snow in the upstream regions, or from the heavy precipitation brought by hurricanes, or both. Normally, the chance of flood in a year, caused by the melting snow only, is 10%. However, during a hurricane there is a 25% probability of flooding. Assume that floods caused by the melting snow and floods caused by hurricanes are independent events.

What is the probability that there will be flooding in this county in a year?

2.41 Before the design of a tunnel through a rocky region, geological exploration was conducted to investigate the joints and the potential slip surfaces that exist in the rock strata (Fig. P2.41). For economic reasons, only portions of the strata are explored. In addition the measurements recorded by the instruments are not perfectly reliable. Thus the geologist can only conclude that the condition of the rock may be either highly fissured (H), medium fissured (M), or slightly fissured (L) with relative likelihoods of 1:1:8. Based on this

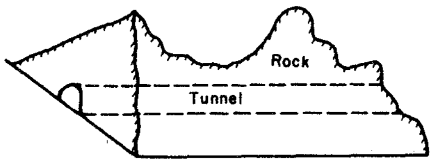


Figure P2.41

information, the engineer designs the tunnel and estimates that if the rock condition is L, the reliability of the proposed design is 99.9%. However, if it turns out that the rock condition is M, the probability of failure will be doubled; similarly, if the rock condition is H, the probability of failure will be 10 times that for condition L.

(a) What is the expected reliability of the proposed tunnel design? *Ans. 0.998.*

(b) A more reliable device is subsequently used to improve the prediction of rock condition. Its results indicate that a highly fissured condition for the rock around the tunnel is practically impossible, but it cannot give better information on the relative likelihood between rock conditions M and L. In light of this new information, what would be the revised reliability of the proposed tunnel design? *Ans. 0.9989.*

(c) If the tunnel collapsed, what should be the updated probabilities of M and L? *Ans. 0.2 ; 0.8.*

2.42 Three research and development groups, A, B, and C, submitted proposals for a research project to be awarded by a research agency of the government. From past performance records, the respective histograms of completion time relative to the scheduled target time t_0 are shown in Fig. P2.42. It is known that groups A and B have about equal chances of getting the project, whereas C is twice as likely as either A or B to win the contract.

Based on past performance records, determine:

(a) The probability that the project will be completed on schedule. *Ans. 0.60.*

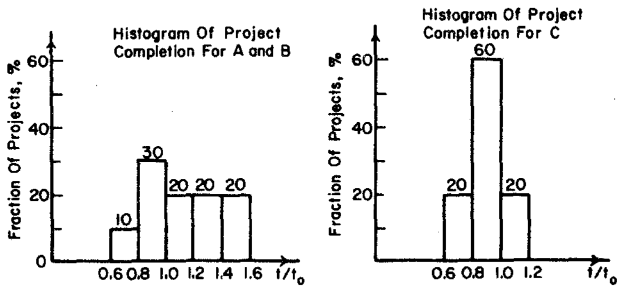


Figure P2.42

(b) If the project completion is delayed, what is the probability that it was originally awarded to C? *Ans. 0.25.*

2.43 Two independent remote sensing devices, A and B, mounted on an airplane are used to determine the locations of diseased trees in a large area of forest land. The detectability of device A is 0.8 (that is, the probability that a group of diseased trees will be detected by device A is 0.8), whereas the detectability of device B is 0.9.

However, when a group of diseased trees has been detected its location may not be pinpointed accurately by either device. Based on a detection from device A alone, the location can be accurately determined with probability 0.7, whereas the corresponding probability with device B alone is only 0.4. If the same group of diseased trees is detected by both devices, its location can be pinpointed with certainty. Determine the following.

(a) The probability that a group of diseased trees will be detected. *Ans. 0.98.*

(b) The probability that a group of diseased trees will be detected by *only one* device. *Ans. 0.26.*

(c) The probability of accurately locating a group of diseased trees. *Ans. 0.848.*

Thus far (and this will continue through Chapter 4), we have been dealing with idealized theoretical models. In particular, we have assumed, tacitly at least, that the probability distribution of a random variable, or its main descriptors, are known. In a real problem, of course, these must be estimated and inferred or derived on the basis of real-world data and conditions. The concepts and methods for these purposes are the subjects of Chapters 5 to 8.

PROBLEMS

Section 3.1

3.1 A contractor is submitting bids to 3 jobs, A, B, and C. The probabilities that he will win each of the three jobs are $P(A) = 0.5$, $P(B) = 0.8$, and $P(C) = 0.2$, respectively. Assume events A, B, C are statistically independent. Let X be the total number of jobs the contractor will win.
  (a) What are the possible values of X? Compute and plot the probability mass function (PMF) of the random variable X.
  (b) Plot the distribution function of X.
  (c) Determine $P(X \leq 2)$. *Ans. 0.92.*
  (d) Determine $P(0 < X \leq 2)$. *Ans. 0.84.*

3.2 The settlement of a structure has the probability density function shown in Fig. P3.2.
  (a) What is the probability that the settlement is less than 2 cm?
  (b) What is the probability that the settlement is between 2 and 4 cm?
  (c) If the settlement is observed to be more than 2 cm, what is the probability that it will be less than 4 cm?

3.3 The bearing capacity of the soil under a column-footing foundation is known to vary between 6 and 15 kips/sq ft. Its probability density within this range is given as

$$f_X(x) = \frac{1}{2.7}\left(1 - \frac{x}{15}\right) \qquad 6 \leq x \leq 15$$
$$= 0 \qquad \text{elsewhere}$$

If the column is designed to carry a load of 7.5 kips/sq ft, what is the probability of failure of the foundation?

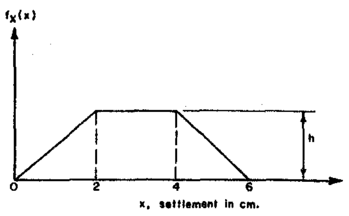


Figure P3.2

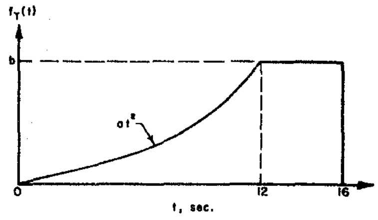


Figure P3.4

3.4 The time duration of a force acting on a structure has been found to be a random variable having the density function shown in Fig. P3.4.
  (a) Determine the appropriate values of a and b for the density function.
  (b) Calculate the mean and median for the variable T.
  (c) Calculate the probability that T will be equal to or greater than 6 sec, that is $P(T \geq 6)$.

3.5 A construction project consisted of building a major bridge across a river and a road linking it to a city (Fig. P3.5*a*). The contractual time for the entire project is 15 months.

The contractor knows that the construction of the road will require between 12 and 18 months, and the bridge could take between 10 and 20 months. The probability density functions of the respective completion times, however, are uniform for the road, and triangular for the bridge, as shown in Figs. P3.5*b* and *c*. Construction of the road and bridge can proceed simultaneously, and the completion of the bridge and the road are statistically independent.

Determine the probability of completing the project within the contractual time.

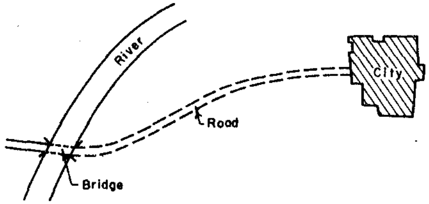


Figure P3.5a

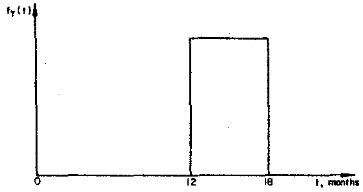


Figure P3.5b

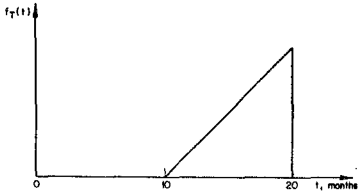


Figure P3.5c

3.6 In order to repair the cracks that may exist in a 10-ft weld, a nondestructive testing (NDT) device is used first to detect the location of cracks. Because cracks may exist in various shapes and sizes, the probability that a crack will be detected by the NDT device is only 0.8. Assume that the events of each crack being detected are statistically independent.
  (a) If there are two cracks in the weld, what is the probability that they would not be detected?
  (b) The actual number of cracks N in the weld is not known. However, its

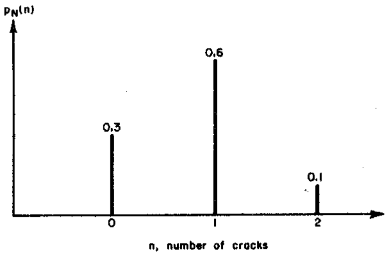


Figure P3.6

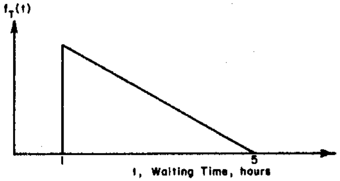


Figure P3.8 PDF of waiting time

PMF is given as in Fig. P3.6. What is the probability that the NDT device will fail to detect any crack in this weld?

(c) Determine the mean, variance, and coefficient of variation of N based on the PMF given in Fig. P3.6.

(d) If the device fails to detect any crack in the weld, what is the probability that the weld is flawless (that is, no crack at all)?

3.7 Suppose the duration (in months) of a construction job can be modeled as a continuous random variable T whose cumulative distribution function (CDF) is given by

$$\begin{aligned} F_T(t) &= t^2 - 2t + 1 && 1 \le t \le 2 \\ &= 0 && t < 1 \\ &= 1 && t > 2 \end{aligned}$$

(a) Determine the corresponding density function $f_T(t)$.

(b) Compute $P(T > 1.5)$.

3.8 The waiting time at airport A of city B has a density function shown in Fig. P3.8. The waiting time is measured from the time a traveler enters the terminal to the time when he is airborne.

The travel time from hotel C to the airport depends on the transportation mode and may be assumed to be 0.75, 1.00, and 1.25 hours corresponding to travel by rapid transit, taxi, and limousine, respectively. The probability of a traveler's taking each mode of transportation is as follows:

$$\begin{aligned} P\,(\text{rapid transit}) &= 0.3 \\ P\,(\text{taxi}) &= 0.4 \\ P\,(\text{limousine}) &= 0.3 \end{aligned}$$

(a) What is the probability that a traveler will be airborne in at most 3 hr after leaving hotel C? *Ans. 0.436.*

(b) Given that the traveler is airborne within 3 hr, what is the probability that he took the limousine? *Ans. 0.234.*

3.9 Two reservoirs are located upstream of a town; the water is held back by two dams A and B. Dam B is 40 m high. (See Fig. P3.9*a*.) During a strong-motion earthquake, dam A will suffer damage and water will flow downstream into the lower reservoir. Depending on the amount of water in the upper

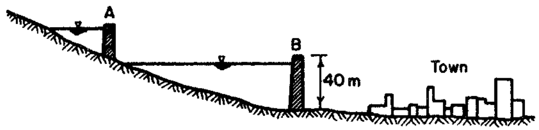


Figure P3.9a

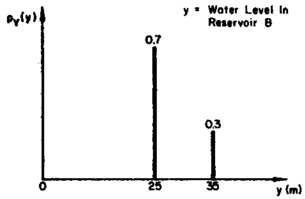


Figure P3.9b

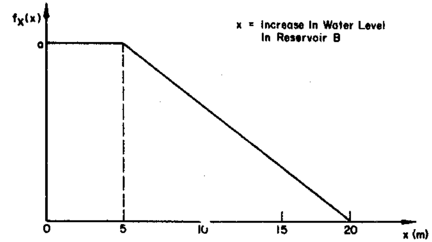


Figure P3.9c

reservoir when such an earthquake occurs, the lower reservoir water may or may not overflow dam B. Suppose that the water level at reservoir B, during an earthquake, is either 25 m or 35 m, as shown in Fig. P3.9*b*; and the increase in the elevation of water level in B caused by the additional water from reservoir A is a continuous random variable with the probability density function given in Fig. P3.9*c*.

(a) Determine the value of a in Fig. P3.9*c*.

(b) What is the probability of overflow at B during a strong-motion earthquake?

(c) If there were no overflow at B during an earthquake, what is the probability that the original water level in reservoir B is 25 m?

3.10 A stretch of an intercity freeway has 3 one-way lanes and 2 convertible lanes. The capacity of the highway when the 3 lanes are used is 100 cars per minute. Its capacity when 5 lanes are used is 140 cars per minute.

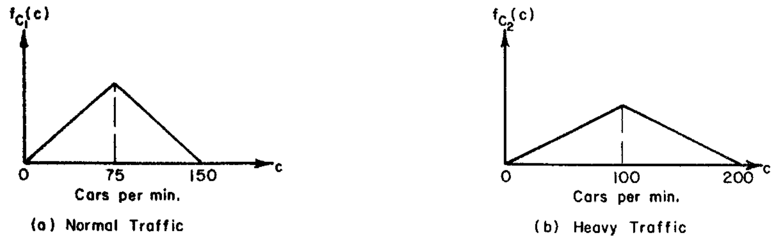


Figure P3.10 PDF of traffic volume. (*a*) Normal traffic. (*b*) Heavy traffic

Three lanes of the freeway is used when there is normal traffic whereas all five lanes will be used whenever there is heavy traffic volume. The density function of the traffic volumes in each case are shown in Figs. P3.10*a* and *b*.

On a given day, if normal traffic is twice as likely as heavy traffic, what is the probability that the capacity of the freeway will be surpassed?

3.11 A traveler going from city A to city C must pass through city B (Fig. P3.11*a*). The quantities T_1 and T_2 are the times of travel from city A to city B and from city B to city C, in hours, respectively, which are statistically independent random variables. The probability mass functions of T_1 and T_2 are as shown in Figs. P3.11*b* and *c*. The time required to go through city B may be considered a deterministic quantity equal to 1 hr.

(a) Calculate the mean, the variance, the standard deviation, and the coefficient of variation of T_1.
(b) Determine the PMF of the total time of travel from city A to city C. Sketch your results graphically.
(c) What is the probability that the travel time from city A to city C will be at least 8 hr?

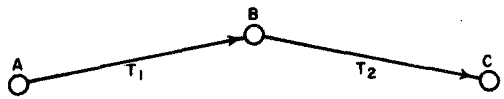


Figure P3.11a

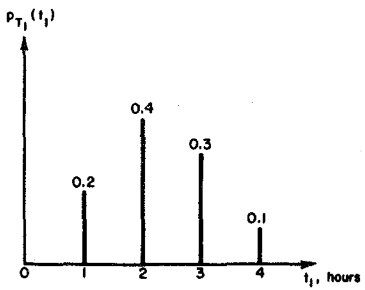


Figure P3.11b

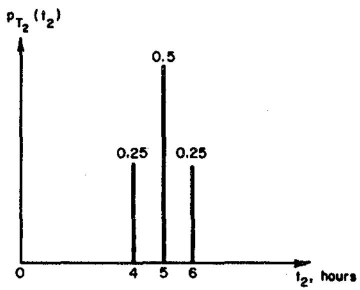


Figure P3.11c

3.12 The hourly volume of traffic for a proposed highway is distributed as in Fig. P3.12.

(a) The traffic engineer may design the highway capacity equal to the following.
  (i) The mode of X.
  (ii) The mean of X.
  (iii) The median of X.
  (iv) $x_{.90}$, the 90-percentile value, which is defined as $F_X(x_{.90}) = 0.90$.

Determine the design capacity of the highway and the corresponding probability of exceedance (that is, capacity is less than traffic volume) for each of the four cases.

(b) Assume that the actual capacity of the highway after it is built is either 300 or 350 vehicles per hr with relative likelihoods of 1 to 4. What is the probability that the capacity will be exceeded?

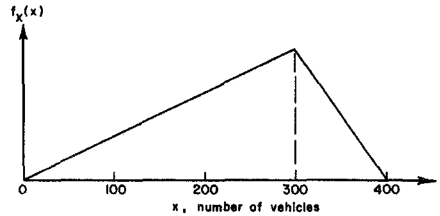


Figure P3.12 PDF of hourly traffic volume

3.13 The lateral resistance of a small building frame is random with the density function

$$f_R(r) = \frac{3}{500}(r-10)(20-r) \qquad 10 \le r \le 20$$
$$= 0 \qquad \text{elsewhere}$$

(a) Plot the density function $f_R(r)$ and the cumulative distribution function $F_R(r)$.
(b) Determine:
  (i) Mean value of R.
  (ii) Median of R.
  (iii) Mode of R.
  (iv) Standard deviation of R. *Ans.* $\sqrt{5}$.
  (v) Coefficient of variation of R. *Ans. 0.149.*
  (vi) Skewness coefficient. *Ans. 0.*

3.14 The delay time of a construction project is described with a random variable X. Suppose that X is a discrete variate with probability mass function given in Table P.3.14*a*. The penalty for late completion of the project depends on the

Table P3.14*a*. PMF of X

x_i in days	$p_X(x_i)$
1	0.5
2	0.3
3	0.1
4	0.1

Table P3.14*b*. Penalty function

x_i (days)	$g(x_i)$ ($100,000)
1	5
2	6
3	7
4	7

number of days of delay; that is, penalty $= g(x_i)$. The penalty function is given in Table P3.14*b* in units of \$100,000.

(a) Calculate the mean penalty for this project. *Ans. \$570,000.*
(b) Calculate the standard deviation of the penalty. *Ans. \$78,000.*

Section 3.2

3.15 If the annual precipitation X in a city is a normal variate with a mean of 50 in. and a coefficient of variation of 0.2, determine the following.

(a) The standard deviation of X.
(b) $P(X < 30)$.
(c) $P(X > 60)$.
(d) $P(40 < X \leq 55)$.
(e) Probability that X is within 5 in. from the mean annual precipitation.
(f) The value x_0 such that the probability of the annual precipitation exceeding x_0 is only 1/4 that of not exceeding x_0.

3.16 The present air traffic volume at an airport (number of landings and takeoffs) during the peak hour is a normal variate with a mean of 200 and a standard deviation of 60 airplanes (Fig. P3.16).

(a) If the present runway capacity (for landings and takeoffs) is 350 planes per hr, what is currently the daily probability of air traffic congestion? Assume there is one peak hour daily. *Ans. 0.0062.*
(b) If no additional airports or expansion is built, what would be the probability of congestion 10 years hence? Assume that the mean traffic volume is increasing linearly at 10% of current volume per year, and the coefficient of variation remains the same. *Ans. 0.662.*
(c) If the projected growth is correct, what airport capacity will be required 10 years from now to maintain the present service condition (that is, the same probability of congestion as now)? *Ans. 700.*

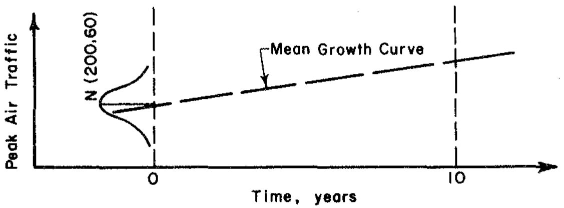


Figure P3.16

3.17 The moment capacity M for the cantilever beam shown in Fig. P3.17 is constant throughout the entire span. Because of uncertainties in material strength, M is assumed to be Gaussian with mean 50 kip-ft and coefficient of variation 20%. Failure occurs if the moment capacity is exceeded anywhere in the beam.

(a) If only a concentrated load 3 kips is applied at the free end, what is the probability that the beam will fail? *Ans. 0.023.*
(b) If only a uniform load of 0.5 kips/ft is applied on the entire beam, what is the probability that the beam will fail? *Ans. 0.006.*

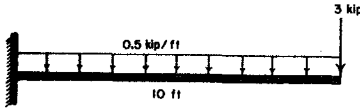


Figure P3.17

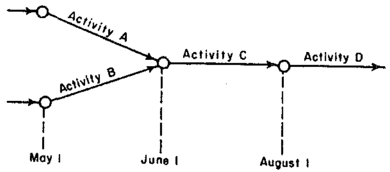


Figure P3.18

(c) In rare cases, the beam may be subjected to the combination of the concentrated load and the uniform load; what will be the reliability (probability of no failure) of the beam when this case occurs? *Ans. 0.308.*
(d) Suppose that the beam had survived under the concentrated load. What will be the probability that it will survive under the combined loads? *Ans. 0.316.*
(e) Suppose that a reliability level of 99.5% is desired, and the beam is subjected only to the uniform load w across the span. What will be the maximum allowable w? *Ans. 0.484 kip/ft.*

3.18 A portion of an activity network is shown in Fig. P3.18; an arrow indicates the starting and ending of an activity. Activity C can start only after completion of both activities A and B, whereas activity D can start only after completion of C. A, B, C, D are statistically independent activities.

The scheduled starting dates are as follows, and an activity cannot start earlier than its scheduled date. (For simplicity, assume all months have 30 days.)

Activities A & B:	May 1
Activity C	: June 1
Activity D	: August 1

The times required to complete each activity are Gaussian random variables as follows.

Activity A:	N(25 days, 5 days)
Activity B:	N(26 days, 4 days)
Activity C:	N(48 days, 12 days)
Activity D:	N(40 days, 8 days)

Assume that both activities A and B started on schedule, that is, on May 1.

(a) Determine the probability that activity C will not start on schedule. *Ans. 0.292.*
(b) The availability of labor is such that unless C is started on schedule the necessary work force will be diverted to another project and thus will be unavailable for this activity for at least 90 days. What is the probability that activity D will start on schedule? *Ans. 0.596.*

3.19 A contractor estimates that the expected time for the completion of job A is 30 days. Because of uncertainties that exist in the labor market, materials supply, bad weather conditions, and so on, he is not sure that he will finish the job in exactly 30 days. However, he is 90% confident that the job will be completed within 40 days. Let X denote the number of days required to complete job A.

(a) Assume X to be a Gaussian random variable; determine μ and σ and also the probability that X will be less than 50, based on the given information. *Ans. 0.9948.*

(b) Recall that a Gaussian random variable ranges from $-\infty$ to $+\infty$. Thus X may take on negative values that are physically impossible. Determine the probability of such an occurrence. Based on this result, is the assumption of the normal distribution for X reasonable? *Ans. 0.00006.*

(c) Let us now assume that X has a log-normal distribution with the same expected value and variance as those in the normal distribution of part (a). Determine the parameters λ and ζ, and also the probability that X will be less than 50. Compare this with the result of part (a). *Ans. 0.9817.*

3.20 From records of repairs of construction equipments, it is found that the failure-free operation time (that is, time between breakdowns) of an equipment may be modeled with a log-normal variate, with a mean of 6 months and a standard deviation of 1.5 months. As the engineer in charge of maintaining the operational condition of a fleet of construction equipment, you wish to have at least a 90% probability that a piece of equipment will be operational at any time.

(a) How often should each piece of equipment be scheduled for maintenance? *Ans. 4.22 months.*

(b) If a particular piece of equipment is still in good operating condition at the time it is scheduled for maintenance, what is the probability that it can operate for at least another month without its regular maintenance? *Ans. 0.749.*

3.21 A system of storm sewers is proposed for a city. In order to evaluate the effectiveness of the sewer system in preventing flooding of the streets, the following information has been gathered. Figure P3.21*a* shows the probability mass function for the number of occurrences of rainstorm each year in the city. Figure P3.21*b* shows the distribution of the maximum runoff rate in each storm, which is log-normal with a median of 7 cfs (cubic feet/sec) and COV of 15%. From hydraulic analysis, the proposed sewer system is shown to be

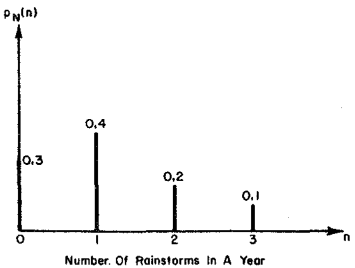


Figure P3.21a

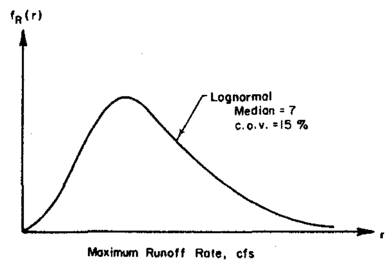


Figure P3.21b

adequate for any storm with runoff rate less than 8 cfs. Assume that the maximum runoff rates between storms are statistically independent.

(a) What is the mean and variance of the number of rainstorms in a year for the city?

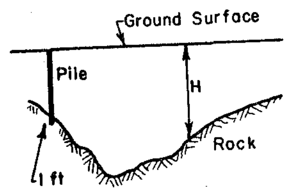
(b) What is the probability of flooding during a rainstorm? *Ans. 0.187.*

(c) What is the probability of flooding in a year? *Ans. 0.189.*

3.22 The depth to which a pile can be driven without hitting the rock stratum is denoted as H (Fig. P3.22*a*). For a certain construction site, suppose that this depth has a log-normal distribution (Fig. P3.22*b*) with mean of 30 ft and COV of 20%. In order to provide satisfactory support, a pile should be embedded 1 ft into the rock stratum.

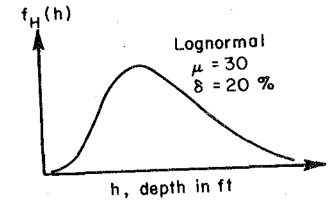
(a) What is the probability that a pile of length 40 ft will not anchor satisfactorily in rock? *Ans. 0.10.*

(b) Suppose a 40-ft pile has been driven 39 ft into the ground and rock has not yet been encountered. What is the probability that an additional 5 ft of pile welded to the original length will be adequate to anchor this pile satisfactorily in rock? *Ans. 0.71.*



(a)

Figure P3.22a



(b)

Figure P3.22b

3.23 A water distribution subsystem consists of pipes AB, BC, and AC as shown in Fig. P3.23. Because of differences in elevation and in hydraulic head loss in the pipes and associated uncertainties, the capacity of each pipe (which is defined as the maximum rate of flow) is given as follows, in cfs (cubic feet/sec):

AB: capacity is Gaussian with mean 5, COV 10%
BC: capacity is log-normal with median 5, COV 10%
AC: capacity equal to 8 or 9 with equal likelihood

(a) Determine the probability that the capacity of the branch ABC will exceed 4 cfs. *Ans. 0.963.*

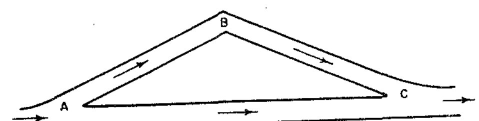


Figure P3.23

(b) Determine the probability that the total capacity of the subsystem shown above will exceed 13 cfs. *Ans. 0.607.* (*Hint.* Use conditional probability.)

3.24 A construction project is at present 30 days away from the scheduled completion date. Depending on the weather condition in the next month, the time required for the remaining construction will have log-normal distributions as follows:

Weather	Time required (days)	
Good	$\mu = 25,$	$\sigma = 4$
Bad	Median $= 30,$	$\sigma = 6$

Based on preliminary investigation, the weather in the next month would be equally likely to be good or bad.

(a) What is the probability that there will be a delay in the completion of the project? *Ans. 0.306.*

(b) A weather specialist is hired to obtain additional information on the weather condition for the next month. However, the specialist is not perfect in his prediction. In general, his predictions are correct 90% of the time, that is $P(PG \mid G) = 0.9$ and $P(PB \mid B) = 0.9$, where PG, PB denote the event that he predicts good and bad weather, respectively, and G, B denote the event that the weather is actually good and bad, respectively. Suppose that the specialist predicted good weather for the next month. What is the updated probability that there will be a delay in the completion of the project? *Ans. 0.150.*

3.25 A compacted subgrade is required to have a specified density of 110 pcf (pounds per cu ft). It will be acceptable if 4 out of 5 cored samples have at least the specified density.

(a) Assuming each sample has a probability of 0.80 of meeting the required density, what is the probability that the subgrade will be acceptable? *Ans. 0.737.*

(b) What should the probability of each sample be in order to achieve a 80% probability of an acceptable subgrade?

3.26 The following is the 20-year record of the annual maximum wind velocity V in town A (in kilometers per hour, kph).

Year	V (kph)	Year	V (kph)
1950	78.2	1960	78.4
1951	75.8	1961	76.4
1952	81.8	1962	72.9
1953	85.2	1963	76.0
1954	75.9	1964	79.3
1955	78.2	1965	77.4
1956	72.3	1966	77.1
1957	69.3	1967	80.8
1958	76.1	1968	70.6
1959	74.8	1969	73.5

(a) Based on this record, estimate the probability that V will exceed 80 kph in any given year.

(b) What is the probability that in the next 10 years there will be exactly 3 years with annual maximum wind velocity exceeding 80 kph?

(c) If a temporary structure is designed to resist a maximum wind velocity of 80 kph, what is the probability that this design wind velocity will be exceeded during the structure's lifetime of 3 years?

(d) How would the answer in part (c) change, if the design wind velocity is increased to 85 kph?

3.27 The sewers in a city are designed for a rainfall having a return period of 10 years.

(a) What is the probability that the sewers will be flooded for the first time in the third year after completion of construction?

(b) What is the probability of flooding within the first 3 years?

(c) What is the probability of flooding in 3 of the first 5 years?

(d) What is the probability of only one flood within 3 years?

3.28 A preliminary planning study on the design of a bridge over a river recommended a permissible probability of 30% of the bridge being inundated by flood in the next 25 years.

(a) If p denotes the probability that the design flood level for the bridge will be exceeded in 1 year, what should the value of p be to satisfy the design criterion given above? [*Hint.* For small value of x, $(1 - x)^n \simeq 1 - nx$.]

(b) What is the return period of this design flood? *Ans. 83.4 years.*

3.29 Figure P3.29 shows a 40-ft soil stratum where boulders are randomly deposited. Piles are designed to be driven to rock. For simplicity, assume that the stratum can be divided into 4 independent layers of 10 ft each, that the probability of hitting a boulder within each 10-ft layer is 0.02, and that the probability of hitting 2 or more boulders within each layer is negligible.

(a) What is the probability that a pile will be successfully driven to rock without hitting any boulder?

(b) What is the probability that it will hit at most 1 boulder on its way to rock?

(c) What is the probability that a pile will hit the first boulder in layer C?

(d) Suppose the foundation of a small building requires a group of 4 such piles driven to rock. What is the probability that no boulders will be

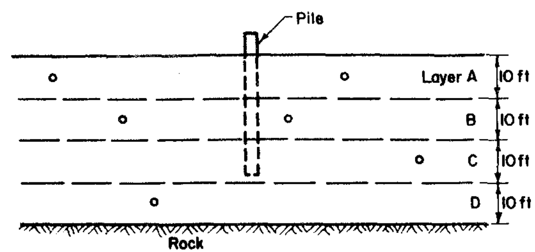


Figure P3.29

encountered in driving the piles? Assume that the pile-driving conditions between piles are statistically independent.

3.30 The useful life per mile of pavement (Fig. P3.30) is described as a log-normal variate with a median of 3 years and COV of 50%. Life means the usable time until repair is required. Assume that the lives between any 2 miles of pavement are statistically independent.
(a) What is the probability that a mile of pavement will require repair in a year?
(b) Suppose that the design life is specified to be the 5-percentile life $x_{.05}$ (that is, the pavement life will be less than the design life with probability 5%). Determine the design life.
(c) What is the probability that there will be no repairs required in the first year of a 4-mile stretch of pavement?
(d) What is the probability that 2 of the 4 miles will need repairs in the first year?
(e) What is the probability of repairs of the 4-mile stretch in the first 3 years of use?
(f) What is the probability that the first repair of the 4-mile stretch will occur in the second year? (Note that the condition in the second year is not independent of the first year.) *Ans. 0.543.*

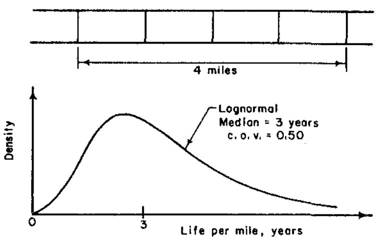


Figure P3.30

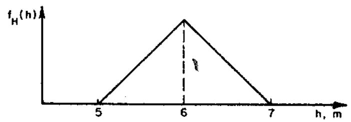


Figure P3.31

3.31 The maximum annual flood level of a river is denoted by H (in meters). Assume that the probability density of H is described by the triangular distribution shown in Fig. P3.31.
(a) Determine the flood height h_{20} which has a mean recurrence interval (return period) of 20 years.
(b) What is the probability that during the next 20 years the river height H will exceed h_{20} at least once?
(c) What is the probability that during the next 5 years the value of h_{20} will be exceeded exactly once?
(d) What is the probability that h_{20} will be exceeded at most twice during the next 5 years?

3.32 For the river in Problem 3.31, a control dam will be constructed according to

the following specification. The height of the dam will be so selected that in the next 3 years this height will be safe against floods with a probability of 94%.
(a) Determine the required return period of the design flood. *Ans. 50 years.*
(b) Determine the design height that will meet this requirement. *Ans. 6.8 m.*

3.33 For quality control purposes, 3 specimens in the form of 6-in.-diameter cylinders are taken at random from a batch of concrete, and each specimen is tested for its compressive strength. A specimen will pass the strength test if it survives an axial compressive load of 11 kips. From previous record, the contractor concludes that the histogram of crushing strength of similar concrete specimens can be satisfactorily modeled by a normal distribution with mean 14.68 kips and standard deviation 2.1 kips, that is, $N(14.68, 2.1)$.
(a) What is the probability that a specimen picked at random will pass the test?
(b) If the specification requires all 3 specimens to pass the test for the batch of concrete to be acceptable, what is the probability that a batch of concrete prepared by this contractor will be rejected?
(c) The contractor prepares a batch of concrete each day. What is the probability that at most one batch of concrete will be rejected for a 2-day period?
(d) Repeat part (b), if the specification is relaxed so that one failure out of the 3 specimens tested is allowed.
(e) The contractor may use a better grade of concrete mix, and together with better workmanship and supervision, he can improve the mean crushing strength of concrete specimen to 16.5 kip, while reducing the coefficient of variation to 90% of its previous value. What is the probability for a batch to be acceptable now? Assume that the crushing strength of the concrete is a normal variate, and no failures are allowed in the 3 specimens tested. *Ans. 0.986.*

3.34 Three flood control dikes are built to prevent flooding of the low plain as shown in Fig. P3.34. The dikes are designed as follows.
(i) Design flood of Dike I is the 20-year flood of river A.
(ii) Design flood of Dike II is the 10-year flood of river A.
(iii) Design flood of Dike III is the 25-year flood of river B.
Assume that the floods in rivers A and B are statistically independent; also, the failures of dikes I and II are statistically independent.
(a) Within a year, determine the probability of flooding of the low plain caused by river A only. *Ans. 0.145.*

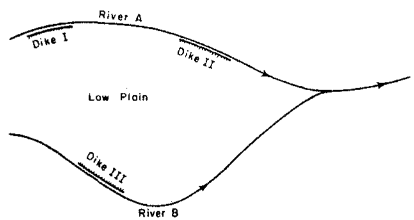


Figure P3.34

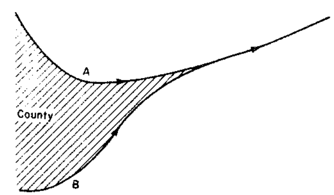


Figure P3.35

(b) What is the probability of flooding of the low plain area in a year? *Ans. 0.179.*

(c) What is the probability of no flooding of the low plains in 4 consecutive years? *Ans. 0.454.*

3.35 A county is bounded by streams A and B (Fig. P3.35). From flow record, the annual maximum flow in A may be modeled by a normal distribution with mean 1000 cfs and COV 20%, whereas that in B may be modeled by a lognormal distribution with mean 800 cfs and COV 20%. The capacities (defined as the maximum flow that can be carried without overflowing) of A and B are 1200 and 1000 cfs, respectively. Assume the stream flows in A and B are statistically independent.

(a) What is the probability that stream A will overflow in a year?
(b) What is the probability that stream B will overflow in a year?
(c) What is the probability that the county will be flooded in a year?
(d) What is the probability that the county will be free of floods in the next 3 years?
(e) If it is decided to reduce the probability of overflow in stream A to 5% a year by enlarging the stream bed at critical locations, what should be the new capacity of A?
(f) Suppose that, because of error in prediction, the capacity of stream B may not be 1000 cfs, and there is a 20% chance that the capacity may be 1100 cfs. In such a case, what is the probability that stream B will overflow in a year?

3.36 A cofferdam is to be built around a proposed bridge pier location so that construction of the pier may be carried out "dry" (see Fig. P3.36).

The height of the cofferdam should protect the site from overflow of wave water during the construction period with a reliability of 95%. The distribution of the monthly maximum wave height is Gaussian $N(5,2)$ ft above mean sea level.

(a) If the construction will take 4 months, what should be the design height of the cofferdam (above mean sea level)? Assume that monthly maximum wave heights are statistically independent. *Ans. 9.46 ft.*
(b) If the time of construction can be shortened by 1 month with an additional cost of \$600, and the cost of constructing the cofferdam is \$2000 per ft (above mean sea level), should the contractor take this alternative? Assume that the same risk of overflow of wave water still applies.

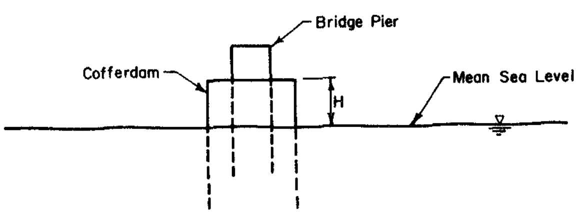


Figure P3.36

3.37 A contractor owns 5 trucks for use in his construction jobs. He decides to institute a new program of truck replacement, using the following procedure:

(i) Any truck that has had more than 1 major breakdown on the job within a year will be evaluated to determine how many miles it gets per gallon of gas.
(ii) Any truck given this special evaluation will be replaced if it gets less than 9 miles per gallon.

From prior experience, the contractor knows two facts with a high degree of confidence: (i) for each truck, the mean rate of major breakdowns is once every 0.8 year; and (ii) the gasoline consumption of trucks that have more than 1 major breakdown is a normal variate $N(10, 2.5)$ in miles per gallon.

(a) What is the probability that a given truck will have more than 1 breakdown within a year?
(b) What is the probability that a truck getting a special evaluation will fail to meet the miles-per-gallon test [see part (ii) above]?
(c) What is the probability that a given truck will be replaced within a year?
(d) What is the probability that the contractor will replace exactly 1 truck within a year?

3.38 On the average 2 damaging earthquakes occur in a certain country every 5 years. Assume the occurrence of earthquakes is a Poisson process in time. For this country, complete the following.

(a) Determine the probability of getting 1 damaging earthquake in 3 years.
(b) Determine the probability of no earthquakes in 3 years.
(c) What is the probability of having at most 2 earthquakes in one year?
(d) What is the probability of having at least 1 earthquake in 5 years?

3.39 (a) The occurrences of flood may be modeled by a Poisson process. If the mean occurrence rate of floods for a certain region A is once every 8 years, determine the probability of no floods in a 10-year period; of 1 flood; of more than 3 floods.

(b) A structure is located in region A. The probability that it will be inundated, when a flood occurs, is 0.05. Compute the probability that the structure will survive if there are no floods; if there is 1 flood; if there are n floods. Assume statistical independence between floods.
(c) Determine the probability that the structure will survive over the 10-year period. *Ans. 0.939.*

3.40 Traffic on a one-way street that leads to a toll bridge is to be studied. The volume of the traffic is found to be 120 vehicles per hr *on the average* and out of which $\frac{2}{3}$ are passenger cars and $\frac{1}{3}$ are trucks. The toll at the bridge is \$0.50 per car and \$2 per truck. Assume that the arrivals of vehicles constitute a Poisson process.

(a) What is the probability that in a period of 1 minute, more than 3 vehicles will arrive at the toll bridge? *Ans. 0.1429.*
(b) What is the *expected* total amount of toll collected at the bridge in a period of 3 hr?

3.41 Strikes among construction workers occur according to the Poisson process; on the average there is one strike every 3 years. The average duration of a strike is 15 days, and the corresponding standard deviation is 5 days.

If it costs (in terms of losses) a contractor \$10,000 per day of strike, answer the following.

(a) What would be the expected loss to the contractor during a strike?
(b) If the strike duration is a normal variate, what is the probability that the contractor may lose in excess of \$20,000 during a strike?
(c) In a job that will take 2 years to complete, what would be the contractor's expected loss from possible strikes? (Remember that the occurrence of strikes is a Poisson process.) *Ans. \$100,000.*

3.42 The service stations along a highway are located according to a Poisson process with an average of 1 service station in 10 miles. Because of a gas shortage, there is a probability of 0.2 that a service station would not have gasoline available. Assume that the availabilities of gasoline at different service stations are statistically independent.
(a) What is the probability that there is at most 1 service station in the next 15 miles of highway?
(b) What is the probability that none of the next 3 stations have gasoline for sale?
(c) A driver on this highway notices that the fuel gauge in his car reads empty; from experience he knows that he can go another 15 miles. What is the probability that he will be stranded on the highway without gasoline?

3.43 Express rapid-transit trains run between two points (for example, between downtown terminal and airport). Suppose that the passengers arriving at the terminal and bound for the airport (Fig. P3.43) constitute a Poisson process with an average rate of 1.5 passengers per minute. If the capacity of the train is 100 passengers, how often should trains leave the terminal so that the probability of overcrowding is no more than 10%?
(a) *Formulate the problem* exactly.
(b) Determine an approximate solution by assuming that the number of airport-bound passengers is Gaussian with the same mean and standard deviation as the preceding Poisson distribution.
(c) If the trains depart from the terminal according to the schedule of part (b), what is the probability that in 5 consecutive departures 1 will be overcrowded? Assume statistical independence.

3.44 A large radio antenna system consisting of a dish mounted on a truss (see Fig. P3.44) is designed against wind load. Since damaging wind storms rarely occur, their occurrences may be modeled by a Poisson process. Local weather records show that during the past 50 years only 10 damaging wind storms have been reported. Assume that if damaging wind storm (or storms) occur in this period, the probabilities that the dish and the truss will be damaged in a storm are 0.2 and 0.05, respectively, and that damage to the

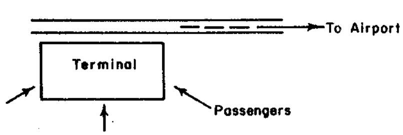


Figure P3.43

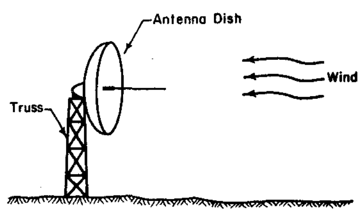


Figure P3.44

dish and truss are statistically independent. Determine the probabilities, during the next 10 years, for the following events.
(a) There will be more than 2 damaging wind storms.
(b) The antenna system will be damaged, assuming the occurrence of at most 2 damaging storms.
(c) The antenna system will be damaged.

3.45 The problem in Example 3.17 may be solved by assuming that whenever the center of a 12-in.-diameter boulder is inside the volume of a cylinder with 15 in. diameter and 50 ft depth, it will be hit by the 3-in. drill hole. On this basis and the assumption that the occurrence of boulders in the soil mass constitutes a Poisson process, develop the corresponding solution procedure for determining the probability of the 3-in. drill hole hitting boulders in a 50-ft depth boring.

3.46 Suppose that the hurricane record for the last 10 years at a certain coastal city in Texas is as follows.

Year	No. of hurricanes
1961	1
1962	0
1963	0
1964	2
1965	1
1966	0
1967	0
1968	2
1969	1
1970	1

The occurrence of hurricanes can be described by a Poisson process. The maximum wind speed of hurricanes usually shows considerable fluctuation. Suppose that those recorded at this city can be fitted satisfactorily by a lognormal distribution with mean = 100 ft/sec and standard deviation = 20 ft/sec.
(a) Based on the available data, find the probability that there will be at least 1 hurricane in this city in the next 2 years. *Ans. 0.798.*
(b) If a structure in this city is designed for a wind speed of 130 ft/sec, what is the probability that the structure will be damaged (design wind speed exceeded) by the next hurricane? *Ans. 0.08.*
(c) What is the probability that there will be at most 2 hurricanes in the next 2 years, and that no structure will be damaged during this period? *Ans. 0.718.*

3.47 Tornadoes may be divided into two types, namely I (strong) and II (weak). From 18 years of record in a city, the number of type I and type II tornadoes are 9 and 54, respectively. The occurrences of each type of tornado are assumed to be statistically independent and constitute a Poisson process.
(a) What is the probability that there will be exactly 2 tornadoes in the city next year?

(b) Assuming that exactly 2 tornadoes actually occurred, and 1 of the 2 is known to be of type I, what is the probability that the other is also type I?

3.48 Figure P3.48*a* shows a record of the earthquake occurrences in a county where a brick masonry tower is to be built to last for 20 years. The tower can withstand an earthquake whose magnitude is 5 or lower. However, if quakes with magnitude more than 5 (defined as damaging quake) occur, there is a likelihood that the tower may fail. The engineer estimated that the probability of failure of the tower depends on the number of damaging quakes occurring during its lifetime, which is described in Fig. P3.48*b*.

(a) What is the probability that the tower will be subjected to less than 3 damaging quakes during its lifetime? Assume earthquake occurrences may be modeled by a Poisson process.

(b) Determine the probability that the tower will not be destroyed by earthquakes within its useful life.

(c) Besides earthquakes, the tower may also be subjected to the attack of tornadoes whose occurrence may be modeled by a Poisson process with mean recurrence time of 200 years. If a tornado hits the tower, the tower will be destroyed. Assume that failures caused by earthquakes and tornadoes are statistically independent. What is the probability that the tower will fail by these natural hazards within its useful life?

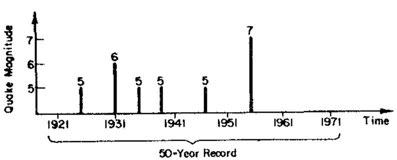


Figure P3.48a

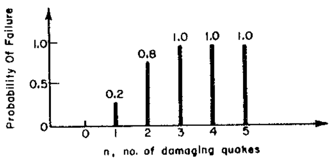


Figure P3.48b

3.49 A skyscraper is located in a region where earthquakes and strong winds may occur. From past record, the mean rate of occurrence of a large earthquake that may cause damage to the building is 1 in 50 years, whereas that for strong wind is 1 in 25 years. The occurrences of earthquake and strong wind may be modeled as independent Poisson processes. Assume that during a strong earthquake, the probability of damage to the building is 0.1, whereas the corresponding probability of damage under strong wind is 0.05. The damages caused by earthquake and wind may be assumed to be independent events.

(a) What is the probability that the skyscraper will be subjected to strong winds but not large earthquakes in a 10-year period? Also, determine the probability of the structure subjected to both large earthquakes and strong winds in the 10-year period.

(b) What is the probability that the building will be damaged in the 10-year period?

3.50 The daily water consumption of a city may be assumed to be a Gaussian random variable with a mean of 500,000 gal/day (gpd), and a standard

deviation of 150,000 gpd. The daily water supply is either 600,000 or 750,000 gallons, with probabilities 0.7 and 0.3, respectively.

(a) What is the probability of water shortage in any given day?

(b) Assuming that the conditions between any consecutive days are statistically independent, what is the probability of shortage in any given week?

(c) On the average, how often would water shortage occur? If the occurrence of water shortage is a Poisson process, what would then be the probability of shortage in a week?

(d) If the city engineer wants the probability of shortage to be no more than 1% in any given day, how much water supply is required?

3.51 Steel construction work on multistory buildings is a potentially hazardous occupation. A building contractor who is building a skyscraper at a steady pace finds that in spite of a strong emphasis on safety measures, he has been experiencing accidents among his large group of steel workers; on the average, about 1 accident occurs every 6 months.

(a) Assuming that the occurrence of a specific accident is not influenced by any previous accident, find the probability that there will be (exactly) 1 accident in the next 4 months.

(b) What is the probability of at least 1 accident in the next 4 months?

(c) What is the mean number of accidents that the contractor can expect in a year? What is the standard deviation for the number of accidents during a period of 1 year?

(d) If the contractor can go through a year without an accident among his steel construction workers, he will qualify for a safety award. What is the probability of his receiving this award next year?

(e) If the contractor's work is to continue at the same pace over the next 5 years, what is the probability that he will win the safety award twice during this 5-year period?

3.52 Two industrial plants are located along a stream (see Fig. P3.52). The solid and liquid wastes that are disposed from the plants into the stream are called effluents. In order to control the quality of the effluent from each plant, there is an effluent standard established for each plant. Assume that each day, the effluent of each plant may exceed this effluent standard with probability $p = 0.2$, during the actual operation. A good measure of the stream quality at A as a result of the pollution from these effluent wastes is given by the dissolved oxygen concentration (DO) at that location. Assume that the DO has a log-normal distribution with the following medians and COV (in mg/l).

Median	COV	
4.2	0.1	when both effluents do not exceed standard
2.1	0.15	when only 1 effluent exceeds standard
1.6	0.18	when both effluents exceed standard

(a) What is the probability that the DO concentration at A will be less than 2 mg/l in any given day?

(b) What is the probability that the DO concentration at A will be less than 2 mg/l in two consecutive days?

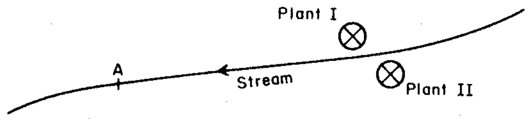


Figure P3.52

(c) It has been proposed as a stream standard that the probability of DO concentration at A falling below 2 mg/l in a day should not exceed 0.1. What should be the allowable maximum value of p (the probability of exceeding the effluent standard for each plant)?

3.53 The daily concentration of a certain pollutant in a stream has the exponential distribution shown in Fig. P3.53.
(a) If the mean daily concentration of the pollutant is 2 mg/10^3 liter, determine the constant c in the exponential distribution.
(b) Suppose that the problem of pollution will occur if the concentration of the pollutant exceeds 6 mg/10^3 liter. What is the probability of pollution problem resulting from this pollutant in a single day?
(c) What is the return period (in days) associated with this concentration level of 6 mg/10^3 liter? Assume that the concentration of the pollutant is statistically independent between days. *Ans. 20 days.*
(d) What is the probability that this pollutant will cause a pollution problem at most once in the next 3 days? *Ans. 0.993.*
(e) If instead of the exponential distribution, the daily pollutant concentration is Gaussian with the same mean and variance, what would be the probability of pollution in a day in this case? *Ans. 0.022.*

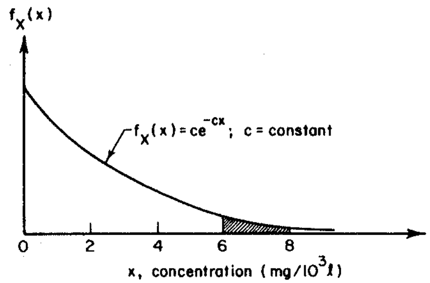


Figure P3.53

3.54 The interarrival times of vehicles on a road follows an exponential distribution with a mean of 15 sec. A gap of 20 sec is required for a car from a side street to cross the road or to join the traffic.
(a) What is the proportion of gaps that are less than 20 sec?
(b) What is the average (mean) interarrival time for all the gaps that are longer than 20 sec?

(c) In 1 hr, what is the expected total time occupied by gaps that are less than 20 sec? (*Hint.* What is the expected number of gaps that are less than 20 sec in 1 hr?)

3.55 The occurrences of tornadoes in a midwestern county may be modeled by a Poisson process with a mean occurrence rate of 2.5 tornadoes per year.
(a) What is the probability that the recurrence time between tornadoes will be longer than 8 months?
(b) Derive the distribution of the time till the occurrence of the second tornado. On the basis of this distribution, determine the probability that a second tornado will occur within a given year.

3.56 The time of operation of a construction equipment until breakdown follows an exponential distribution with a mean of 24 months. The present inspection program is scheduled at every 5 months.
(a) What is the probability that an equipment will need repair at the first scheduled inspection date?
(b) If an equipment has not broken down by the first scheduled inspection date, what is the probability that it will be operational beyond the next scheduled inspection date?
(c) The company owns 5 pieces of a certain type of equipment; assuming that the service lives of equipments are statistically independent, determine the probability that at most 1 piece of equipment will need repair at the scheduled inspection date.
(d) If it is desired to limit the probability of repair at each scheduled inspection date to not more than 10%, what should be the inspection interval? The conditions of part (c) remains valid.

3.57 The cost for the facilities to release and refill water for a navigation lock in a canal increases with decreasing time required for each cycle of operation. For purposes of design, it has been observed that the time of arrival of boats follows an exponential distribution with a mean interarrival time of 0.5 hr. Assume that the navigation lock is to be designed so that 80% of the incoming traffic can pass through the lock without waiting.
(a) What should be the design time of each cycle of operation? *Ans. 0.11 hr.*
(b) What is the probability that of 4 successive arrivals, none of them have to wait at the lock? *Ans. 0.41.*
(c) Suppose that one boat leaves town A every 8 hr, and has to go through the lock to reach its destination. What is the probability that at least 1 of the boats leaving town A in a 24-hr day has to wait at the lock? *Ans. 0.488.*

3.58 A pipe carrying water is supported on short concrete piers that are spaced 20 ft apart as shown in Fig. P3.58*a*. The pipe is saddled on the piers as shown in Fig. P3.58*b*. When subjected to lateral earthquake motions, there is a horizontal inertia force that will tend to dislodge the pipe from its supports. The maximum lateral inertia force F at each pier may be estimated as

$$F = \frac{w}{g} \cdot a$$

where

w = the weight of the pipe and water for a 20-ft section;
g = acceleration of gravity = 32.2 ft/sec^2;
a = maximum horizontal earthquake acceleration.

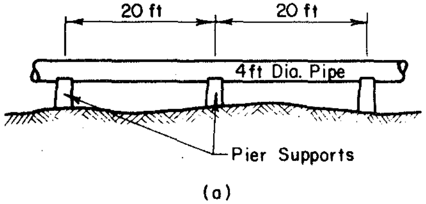


Figure P3.58a

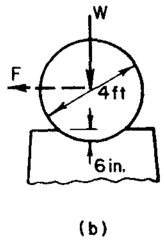


Figure P3.58b

The pipe has a diameter of 4 ft, so that the total weight per foot of pipe and contents is 800 lb per ft. Assume that the maximum acceleration during a strong-motion earthquake is a log-normal variate with a mean of $0.4g$ and a COV of 25%.

(a) What is the probability that during such an earthquake, the pipe will be dislodged from a pier support (by rolling out of the saddle)?
(b) If there are 5 piers supporting the pipe over a ravine, what is the probability that the pipe will not be dislodged anywhere? Assume the conditions between supports to be statistically independent.
(c) If the occurrence of strong-motion earthquakes is a Poisson process, and such earthquakes are expected (on the average) once every 3 years, what is the probability that the pipe may be dislodged from its supports over a period of 10 years?

3.59 Ten percent of the 200 tendons required to prestress a nuclear reactor structure have been corroded during the last year. Suppose that 10 tendons were selected at random and inspected for corrosion; what is the probability that none of the tendons inspected show signs of corrosion? What is the probability that there will be at least one corroded tendon among those inspected?

3.60 The fill in an earth embankment is compacted to a specified CBR (California Bearing Ratio). The entire embankment can be divided into 100 sections, of which 10 do not meet the required CBR.

(a) Suppose that 5 sections are selected at random and tested for their CBR, and acceptance here requires all 5 sections to meet the CBR limit. What is the probability that the compaction of the embankment will be accepted?
(b) If, instead of 5, 10 sections will be inspected and acceptance requires all 10 sections meeting the CBR limit. What is the probability of acceptance now?

Section 3.3

3.61 Both east and west bound rush-hour traffic on a toll bridge are counted at 10-sec intervals. The following table shows the number of observations for

each combination of east and west bound traffic counts:

Number of eastbound vehicles \ Number of westbound vehicles	0	1	2	3	4
0	2	5	15	40	58
1	1	6	15	35	62
2	18	15	28	30	30
3	45	32	25	15	10
4	65	58	35	15	5

Total number of observations = 665

Let X = number of eastbound vehicles in a 10-sec interval.
Y = number of westbound vehicles in a 10-sec interval.

(a) Compute and plot the joint probability mass function of X and Y.
(b) Determine the marginal PMF of X.
(c) If there are 3 eastbound vehicles on the bridge in a 10-sec interval, determine the PMF of westbound vehicles in the same interval.
(d) In a 10-sec interval, what is the probability that 4 vehicles are going east if there are also 4 vehicles going west at the same time?
(e) Determine the covariance Cov (X, Y), and evaluate the corresponding correlation coefficient between X and Y.

3.62 The joint density function of the material and labor cost of a construction project is modeled as follows:

$$f_{X,Y}(x, y) = 2ye^{-y(2+x)} \qquad x, y \geq 0$$
$$= 0 \qquad \text{elsewhere}$$

where X = material cost in \$100,000
Y = labor cost in \$100,000

(a) What is the probability that the material and labor costs of the next construction project will be less than \$100,000 and \$200,000, respectively?
(b) Determine the marginal density function of material cost in a project.
(c) Determine the marginal density function of labor cost in a project.
(d) Are the material and labor costs in the construction project statistically independent? Why?
(e) If it is known that the cost of material in the project is \$200,000, what is the probability that its labor cost will exceed \$200,000?

EXAMPLE 4.17

The capital cost (in $1000) of a combined municipal activated sludge plant may be estimated as follows:

$$C_c = 583Q^{0.84} + (110 + 37Q)\frac{S_o}{200} + (77 + 23Q)\left(\frac{S_s}{200} - 1\right)$$

in which Q is the flow rate in million gallons per day (mgd); S_o is the biological concentration of influent BOD (biological oxygen demand) in milligram per liter (mg/l); and S_s is the concentration of suspended solids (in mg/l).

Suppose that a waste water treatment plant is needed for the following conditions:

mean flow rate, $\bar{Q} = 5$ mgd
mean BOD concentration, $\bar{S}_o = 600$ mg/l
mean concentration of suspended solids, $\bar{S}_s = 200$ mg/l
with coefficients of variation 30%, 20%, and 15%, respectively

Determine the average capital cost of the plant, and corresponding standard deviation.

$$\bar{C}_c \simeq 583(5)^{0.84} + (110 + 37 \times 5)\left(\frac{600}{200}\right) + (77 + 23 \times 5)\left(\frac{200}{200} - 1\right)$$
$$= \$3{,}138{,}219$$

$$\sigma_{C_c}^2 \simeq \left[583(0.84)\bar{Q}^{-0.16} + \frac{37\bar{S}_o}{200} + 23\left(\frac{\bar{S}_s}{200} - 1\right)\right]^2 \sigma_Q^2 + \left(\frac{110 + 37\bar{Q}}{200}\right)^2 \sigma_{S_o}^2 + \left(\frac{77 + 23\bar{Q}}{200}\right)^2 \sigma_{S_s}^2$$
$$= \left[583(0.84)(5)^{-0.16} + \frac{37 \times 600}{200}\right]^2 (1.5)^2 + \left(\frac{110 + 37 \times 5}{200}\right)^2 (120)^2 + \left(\frac{77 + 23 \times 5}{200}\right)^2 (30)^2$$
$$= 539{,}213 + 31{,}329 + 829$$
$$= 571{,}371$$

Therefore, $\sigma_{C_c} \simeq \$756{,}000$ and the COV is $\delta_{C_c} = 0.24$.

4.4. CONCLUDING REMARKS

In this chapter, we saw that the probabilistic characteristics of a function of random variables may be derived from those of the basic constituent variables. These include, in particular, the probability distribution and the main descriptors (mean and variance) of the function. The derivation of the distribution, however, may be complicated mathematically, especially for nonlinear functions of multiple variables. Therefore, even though the required distribution may (theoretically) be derived, they are often impractical to use, except for special cases (for instance, linear functions of independent normal variates). In view of this, it is often necessary, in many applications, to describe the function in terms only of its mean and variance. Even then, the mean and variance of linear functions are amenable to exact evaluation; however, for a general nonlinear function, (first-order) approximations must often be resorted to. In this chapter, we have introduced and developed the elements for such first-order analysis; these concepts will form the basis for the formal analysis of uncertainty covered in Vol. II.

PROBLEMS

Section 4.2

4.1 The force in the cable of the truss shown in Fig. P4.1, when subjected to a load W, is given by

$$F_{ac} = \frac{\sqrt{h^2 + l^2}}{h} W$$

(a) If the load W is a normal variate $N(\mu_W, \sigma_W)$, derive the density function of the force F_{ac}.
(b) If $\mu_W = 20$ metric tons, $\sigma_W = 5$ metric tons, and $h = \frac{1}{2}l$, what is the probability that the force F_{ac} will exceed 30 tons? *Ans. 0.907.*

4.2 A dike is proposed to be built to protect a coastal area from ocean waves (see Fig. P4.2). Assume that the wave height H is related to the wind velocity by the equation

$$H = 0.2V$$

where H is in meters and V is velocity in kilometers per hour (kph). The annual maximum wind velocity is assumed to have a log-normal distribution with a mean of 80 kph and a coefficient of variation of 15%.
(a) Determine the probability distribution of the annual maximum wave height and its parameters.
(b) If the dike is designed for a 20-year wave height, what is the design height of the dike?
(c) With this design, what is the probability that the dike will be topped by waves within the first three years?

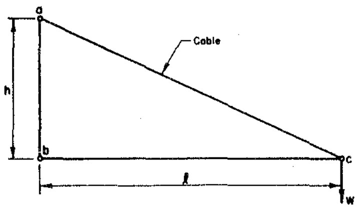


Figure P4.1

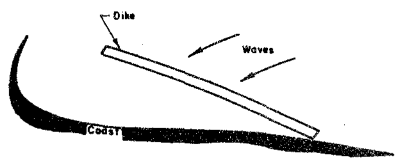


Figure P4.2

4.3 In Example 4.14, the maximum wave pressure on structures is given as

$$p_{max} = 2.7 \frac{\rho K}{D} U^2$$

where U is the horizontal velocity of the advancing wave.

(a) If U has a log-normal distribution with parameters λ_U and ζ_U, derive the distribution of p_{max} using Eq. 4.8.

(b) Using the data given in Example 4.14, determine the probability that the maximum impact pressure will exceed 40 psi. *Ans. 0.121.*

4.4 The hydraulic head loss h_L in a pipe due to friction may be given by the Darcy-Weisbach equation

$$h_L = f \frac{L}{2gD} V^2$$

where L and D are, respectively, the length and diameter of the pipe; f is the friction factor; and V is the velocity of flow in the pipe. If V has an exponential distribution with a mean velocity v_0, derive the density function for the head loss h_L.

4.5 From the statistics collected for towns and cities in Illinois the average consumption of water, in gallons per capita per day, is found to increase with the size of population P as follows:

$$X = 19.5 \ln \frac{P}{40} - 17 \qquad \text{for} \quad P > 1000$$

Suppose that the population in 1974 for a certain developing town can be described by a log-normal distribution with a mean of 10,000 and a COV of 5%. It is expected that the median of the population will grow at 10% (of the 1974 population) per year, while the COV will remain roughly constant (see Fig. P4.5).

(a) Assume that the distribution of population is log-normal at any future time; determine the distribution of X, the average per capita water consumption in 1984.

(b) Determine approximately the mean and variance of D, the average total daily demand of water in 1984. *Ans.* 2.08×10^6; 1.53×10^{10}.

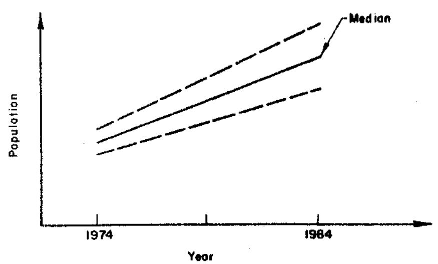


Figure P4.5

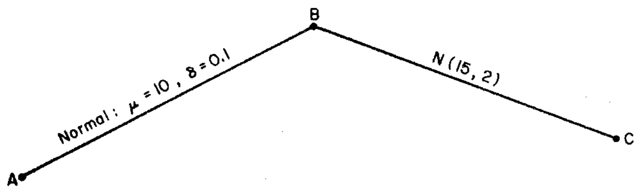


Figure P4.6

4.6 The time of travel between cities A and B is a normal variate with a mean value of 10 hr and a coefficient of variation of 0.10 (see Fig. P4.6). The time of travel between cities B and C is also normal with a mean and standard deviation of 15 and 2 hr, respectively. Assume that these two travel times are statistically independent.

(a) Determine the density function of the travel time between cities A and C going through B if there is exactly 2 hr of waiting in city B.

(b) What is the probability that the time of travel between A and C will exceed 30 hr; will be less than 20 hr?

4.7 A simple structure consisting of a cantilever beam AB and a cable BC is used to carry a load S (see Fig. P4.7). The magnitude of the load varies daily, and its monthly maximum has been observed to be Gaussian with a mean of 25,000 kg, and a coefficient of variation of 30%.

(a) If the cable BC and beam AB are designed to withstand a 10-month maximum load (that is, a maximum load with a return period of 10 months) with factors of safety of 1.25 and 1.40, respectively, what are the probabilities of failure of the cable and of the beam?

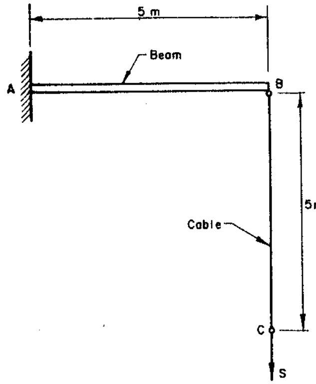


Figure P4.7

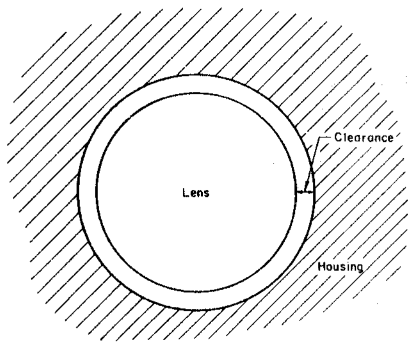


Figure P4.9

(b) Assuming statistical independence between the failures of the beam and cable, what is the probability of failure of the structure (that is, that it will be unable to carry the load)?
(c) If (instead of part [a]) the strength of the cable were random $N(50{,}000$ kg; $10{,}000$ kg$)$, what would be its failure probability under the load S?

4.8 The occurrences of hurricanes in a Texas county is described by a Poisson process. Suppose that 32 hurricanes have occurred in the last 50 years; 28 of the 32 hurricanes occurred in the hurricane season (August 1 to November 30).
(a) For this Texas county, estimate the mean rate of occurrence of hurricanes (i) per year; (ii) per month in the hurricane season; (iii) per month in the nonhurricane season.
(b) A temporary offshore structure is to be located off the coast of this county and it is expected that the structure will operate for 19 months between April 1 and October 31 of the following year. What is the mean number of hurricanes that will occur in this period of time?
(c) What is the probability that this structure will be hit by hurricanes during its period of operation?
(d) Suppose that whenever a hurricane occurs, the owner of the structure will incur a loss of \$10,000, which includes repairs for damage, loss of revenue, and so on. What is the owner's *expected total loss* from hurricanes? The total loss T (in dollars) is given by

$$T = 10{,}000\, N$$

where N is the number of hurricanes during the period of operation, which is assumed to be a Poisson random variable.
(e) What is the probability that the total loss T will *exceed* \$10,000?

4.9 To insure proper mounting of a lens in its housing in an aerial camera, a clearance of not less than 0.10 cm and not greater than 0.35 cm is to be allowed. The clearance is the difference between the radius of the housing and the radius of the lens (see Fig. P4.9).
A lens was produced in a grinder whose past records indicate that the radii of such lenses can be regarded as a normal variate with mean of 20.00 cm and a coefficient of variation of 1%.
A housing was manufactured in a machine whose past records indicate that the radii of such housing can be regarded as a normal variate with mean of 20.20 cm and a coefficient of variation of 2%. What is the probability that the specified clearance will be met for this pair of lens and housing? *Ans. 0.216.*

4.10 The safety of a proposed design for the slope shown in Fig. P4.10 is to be analyzed. Suppose that the circular arc AB (with center at 0) represents the potential failure surface and that the wedge of soil contained within the arc will slide if the clockwise moment about point 0 due to the weight of the soil W exceeds the counterclockwise moment provided by the frictional forces F_1 and F_2. The following information is given:

	Mean (kips)	Standard deviation (kips)
W	400	60
F_1	100	30
F_2	300	60

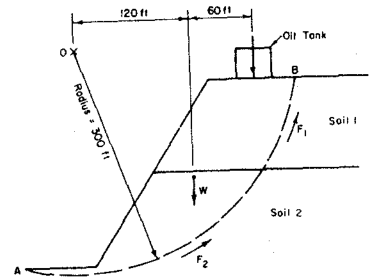


Figure P4.10

(a) Let M_R = total resisting (counterclockwise) moment. Determine $E(M_R)$, $Var\,(M_R)$.
(b) What is the probability that sliding along the arc AB will occur? Assume that W, F_1, F_2 are statistically independent normal random variables. *Ans. 0.000376.*
(c) An oil tank is proposed to be located as shown in Fig. P4.10. If the maximum permissible probability of sliding failure is 0.01, how heavy can the oil tank be? *Ans. 123.3 kips.*

4.11 The water supply to a city comes from two sources—namely, from the reservoir and from pumping underground water, as shown in Fig. P4.11. For the next 3 months, the amounts of water available from each source are independently Gaussian $N(30, 3)$ and $N(15, 4)$, respectively, in million gallons. Suppose that the demand in the next 3 months can be described by the probability mass function given in Fig. P4.11.
(a) Determine the probability that there will be insufficient supply of water in the next 3 months.
(b) Repeat part (a) if the demand is also Gaussian with the same mean and variance as those of Fig. P4.11.

4.12 The traffic on a bridge may be described by a Poisson process with mean

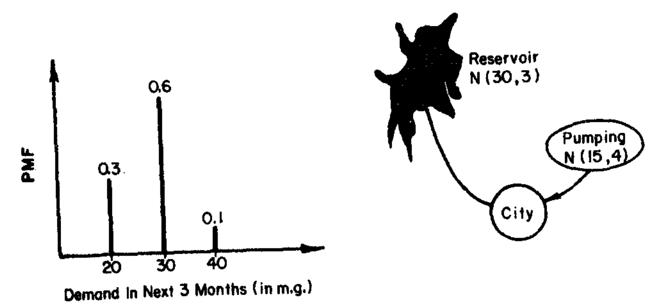


Figure P4.11

arrival rate of 18 vehicles per minute. The vehicles may be divided into two types: trucks and passenger cars. The weight of a truck is $N(15, 5)$ when empty and $N(30, 7)$ when loaded, whereas that of a passenger car is $N(2, 1)$. All units are in tons. Trucks make up only 20% of the total traffic and half the trucks are loaded. The weights of the vehicles are statistically independent.

(a) What is the probability that a vehicle observed at random will not exceed 5 tons? *Ans. 0.801.*

(b) What is the probability that 3 vehicles in a row will each exceed 5 tons?

(c) How would the probability in part (b) change if it is known that 2 vehicles are passenger cars and the remaining one is a truck?

(d) If there are 3 passenger cars and one empty truck on the bridge, what is the probability that the total vehicle load on the bridge will exceed 30 tons? *Ans. 0.0446.*

(e) What is the probability that there will be exactly one truck but no passenger cars arriving within a 10-second interval? *Ans. 0.03.*

4.13 The pole shown in Fig. P4.13 is acted upon by two loads P_1 and P_2 so that the bending moment at the bottom of the pole is

$$M_A = 30P_1 - 20P_2$$

Here, P_1 and P_2 are independent Gaussian random variables with the following parameters.

Load	Mean (kips)	Standard deviation (kips)
P_1	50	5
P_2	20	3

(a) Determine the mean and standard deviation of the moment M_A at the base of the pole.

(b) If the moment-resisting capacity at the bottom of the pole is M_R, a Gaussian random variable with a mean of 1750 ft-kips and a standard deviation of 150 ft-kips, what is the probability that the pole will fail under the loads P_1 and P_2?

(c) Five such poles are arranged in line to support a bank of critical electrical equipment. Adequate support of the equipment requires at least 3 adjacent poles. What is the probability of survival of the system?

(d) In contrast to part (c), what is the probability that exactly 3 poles will fail (regardless of the positions of the poles)?

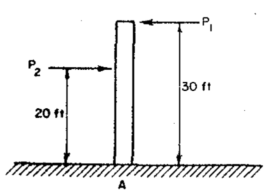


Figure P4.13

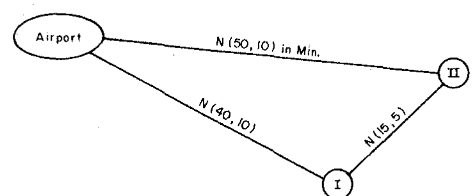


Figure P4.15

4.14 A traffic survey on the various modes of transportation between New York and Boston shows that the percentage of total trips by air, rail, bus, and car are 15.3, 10.6, 9.4, and 64.7 percent, respectively. The distributions of the trip times in each mode are approximately Gaussian with means 0.9, 4.5, 4.8, 4.5 hr, and coefficients of variation 0.15, 0.1, 0.15, 0.2, respectively.

(a) What proportion of trips between New York and Boston can be completed in 4 hr?

(b) What is the probability that transportation by bus is faster than by car between these two cities?

4.15 The feasibility of an airport location is to be evaluated. Among many other criteria, one of them is to minimize the travel time from the city to the airport. For simplicity, assume that the city may be subdivided into 2 regions, I and II, each with independent Gaussian travel times to the airport as indicated in Fig. P4.15 in minutes. The ratio of air passengers originating from the two regions is 7 to 3.

(a) What percentage of the passengers will take more than 1 hr to get to the airport? *Ans. 0.0635*

(b) A limousine service departs from the airport and picks up or unloads passengers at I and II, consecutively, before returning to the airport. Assume that the travel time for the limousine between stops is also Gaussian as shown in Fig. P4.15. What is the probability that the limousine will complete a round trip within 2 hr? Is this equal to the probability of making 2 rounds within 4 hours? Justify your answer. Assume that the travel times between rounds are also statistically independent.

(c) A passenger is waiting at the airport for his friend so that they may leave together at a 9 A.M. flight. At 8:50 A.M., he still has not seen his friend. He becomes impatient and calls his friend's home at region II. If his friend had left home at 7:50 A.M., what is the probability that his friend will arrive at the airport in the next 10 minutes? *Ans. 0.857.*

4.16 The existing sewer network shown in Fig. P4.16 consists of pipes AC, BC, CD, ED, DF. The mean inflows from A, B, E are 30, 10, 20 cfs, respectively. Suppose that the flow capacity of pipe DF is 70 cfs, and all the other pipes can adequately handle their respective flows. Assume that the inflows are statistically independent normal variates with 10% COV.

(a) What is the probability that the capacity of pipe DF will be exceeded in the existing network?

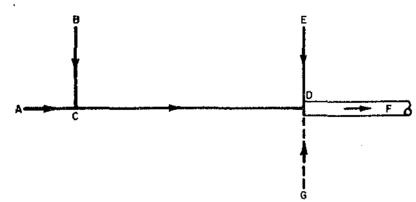


Figure P4.16

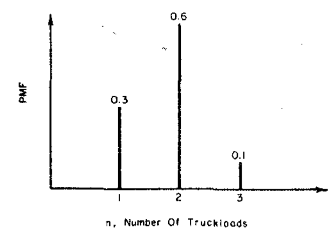


Figure P4.17

(b) Suppose that the sewer from a newly developed area is proposed to be hooked up to the present system at D, and this additional inflow is also Gaussian with a mean of 30 cfs and COV 10%. How should pipe DF be expanded (that is, what should be the new capacity of pipe DF) so that the risk of flow exceedance remains the same as that in the present system?

4.17 The number of truck loads of solid waste arriving at a waste treatment plant in the next hour is random with the PMF given in Fig. P4.17.
The time required for processing each truckload of solid waste is Gaussian $N(10, 2)$ in minutes.

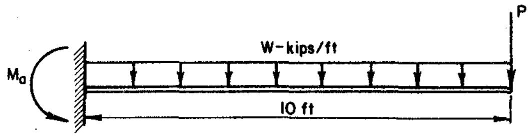


Figure P4.18

What is the probability that the total time needed for processing the solid waste arriving in the next hour will be less than 25 minutes? Assume that the processing times are statistically independent. *Ans. 0.884.*

4.18 The cantilever beam shown in Fig. P4.18 is subjected to a random concentrated load P and a random distributed load W.
Assume

$$P \text{ is } N(5, 1), \text{ in kips}$$
$$W \text{ is } N(1, 0.2), \text{ in kips/ft}$$

(a) Determine the mean and variance of the applied bending moment $M_a = 50W + 10P$. Assume that $\rho_{W,P} = 0.5$ (that is, the loads are correlated).

(b) The resisting moment of the beam M_r, which is statistically independent of the applied moment M_a, is also Gaussian $N(200, 50)$ in ft-kips. Determine the probability of failure of the beam, $P(M_r < M_a)$ assuming that M_a is Gaussian.

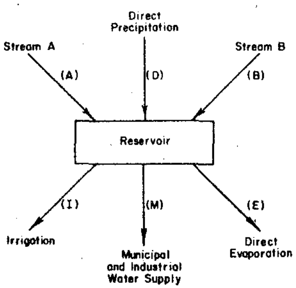


Figure P4.19

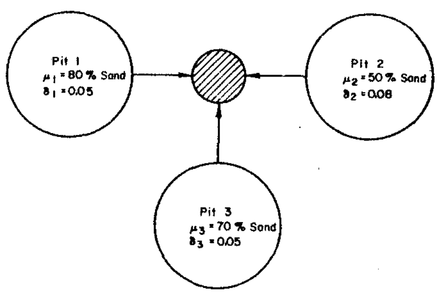


Figure P4.20

4.19 There are three sources of inflow into the reservoir, namely, streams A and B and direct precipitation D, as shown in Fig. P4.19. Each of these three sources depends on the total rainfall R in the watershed surrounding the reservoir. The following are relationships between A, B, D, and R:

$$A = 0.2R + 0.3$$
$$B = 0.15R + 0.4$$
$$D = 0.03R$$

All are in million gallons (mg). The rainfall R for the next 3-month period is assumed to have a normal distribution $N(15, 2)$ in inches. The outflows from the reservoir consist of irrigation I, municipal and industrial use M, and loss through direct evaporation E. During the next 3 months, each of these three outflows is a normal random variable with

$$I = N(1.5, 0.3)$$
$$M = N(1, 0.1) \qquad \text{all in mg}$$
$$E = N(2.5, 0.4)$$

Let T denote the total inflow into the reservoir for the next 3-month period. Assume that T, I, M, E are statistically independent.

(a) Are A, B, D statistically independent? Why?

(b) Determine $E(T)$, $Var(T)$. What is the distribution of T?

(c) Assume that the present storage of the reservoir is 30 mg. Let S be the storage of the reservoir 3 months from now. Determine: $E(S)$, $Var(S)$. What is the distribution of S?

(d) What is the probability that there will be an increase in the reservoir storage 3 months from now?

4.20 A concrete mixing plant obtains sand and gravel mixtures from 3 gravel pits. The mean percentages of sand by weight in each pit are 80, 50, and 70, respectively, and the coefficients of variation of the percentage of sand are 0.05, 0.08, and 0.05, respectively (see Fig. P4.20). Assume that gravel makes up the remaining percentage by weight. Two, three, and five units of sand-gravel mixture are delivered, respectively, from the three pits and are mixed together. What is the probability that in the resultant mixture, the ratio of sand to gravel by weight does not exceed 2.5 to 1 and also does not fall below 1.5 to 1? Note that these two limits may represent the tolerable sand-to-gravel ratio for acceptable concrete aggregate. Assume that the contents of the pits are statistically independent. *Ans. 0.987.*

4.21 A catch basin is used to control flooding of a region. Aside from serving the immediate neighborhood, it also receives the storm water from another district through a storm sewer.
Suppose that the catch basin has a storage capacity of 50 in. of water; also, any water in the basin is drained at the average rate of 2.5 in. per minute with a COV of 20%.
The rate of inflow from the two sources of drainage water during a rainstorm is as follows.

	Mean rate (in. per minute)	Standard deviation (in. per minute)
Immediate neighborhood:	2	1
Distant district:	1.5	0.5

Assume that all variates are independent and normal, and that the rates of inflow and outflow are constant with time.

(a) Determine the mean and standard deviation of the rate of filling (per minute) of the catch basin.
(b) Determine the probability of flooding (basin capacity exceeded) in 30 minutes of rain (assume that the basin is dry before it rains).
(c) The probability of flooding may be decreased by increasing the capacity of the catch basin. If it is decided to decrease this probability to no more than 10% during a 30-minute rain, what should the catch basin capacity be?

4.22 A plain concrete column is subjected to an axial load W that is a log-normal variate with mean $\bar{W} = 3000$ kN (kilo Newton) and COV $\delta_W = 0.20$ (see Fig. P4.22).
The mean crushing strength of the concrete is $\bar{s}_c = 35{,}000\ \text{kN/m}^2$ (kilo Newtons/square meter) with COV $\delta_{s_c} = 0.20$. Assume uniform compressive stress over the cross-sectional area of the column, so that the applied stress is

$$s = \frac{W}{A}$$

where A = cross-sectional area of column.

(a) What is the density function of the applied stress?
(b) Determine the probability of crushing of a 0.40 m × 0.40 m column. Assume a convenient probability distribution for s_c.
(c) If a failure (crushing) probability of 10^{-3} is permitted, determine the required cross-sectional area of the column.
(d) Derive the expression for the allowable design stress corresponding to a permissible risk or failure probability p_F (see Example 4.11).

4.23 Figure P4.23 shows a schematic procedure of the treatment system for the waste from a factory before it is dumped into a nearby river. Here X denotes the concentration of a pollutant feeding into the treatment system, and Y denotes the concentration of the same pollutant leaving the system. Suppose that for a normal day, X has a log-normal distribution with median 4 mg/l and the COV is 20%. Because of the erratic nature of biological and chemical reactions, the efficiency of the treatment system is unpredictable. Hence the fraction of the influent pollutant remaining untreated, denoted by F, is also a random variable. Assume F is also a log-normal variate with a median of 0.15 and COV of 10%. Assume X and F are statistically independent.

(a) Determine the distribution of Y and the values of its parameters. Note that

$$Y = FX$$

(b) Suppose that the maximum concentration of the pollutant permitted to

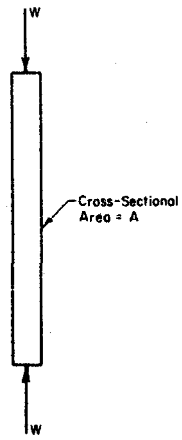


Figure P4.22

Figure P4.23

be dumped into the river is specified to be 1 mg/l. What is the probability that this specified standard will be exceeded on a normal day?

(c) On some working days, owing to heavy production in the factory, the influent X will have a median of 5 mg/l instead. Assume that the distribution of X is still log-normal with the same COV and that the efficiency of the treatment system does not change statistically. Suppose that such a heavy work day happens only 10% of the time. Then, on a given day selected at random, what is the probability that the specified standard of 1 mg/l for Y will be exceeded?

4.24 You are taking a plane from O'Hare to Kennedy Airport. Being conscious of the congested conditions at O'Hare, you would like to find out your chance of delay. Based on available data, the delay time (beyond the scheduled departure time) at O'Hare is an exponential random variable; its mean value depends on the weather condition as shown in Fig. P4.24.

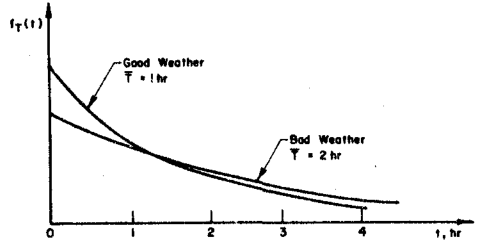


Figure P4.24 Waiting time at O'Hare

The relative likelihood of good and bad weather conditions at O'Hare is about 3 to 1.

(a) What is the probability that your delay at O'Hare will be at least 1.5 hr beyond the scheduled departure time? *Ans. 0.285.*

(b) The delay in landing at Kennedy Airport on a good weather day also has an exponential distribution with a mean of 1/2 hr. What is the probability that the total delay in arrival would be more than 2 hr if the weather is good at both O'Hare and Kennedy airports? Assume that there is no delay in flight. (*Hint.* Derive the density function for the sum of two exponential variates using Eq. 4.16*a*.) *Ans. 0.252.*

4.25 A construction project consists of two major phases, namely, the construction of the foundation and the superstructure. Let T_1 and T_2 denote the respective durations of each phase. Assume that T_1 and T_2 are independent exponential random variables with $E(T_1) = 2$ months and $E(T_2) = 3$ months. Assume also that the superstructure phase will start only after the foundation phase has been completed.

(a) What is the probability that the project will be completed within 6 months?

(b) If T_1 and T_2 have the same exponential distribution with $E(T_1) = E(T_2) = \lambda$, show that the project duration has a gamma distribution.

4.26 The mean number of arrivals at an airport during rush hour is 20 planes per hour whereas the mean number of departures is 30 planes per hour. Suppose that the arrivals and departures can each be described by a respective Poisson process. The number of passengers in each arrival or departure has a mean of 100 and COV of 40%.

(a) What is the probability that there will be a total of two arrivals and/or departures in a 6-minute period?

(b) Suppose that in the last hour there have been 25 arrivals.

(i) What is the mean and variance of the total number of arriving passengers in the last hour?

(ii) What is the probability that the total number of arriving passengers exceeded 3000 in the last hour? State and justify any assumption you may make.

4.27 The total distance between A and B is composed of the sum of 124 independent measurements. See Fig. P4.27. The random error E in each measurement is uniformly distributed between ± 1 in. If the total distance AB is approximately 2 miles and the lengths of the segments are approximately equal, compute the following.

(a) The mean and variance of the error in each measurement.

(b) The probability that the error in each measurement is not more than 0.01% of the actual length of the segment.

(c) The mean and variance of the total error in the distance AB.

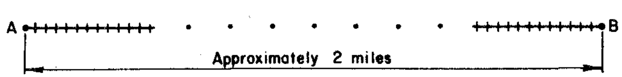


Figure P4.27

(d) The probability that the total error is not more than 0.01% of the actual distance AB.

Section 4.3

4.28 A cylindrical volume has an average measured outside diameter of $D_o = 5$ m and an average inside diameter of $D_i = 3$ m, and a height of $H = 10$ m (see Fig. P4.28). If the COV of these measurements are, respectively, 2%, 1%, and 1%, what are the mean and variance of the volume, if D_o and D_i are perfectly correlated (that is, $\rho_{D_o,D_i} = 1.0$) whereas these are statistically independent of H?

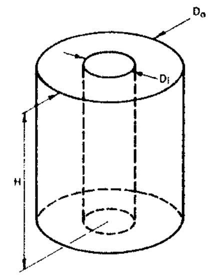


Figure P4.28

4.29 Small flaws (cracks) in metals grow when subjected to cyclic stresses. The rate of crack growth (per load cycle) may be given by

$$\frac{dA}{dn} = C(\Delta K)^m$$

where

$$\Delta K = \Delta S\sqrt{\pi A}$$
$$A = \text{existing crack size}$$
$$\Delta S = \text{applied stress increment}$$

and C and m are constants. If

$$C = 0.5 \times 10^{-5} \qquad m = 2$$

and

$$\bar{A} = 0.1 \text{ in.} \qquad \text{with } \delta_A = 20\%$$
$$\overline{\Delta S} = 50 \text{ ksi} \qquad \text{with } \delta_{\Delta S} = 30\%$$

determine the mean and COV of the crack growth rate per load cycle. Assume that A and ΔS are statistically independent. *Ans. 0.00392; 0.633.*

4.30 The range R of a projectile is given by the following:

$$R = \frac{v_0^2}{g}\sin 2\phi$$

where g is the gravitational acceleration, v_0 is the initial velocity of the

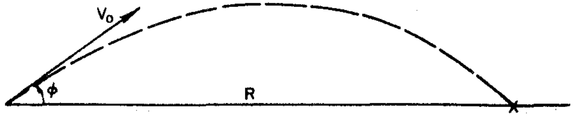


Figure P4.30

projectile, and ϕ is its direction from the horizontal (see Fig. P4.30). If

$$\bar{\phi} = 30^\circ \quad \text{and} \quad \delta_\phi = 5\%$$
$$\bar{v}_0 = 500 \text{ ft/sec} \quad \text{and} \quad \sigma_{v_0} = 50 \text{ ft/sec}$$

determine the first-order mean and standard deviation of the range R. Assume that $g = 32.2$ ft/sec^2; ϕ and v_0 are statistically independent. Evaluate also the second-order mean range. *Ans. 6723.6; 1359.8; 6781.6 ft.*

4.31 The number of airplanes arriving over Chicago O'Hare Airport during the peak hour from various major cities in the United States are listed below.

City	Average number of arrivals	Standard deviation
New York	5	2
Miami	3	1
Los Angeles	4	2
Washington, D.C.	4	1
San Francisco	4	2
Dallas	2	0
Seattle	3	1

Suppose (hypothetically) that the holding time T (in minutes) at O'Hare Airport is a function of the number of arrivals from the above cities; specifically,

$$T = 4\sqrt{N_A}, \text{ in minutes}$$

where N_A is the total number of arrivals (during the peak hour) from the cities listed above.
Assuming a log-normal distribution for T, determine the probability that the holding time will exceed 25 minutes. Assume that arrivals from different cities are statistically independent.

4.32 In a study of noise pollution, the noise level at C transmitted from two noise sources as shown in Fig. P4.32 is analyzed. Suppose that the intensities of the noise originating from A and B are statistically independent and denoted as I_A and I_B, with mean value 1000 and 2000 units, respectively, and the coefficient of variation is 10% for both I_A and I_B. Since the noise intensity decreases with distance from the source, the following equation has been suggested:

$$I(x) = \frac{I}{(x+1)^2}$$

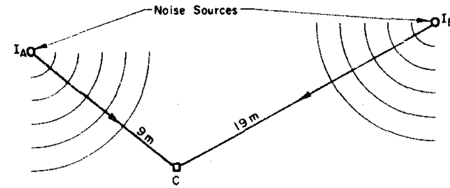


Figure P4.32

where

$$I = \text{intensity generated at a source}$$
$$I(x) = \text{intensity at distance } x \text{ from the source}$$

(a) Let I_C be the noise intensity at C, which is the sum of the two intensities transmitted from A and B. Determine $E(I_C)$, $Var(I_C)$.

(b) A common measure of noise intensity is in terms of decibels. Suppose that the number of decibels D is expressed as a function of intensity I as

$$D = 40 \ln 2I$$

Determine the approximate mean and variance of D_C, that is, the number of decibels at C.

4.33 The velocity of uniform flow, in feet per second, in an open channel is given by the Manning equation

$$V = \frac{1.49}{n} R^{2/3} S^{1/2}$$

where

$$S = \text{slope of the energy line}$$
$$R = \text{hydraulic radius, in feet}$$
$$n = \text{roughness coefficient of the channel}$$

Consider a rectangular open channel with concrete surface ($\bar{n} = 0.013$); R is estimated to be 2 ft (average value) and the average slope S is 1%. Because the determinations of R, S, and n are not very precise, the uncertainties associated with these values, expressed in terms of COV, are as follows: $\delta_R = 0.05$, $\delta_S = 0.10$, and $\delta_n = 0.30$.
Determine the first-order mean value of the flow velocity V, and the underlying uncertainty in terms of COV. Evaluate also the second-order approximation for the mean flow velocity.

4.34 The settlement of a column footing, shown in Fig. P4.34*a*,

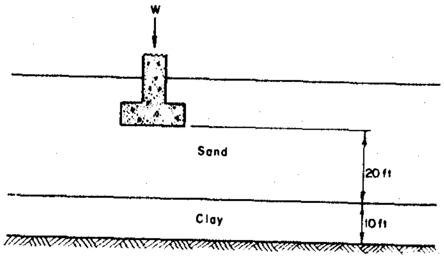


Figure P4.34a

is composed of two components—the settlements of the sand and clay strata. The flexibilities (that is, inches settlement per foot of strata per ton of applied load) of the two strata, denoted F_S and F_C, are independent normal variates $N(0.001, 0.0002)$ and $N(0.008, 0.002)$, respectively. The total column load is W, which may be assumed to be statistically independent of F_S and F_C.

(a) If $W = 20$ tons, what is the probability that the total settlement will exceed 3 in.? *Ans. 0.007.*

(b) Suppose the load W is also a random variable with the PMF given in Fig. P4.34*b*.

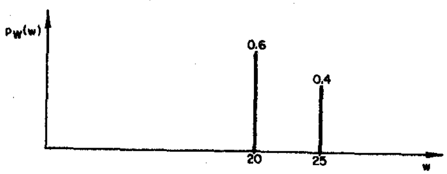


Figure P4.34b

With this PMF of W, determine the mean and variance of the total settlement by first-order approximation. In this case, what would be the probabily that the settlement will exceed 3 in. assuming that settlement follows a normal distribution? *Ans. 2.2; 0.51; 0.058.*

5. Estimating Parameters From Observational Data

5.1. THE ROLE OF STATISTICAL INFERENCE IN ENGINEERING

We have seen in the previous chapters that once we know (or assume) the distribution function of a random variable and the values of its parameters, the probabilities associated with events defined by values of the random variable can be computed. The calculated probability is clearly a function of the values of the parameters, as well as of the assumed form of distribution. Naturally, questions pertaining to the determination of the parameters, such as the mean value μ and variance σ^2, and the choice of specific distributions are of interest.

Answers to these questions often require observational data. For example, in determining the maximum wind speed for the design of a tall building, past records of measured wind velocities at or near the building site are pertinent and important; similarly, in designing a left-turn lane at an existing highway crossing, a traffic count of left turns at the intersection may be required. Based on these observations, information about the probability distribution may be inferred, and its parameters estimated statistically.

In many geographic regions, data on natural processes, such as rainfall intensities, flood levels, wind velocity, earthquake frequencies and magnitudes, traffic volumes, pollutant concentrations, ocean wave heights and forces, have been and continue to be collected and reported in published records. Field and laboratory data on the variabilities of concrete strength, yield strength of steel, fatigue lives of materials, shear strength of soils, efficiency of construction crews and equipment, measurement errors in surveying, and many others, continue to be collected. These statistical data provide the information from which the probability model and the corresponding parameters required in engineering design may be developed or evaluated.

The techniques of deriving probabilistic information and of estimating parameter values from observational data are embodied in the methods of

Then the confidence interval for p is obtained from

$$P\left(-k_{\alpha/2} < \frac{\hat{p} - p}{\sqrt{\hat{p}(1-\hat{p})/n}} \leq k_{\alpha/2}\right) = 1 - \alpha \tag{5.46}$$

giving the $(1 - \alpha)$ confidence interval as

$$\langle p \rangle_{1-\alpha} = \left(\hat{p} - k_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\ ;\ \hat{p} + k_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) \tag{5.47}$$

Figure 5.10 is a graph showing the 95% confidence interval for p as a function of the observed proportion $\hat{p}$ for different sample size n.

EXAMPLE 5.14

In inspecting the quality of soil compaction in a highway project, 10 out of 50 specimens inspected do not pass the CBR requirement. It is desired to estimate the actual proportion p of embankment that will be well compacted (that is, meet CBR requirement) and also establish a 95% confidence interval on p.

The point estimate for p is given by Eq. 5.43 as

$$\hat{p} = \frac{40}{50} = 0.8$$

The corresponding 95% confidence interval is, according to Eq. 5.47,

$$\langle p \rangle_{0.95} = \left\{0.8 - 1.96\sqrt{\frac{0.8(1-0.8)}{50}}\ ;\ 0.8 + 1.96\sqrt{\frac{0.8(1-0.8)}{50}}\right\}$$
$$= \{0.69; 0.91\}$$

5.3. CONCLUDING REMARKS

In modeling real-world situations, the form of the probability distribution of a random variable may be deduced theoretically on the basis of physical considerations or inferred empirically on the basis of observational data. However, the parameters of the distribution or the main descriptors (mean and variance) of the random variable must necessarily be related empirically to the real world; therefore, estimation based on factual data is required. Classical methods of parameter estimation are presented in this chapter; in Chapter 6 empirical and inferential methods for determining probability distributions are described.

Classical methods of estimation are of two types—*point and interval* estimations. The common methods of point estimation are the *method of maximum likelihood* and the *method of moments*; the former derives the estimator directly; the latter evaluates a parameter by first estimating the moments (usually the mean and variance) of the variate through the corresponding sample moments. Interval estimation includes a determination of the interval that contains the parameter value with a prescribed level of confidence.

It should be recognized that when population parameters are estimated on the basis of finite samples, errors of estimation are unavoidable. Within the classical methods of estimation, the significance of such errors are not reflected in the (point) estimates of the parameters; they can only be expressed in terms of appropriate confidence intervals. Explicit consideration of these errors is embodied in the Bayesian approach to estimation, which is the subject of Chapter 8.

PROBLEMS

5.1 In the measurement of daily dissolved oxygen (DO) concentrations in a stream, let p denote the probability that the DO concentration will fall below the required standard on a single day. DO concentration is measured daily until unsatisfactory stream quality is encountered, and the number of days in this sequence of measurement is recorded. Suppose 10 sequences have been observed and the length of each sequence is

2, 5, 6, 4, 6, 6, 8, 5, 10, 1 (days)

(a) Determine the maximum likelihood estimator for p, and estimate p on the basis of the observed data.
(b) Estimate p by the method of moments. (*Hint*. Use the relations in Table 5.1).

5.2 For the concrete crushing strength data tabulated in Table E5.1 in Example 5.1, determine the point estimates for μ and σ by the method of maximum likelihood. Assume that concrete strength follows a Gaussian distribution.

5.3 The distribution of wave height has been suggested to follow a Rayleigh density function,

$$f_H(h) = \frac{h}{\alpha^2} e^{-\frac{1}{2}(h/\alpha)^2} \quad h \geq 0$$

with parameter α. Suppose the following measurements on wave heights were recorded: 1.5, 2.8, 2.5, 3.2, 1.9, 4.1, 3.6, 2.6, 2.9, 2.3 m.
Estimate the parameter α by the method of maximum likelihood.

5.4 Data on rainfall intensities (in inches) collected between 1918 and 1946 for the watershed of Esopus Creek, N.Y., are tabulated below as follows:

1918—43.30	1925—43.93	1932—50.37	1939—42.96
1919—53.02	1926—46.77	1933—54.91	1940—55.77
1920—63.52	1927—59.12	1934—51.28	1941—41.31
1921—45.93	1928—54.49	1935—39.91	1942—58.83
1922—48.26	1929—47.38	1936—53.29	1943—48.21
1923—50.51	1930—40.78	1937—67.59	1944—44.67
1924—49.57	1931—45.05	1938—58.71	1945—67.72
			1946—43.11

(a) Determine the point estimates for the mean μ and variance σ^2.
(b) Determine the 95% confidence interval for the mean μ. Assume the annual rainfall intensity is Gaussian, and $\sigma \simeq s$.

5.5 Consider the annual maximum wind velocity (V) data given in Problem 3.25.
(a) Calculate the sample mean and sample variance of V.
(b) Determine the 99% confidence interval for the mean velocity. Assume that the true standard deviation of V, σ_V, is satisfactorily given by the sample standard deviation s_v.
(c) Assume that V has a log-normal distribution; determine the point estimates for the corresponding parameters λ_V and ζ_V.

5.6 A structure is designed to rest on 100 piles. Nine test piles were driven at random locations into the supporting soil stratum and loaded until failure occurred. Results are tabulated as follows.

Test pile	Pile capacity (tons)
1	82
2	75
3	95
4	90
5	88
6	92
7	78
8	85
9	80

(a) Estimate the mean and standard deviation of the individual pile capacity to be used at the site.
(b) Establish the 98% confidence interval for the mean pile capacity, assuming known $\sigma = s$.
(c) Determine the 98% confidence interval for the mean pile capacity on the basis of unknown variance.

5.7 The daily dissolved oxygen concentration (DO) for a location A downstream from an industrial plant has been recorded for 10 consecutive days.

Day	DO (mg/l)
1	1.8
2	2.0
3	2.1
4	1.7
5	1.2
6	2.3
7	2.5
8	2.9
9	1.6
10	2.2

(a) Assume that the daily DO concentration has a normal distribution $N(\mu, \sigma)$; estimate the values of μ and σ.

(b) Determine the 95% confidence interval for the true mean μ.
(c) Determine the 95% lower confidence limit of μ.

5.8 A river has the following record on the levels of floods that occurred each year between 1960 through 1970.

Year	Flood level (m) (above mean flow)
1960	3.7
1961	2.3
1962	5.1, 3.5
1963	5.2
1964	4.7, 6.1, 5.2
1965	3.4, 7.2, 1.5
1966	1.5, 3.6
1967	5.2, 1.4
1968	1.3, 4.5
1969	3.4
1970	4.4, 2.4

(a) Draw the histogram of flood levels at 1-m interval.
(b) Draw the histogram of the *annual maximum flood levels* at 1-m interval.
(c) Based on the histogram, what is the return period for a 7-m flood?
(d) Compute sample mean and sample variance of the annual maximum flood.
(e) Establish the 99% 2-sided confidence interval for the mean annual maximum flood.
(f) Assume that the annual maximum flood level has a log-normal distribution with the mean and variance computed in part (d); on this basis, determine the return period for a 7-m flood of this river.

5.9 From a set of data on the daily BOD level at a certain station for 30 days, the following have been computed:

$$\bar{x} = 3.5 \text{ (mg/l)}$$
$$s^2 = 0.184 \text{ (mg/l)}^2$$

Assume that the daily BOD level is a Gaussian variable.
(a) Estimate the mean and standard deviation of the BOD level.
(b) Determine the 99.5% confidence interval for the mean BOD.
(c) If the engineer is not satisfied with the width of the confidence interval established in part (b), and would like to reduce this interval by 10%, keeping the 99.5% confidence level, how many *additional* daily measurements have to be gathered? *Ans. 7.*

5.10 Suppose that a sample of 9 steel reinforcing bars were tested for yield strength, and the sample mean was found to be 20 kips.
(a) What is the 90% confidence interval for the population mean, if the standard deviation is assumed to be equal to 3 kips?
(b) How many additional bars must be tested to increase the confidence of the same interval to 95%? *Ans. 4.*

(c) If the standard deviation is not known, but the 9 measurements yielded

$$\sum_{i=1}^{9}(x_i - 20)^2 = 84.5$$

then what would be the 90% confidence interval for the mean yield strength? Assume that the yield strength is a normal variate.

5.11 A 20-year data series for the annual maximum wind velocity V for a city in Illinois yielded the following quantities:

$$\bar{v} = 76.5 \text{ mph}$$

$$\sum_{i=1}^{20}(v_i - \bar{v})^2 = 2640 \text{ (mph)}^2$$

(a) Determine the sample standard deviation s_v.
(b) Determine the 95% upper confidence limit for μ_V; that is,

$$P(\mu_V < \text{Limit}) = 0.95$$

assume $\sigma_V = s_v$ from part (a).
(c) Assume that the annual maximum wind velocity is a log-normal variate with $\mu_V = 76.5$ mph and $\sigma_V = s_v$ from part (a). Estimate the distribution parameters λ_V and ζ_V.

5.12 The height H of a radio tower is being determined by measuring the horizontal distance L from the center of its base to the instrument and the vertical angle β as shown in Fig. P5.12.
(a) The distance L is measured 3 times, and the readings are: 124.3, 124.2, 124.4 ft.
Determine the estimated distance, and its standard error. *Ans. 124.3 ft; 0.0577 ft.*
(b) The angle β is measured 5 times and the readings are: 40° 24.6′, 40° 25.0′, 40° 25.5′, 40° 24.7′, 40° 25.2′.
Determine the estimated angle, and its standard error. *Ans. 40°25′; 0.164′.*

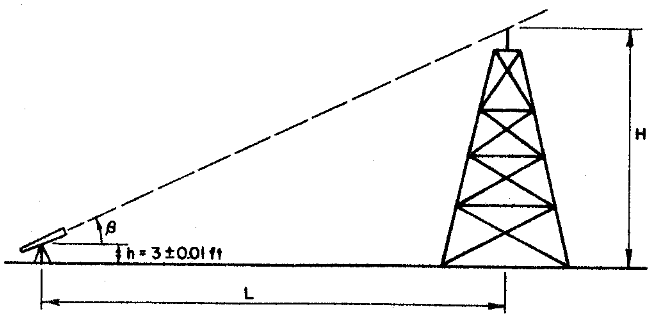


Figure P5.12

(c) Estimate the height of the tower $\bar{H}$. Assume the instrument is 3 ft high with a standard deviation of 0.01 ft. *Ans. 108.85 ft.*
(d) Compute the standard error of the estimated height of the tower, $\sigma_{\bar{H}}$. *Ans. 0.051 ft.*
(e) Determine the 98% confidence interval of the actual height of the tower H. Assume that $\bar{H}$ is normally distributed about the actual height H. *Ans. (108.73 ft; 108.97 ft).*

5.13 To determine the area of a rectangular tract of land shown in Fig. P5.13, the sides b and c were measured 5 times each. Following are the 5 independent measurements made on b and c:

Side b (m)	Side c (m)
500.5	299.8
499.5	300.3
500.0	300.2
500.2	299.7
499.8	300.0

The area of the tract is computed as

$$\bar{A} = \bar{b} \cdot \bar{c}$$

where $\bar{b}$ and $\bar{c}$ are the sample means of the respective measurements. Estimate the 95% confidence interval for the actual area A.

5.14 The following five repeated independent observations (measurements) were made on each of the outer and inner radii of a circular ring shown in Fig. P5.14.

outer radius r_1: 2.5, 2.4, 2.6, 2.6, 2.4 cm
inner radius r_2: 1.6, 1.5, 1.6, 1.4, 1.4 cm

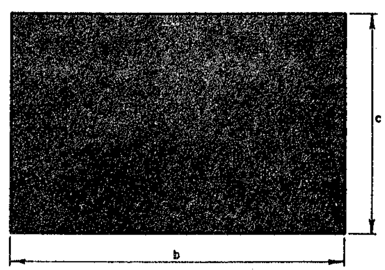


Figure P5.13

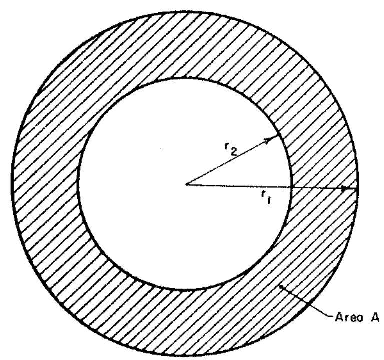


Figure P5.14

(a) Determine the best estimates of the outer and inner radii, and corresponding standard errors.
(b) The shaded area between the two concentric circles is computed based on the mean values of the measured outer and inner radii; namely $\bar{A} = \pi(\bar{r}_1^2 - \bar{r}_2^2)$. What is the computed area? *Ans. 12.57* cm^2.
(c) Determine the standard deviation (standard error) of the computed area. *Ans. 0.819* cm^2.
(d) If it is desired to determine the sample mean of r_1 within ± 0.07 cm with 99% confidence, how many additional independent measurements must be made on r_1? Assume that all measurements are independent and taken with the same care and skill. *Ans. 12.*

5.15 The distance between A and C is measured in 2 stages: namely, AB and BC as shown in Fig. P5.15. Measurements on AB and BC are recorded as follows:

AB: 100.5, 99.6, 100.1, 100.3, 99.5 ft
BC: 50.2, 49.8, 50.0 ft

(a) Compute the sample mean and sample variance of the measured distances for AB.
(b) Compute the standard error of the estimated distance of AB, that is, $s_{\overline{AB}}$.
(c) Establish the 98% confidence interval for the actual distance AB.
(d) If the distance AC is given by the sum of the estimated distances $\overline{AB}$ and $\overline{BC}$, that is,

$$AC = \overline{AB} + \overline{BC}$$

what is the standard error of the estimated total distance between A and C?
(e) Establish the 98% confidence interval on the actual length AC.

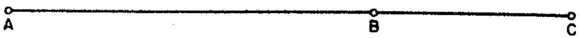


Figure P5.15

6. Empirical Determination of Distribution Models

6.1. INTRODUCTION

The probabilistic characteristics of a random phenomenon is sometimes difficult to discern or define, such that the appropriate probability model needed to describe these characteristics is not readily amenable to theoretical deduction or formulation. In particular, the functional form of the required probability distribution may not be easy to derive or ascertain. Under certain circumstances, the basis or properties of the physical process may suggest the form of the required distribution. For example, if a process is composed of the sum of many individual effects, the Gaussian distribution may be appropriate on the basis of the central limit theorem; whereas, if the extremal conditions of a physical process are of interest, an extreme-value distribution may be a suitable model.

Nevertheless, there are occasions when the required probability distribution has to be determined empirically (that is, based entirely on available observational data). For example, if the frequency diagram for a set of data can be constructed, the required distribution model may be determined by visually comparing a density function with the frequency diagram (see for example, Figs. 1.5 through 1.7). Alternatively, the data may be plotted on *probability papers* prepared for specific distributions (see Section 6.2 below). If the data points plot approximately on a straight line on one of these papers, the distribution corresponding to this paper may be an appropriate distribution model.

Furthermore, an assumed probability distribution (perhaps determined empirically as described above, or developed theoretically on the basis of prior assumptions) may be verified, or disproved, in the light of available data using certain statistical tests, known as *goodness-of-fit* tests for distribution. Moreover, when two or more distributions appear to be plausible probability distribution models, such tests can be used to delineate the relative degree of validity of the different distributions. Two such tests are commonly used for these purposes—the *chi-square* (χ^2) and the *Kolmogorov-Smirnov* (K-S) tests.

The theoretical distribution function for the proposed normal model N (77, 4.6) is also shown in the same figure. The maximum discrepancy between the two functions is $D_n = 0.16$ and occurs at $K_{Ic} = 77$ ksi $\sqrt{\text{in.}}$ In this case, there are 26 observed data points; hence the critical value of D_n^α at the 5% significance level is $D_n^{.05} = 0.265$ (obtained from Table A.4). Since the maximum discrepancy of 0.16 is less than $D_{26}^{.05} = 0.265$, the normal model N (77, 4.6) is verified at the 5% significance level.

EXAMPLE 6.9

Data for stream temperature at mile 41.83 of the Little Deschutes River in Oregon measured at 3-hr intervals over a 3-day period (August 1–3, 1969), are shown plotted in Fig. E6.9 in accordance with Eq. 6.2. The distribution function of the proposed theoretical model is also shown in the same figure.

In this case the maximum difference between $S_n(x)$ and $F(x)$ is observed to be $D_n = 0.174$ at the temperature of 70.9°F.

With $n = 23$, the critical value D_n^α at the 5% significance level, obtained from Table A.4, is $D_{23}^{.05} = 0.273$. Since $D_n < D_n^\alpha$, the proposed theoretical distribution is suitable for modeling the stream temperature of this river at the significance level of $\alpha = 5\%$.

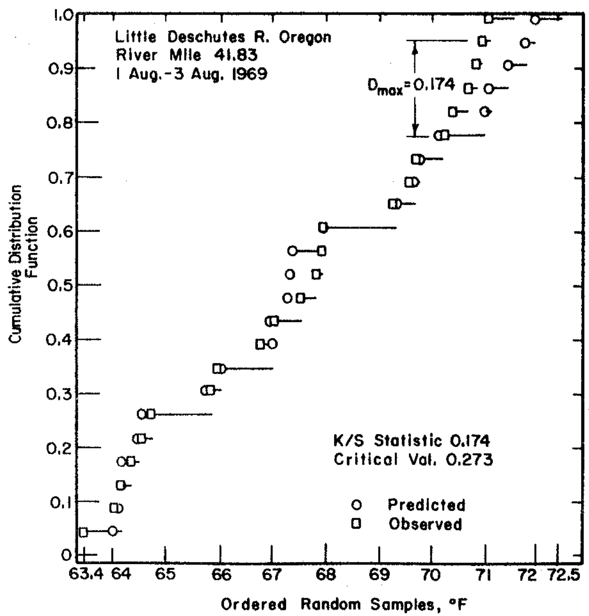


Figure E6.9 Kolmogorov-Smirnov test for proposed stream temperature prediction model (after Morse, 1972)

6.4. CONCLUDING REMARKS

Whereas Chapter 5 was concerned with the statistical estimation of parameters of a distribution, this chapter is concerned with the determination of the probability distribution for a random variable, and with questions related to the validity of an assumed distribution, based on finite samples of the population. Unless developed theoretically from physical considerations, the required distribution model may be determined empirically. One way of doing this is through the use of probability papers constructed for specific distributions. The linearity, or lack of linearity, of sample data plotted on such papers would suggest the appropriateness of a given distribution for modeling the population.

The validity of an assumed distribution may also be appraised by goodness-of-fit tests, including specifically the *chi-square* (χ^2) and the *Kolmogorov-Smirnov* (K-S) tests. Such tests, however, depend on the prescribed level of significance, the choice of which is largely a subjective matter. Nevertheless, these tests are useful for determining (in the light of sample data) the relative appropriateness of several potentially possible distribution models.

PROBLEMS

6.1 Plot the data in Example 6.1 on log-normal probability paper. Estimate the median and COV from the straight line drawn through the data points.

6.2 The ultimate strains (ε_u, in %) of 15 No. 5 steel reinforcing bars were measured. The results are as follows (data from Allen, 1972):

19.4	17.9	16.1
16.0	17.8	16.8
16.6	18.8	17.0
17.3	20.1	18.1
18.4	19.1	18.6

Plot these data on both the normal and the log-normal probability papers, and discuss the results.

6.3 The shear strengths (in kips per square feet, ksf) of 13 undisturbed samples of clay from the Chicago subway project are tabulated as follows (data from Peck, 1940):

0.35	0.42	0.49	0.70	0.96
0.40	0.43	0.58	0.75	
0.41	0.48	0.68	0.87	

Plot the data on log-normal probability paper. Estimate the parameters of the log-normal distribution to describe the shear strength of Chicago clay.

6.4 For the wind velocity data in Problem 3.26 (of Chapter 3), plot the data on normal probability paper. Determine the normal distribution for describing the wind velocity.

6.5 A random variable with a triangular distribution between a and $a + r$, as shown in Fig. P6.5, is given by the density function

$$f_X(x) = \frac{2(x-a)}{r^2}; \qquad a \le x \le a + r$$
$$= 0; \qquad \text{elsewhere.}$$

(a) Determine the appropriate standard variate S for this distribution.
(b) Construct the corresponding probability paper. What do the values of X at $F_S(0)$ and $F_S(1.0)$ on this paper mean?
(c) Suppose the following sample values were observed for X.

36	32	34	71
18	69	45	66
56	71	53	58
64	50	55	53
72	28	62	48
			75

Plot the above set of data on the triangular probability paper. From this plot estimate the minimum and maximum values of X.

6.6 The density function of the Rayleigh distribution is given by

$$f_X(x) = \frac{x}{\alpha^2} e^{-\frac{1}{2}(x/\alpha)^2}; \qquad x \ge 0$$
$$= 0; \qquad x < 0$$

in which the parameter α is the modal (or most probable) value of X.
(a) Construct the probability paper for this distribution. What does the slope of a straight line on this paper represent?
(b) The following is a set of data for strain range induced by vehicle loads on highway bridge members.

Strain Range, in micro in./in. (Data courtesy of W. H. Walker)

48.4	52.7	42.4
47.1	44.5	146.2
49.5	84.8	115.2
116.0	52.6	43.0
84.1	53.6	103.6
99.3	33.5	64.7
108.1	43.8	69.8
47.3	56.3	44.0
93.7	34.5	36.2
36.3	62.8	50.6
122.5	180.5	167.0

Plot this set of data on the Rayleigh probability paper constructed in part (a).
(c) What inference can you draw regarding the Rayleigh distribution as a model for live-load stress range in highway bridges, in light of the data plotted above? Determine the most probable strain range (if possible) from the results of part (b).

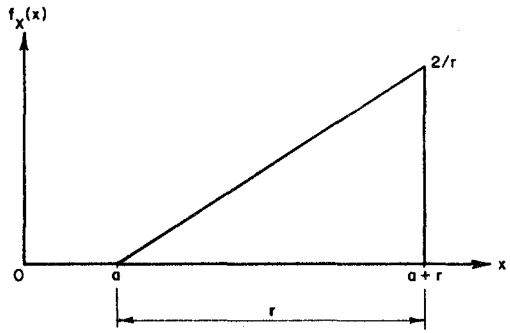


Figure P6.5

6.7 Time-to-failure (or malfunction) of a certain type of diesel engine has been recorded as follows (in hours).

0.13	121.58	2959.47	102.34
0.78	672.87	124.09	393.37
3.55	62.09	85.28	184.09
14.29	656.04	380.00	1646.01
54.85	735.89	298.58	412.03
216.40	895.80	678.13	813.00
1296.93	1057.57	861.93	239.10
952.65	470.97	1885.22	2633.98
8.82	151.44	862.93	658.38
29.75	163.95	1407.52	855.95

(a) Construct the exponential probability paper (see Example 6.4) and plot on it the data given above.
(b) On the basis of the results of part (a), estimate the minimum and mean time-to-failure of such engines.
(c) Perform a chi-square test to determine the validity of the exponential distribution at the 1% significance level.

6.8 The following are observations of the number of vehicles per minute arriving at an intersection from a one-way street:

0, 3, 1, 2, 0, 1, 1, 1, 2, 0, 1, 4, 3, 1, 1, 0, 0, 1, 0, 2

Perform a chi-square test to determine if the arrivals can be modeled by a Poisson process, at the 1% significance level.

6.9 Cars coming toward an intersection are required to stop at the stop sign before they find a gap large enough to cross or to make a turn. This acceptance gap G, measured in seconds, varies from driver to driver, since some drivers may be more alert or more risk-taking than others. The following are measurements taken for several similar intersections.

Acceptance gap size (sec.)	Observed number of drivers
0.5–1.5	0
1.5–2.5	6
2.5–3.5	34
3.5–4.5	132
4.5–5.5	179
5.5–6.5	218
6.5–7.5	183
7.5–8.5	146
8.5–9.5	69
9.5–10.5	30
10.5–11.5	3
11.5–12.5	0

(a) Plot a histogram for the acceptance gap size.
(b) Assume that the distribution of G is normal; estimate its mean and variance. You may assume that all observations in each interval have the gap length equal to the average gap length for that interval. For example, for the interval 1.5–2.5, it may be assumed that there are 6 observations with gap length of 2.0 sec.
(c) Perform a chi-square goodness-of-fit test at the 1% significance level.

6.10 An extensive series of ultimate load tests on reinforced concrete columns was carried out at the University of Illinois (Hognestad, 1951). The ratio ϕ of the actual ultimate load to that computed by the appropriate ACI 318-63 formula, without consideration of the understrength factor in the ACI code, is tabulated below (for part of the 84 square tied columns tested).

Table of ratio ϕ	
0.79976	0.99410
0.82395	0.93811
0.99938	0.81649
0.78017	0.87551
0.91342	0.95705
0.90304	0.92863
0.86011	0.93054
1.01836	1.03065
0.90133	

(a) Plot the data on normal probability paper, and estimate (if possible) the mean and standard deviation from this plot.
(b) Perform a chi-square test at the 5% significance level on the fitted normal distribution.
(c) Repeat part (b) using the Kolmogorov-Smirnov test.

6.11 Data on the rate of oxygenation K in streams at 20°C have been observed at the Cincinnati Pool, Ohio River, and summarized in the following table (data from Kothandaraman, 1968).

K (per day)	Observed frequency
0.000 to 0.049	1
0.050 to 0.099	11
0.100 to 0.149	20
0.150 to 0.199	23
0.200 to 0.249	15
0.250 to 0.299	11
0.300 to 0.349	2

A normal distribution with a mean oxygenation rate of 0.173 per day and a standard deviation of 0.066 per day (both values are estimated from observed data) is proposed to model the oxygenation rate at the Cincinnati Pool, Ohio River.

Perform a chi-square test on the goodness of fit of the proposed distribution at the 5% significance level.

6.12 On the basis of the data given in Problem 6.2 and using the Kolmogorov-Smirnov method, determine which of the two distributions (normal and log-normal) considered in Problem 6.2 is a better model for the distribution of the ultimate strains of steel reinforcing bars.

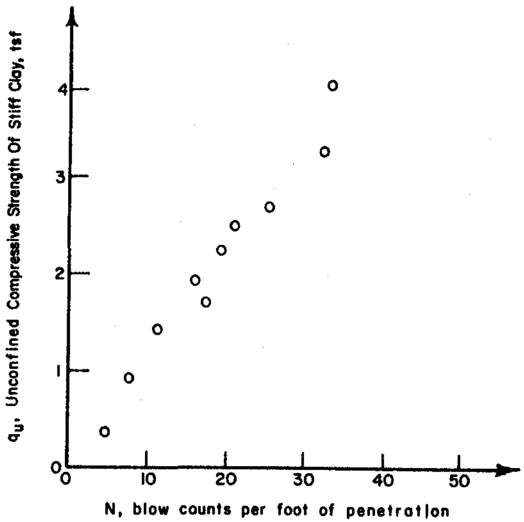


Figure E7.7 Unconfined compressive strength vs. blow counts for stiff clay

confined compressive strength of stiff clay; on this basis, therefore, the blow count may be used to estimate the unconfined strength of stiff clay.

EXAMPLE 7.8

For the data recorded on the Monocacy River (described earlier in Examples 5.8 and 7.2), estimate the correlation coefficient between runoff and precipitation.

Based on the computations tabulated earlier for Example 5.8, we obtain the sample variance of precipitation $s_x^2 = 1.53$; and the sample variance of runoff $s_y^2 = 0.36$. From the calculations in Example 7.2, we also have

$$\sum_{i=1}^{25} x_i y_i - 25\bar{x}\bar{y} = 59.24 - 25(2.16)(0.80) = 16.04$$

Hence

$$\hat{\rho} = \frac{(1/24)(16.04)}{\sqrt{1.53}\sqrt{0.36}} = 0.90$$

The correlation coefficient is required when calculating the joint probabilities of two or more random variables that are jointly normal (see Example 3.25). However, for non-normal variates the quantitative role of the correlation coefficient in the computation of joint probabilities is seldom defined. Nevertheless, the correlation coefficient is a measure of linear interdependency between two random variables irrespective of their distributions.

Multiple correlation. When more than two random variables are involved, as in the case of multiple linear regression of Eq. 7.16, any pair of variables may be mutually correlated, for example, between X_i and X_j, or between Y and X_i; the corresponding correlation coefficients are

$$\rho_{X_i,X_j} = \frac{E[(X_i - \mu_{X_i})(X_j - \mu_{X_j})]}{\sigma_{X_i}\sigma_{X_j}} \tag{7.25}$$

and can be estimated as

$$\hat{\rho}_{X_i,X_j} = \frac{1}{n-1}\,\frac{\left(\sum_{k=1}^{n} x_{ik}x_{jk} - n\bar{x}_i\bar{x}_j\right)}{s_{x_i}s_{x_j}} \tag{7.26}$$

7.6. CONCLUDING REMARKS

The statistical method for determining the mean and variance of one random variable as a function of the values of other variables is known as *regression analysis*. On the basis of the least-squares criterion, regression analysis provides a systematic approach for the empirical determination of the underlying relationships among the random variables. Furthermore, the associated correlation analysis determines the degree of linear interrelationship between the variables (in terms of the correlation coefficient); a high correlation means the existence of a strong linear relationship between the variables, whereas a low correlation would mean the lack of linear relationship (however, there could be a nonlinear relationship).

Regression and correlation analyses have applications in many areas of engineering, and are especially significant in situations where the necessary relationships must be developed empirically.

PROBLEMS

7.1 Assume hypothetically that the concentration of dissolved solids and the turbidity of a stream are measured simultaneously for five separate days, selected at random throughout a year. The data are as follows.

Day	Dissolved solids (mg/l)	Turbidity (JTU)
1	400	5
2	550	30
3	700	32
4	800	58
5	500	20

Because turbidity is easier to measure, a regression equation may be used to predict the concentration of dissolved solids on the basis of known turbidity. Assume that the variance of dissolved solid concentration is constant with turbidity.

(a) What are the values of the intercept and slope parameters (α and β) of the regression line? *Ans. 364.1; 7.79.*

(b) Estimate the standard deviation of dissolved solid concentration about the regression line. *Ans. 58.8.*

7.2 Suppose that data on the consumption of water per capita per day have been collected for four towns in the Midwest and tabulated as follows (see also Fig. P7.2).

Town	x Population (in 10^4)	y Per capita water consumption (in 100 gal/day)
1	1.0	1.0
2	4.0	1.3
3	6.0	1.3
4	9.0	1.4

(a) If the effect of population size of a town on the per capita consumption is neglected, determine the sample variance s_y^2.

(b) From the observed data, there seems to be a general trend that the per capita water consumption increases with the population of the town. Suppose it is assumed that

$$E(Y \mid x) = \alpha + \beta x$$

and Var $(Y \mid x)$ is constant for all x.

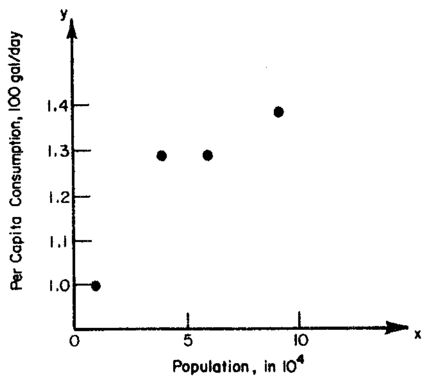


Figure P7.2

(i) Determine the least-squares estimates for α and β.

(ii) Estimate $s^2_{Y|x}$.

(c) An engineer is interested in studying the consumption of water in Urbana (a town with 50,000 population). Assume a normal distribution for Y; determine the probability that the demand for water in Urbana will exceed 7,000,000 gal/day.

7.3 Dissolved oxygen (DO) concentration (in parts per million, ppm) in a stream is found to decrease with the time of travel downstream (Thayer and Krutchkoff, 1966). Assume a linear relationship between the mean DO and the time of travel t. Determine the least-squares regression equation and estimate the standard deviation about the regression line from the following set of observations.

DO (ppm)	Time of travel t (days)
0.28	0.5
0.29	1.0
0.29	1.6
0.18	1.8
0.17	2.6
0.18	3.2
0.10	3.8
0.12	4.7

7.4 From a survey of the effect of fare increase on the loss in ridership for transit systems throughout the United States, the following data were obtained.

X Fare increase (%)	Y Loss in ridership (%)
5	1.5
35	12.0
20	7.5
15	6.3
4	1.2
6	1.7
18	7.2
23	8.0
38	11.1
8	3.6
12	3.7
17	6.6
17	4.4
13	4.5
7	2.8
23	8.0

(a) Plot the percent loss in ridership versus the percent fare increase.
(b) Perform a linear regression analysis to determine the expected percent loss in ridership as a function of the percent fare increase.
(c) Estimate the conditional standard deviation $s_{Y|x}$. *Ans. 0.82.*
(d) Evaluate the correlation coefficient between X and Y. *Ans. 0.97.*

7.5 Data for per capita energy consumption and per capita GNP output for eight different countries are tabulated below (data extracted from Meadows et al., 1972).

X	Y
600	1,000
2,700	700
2,900	1,400
4,200	2,000
3,100	2,500
5,400	2,700
8,600	2,500
10,300	4,000

Note that

X = GNP in U.S. dollar equivalent per person per year
Y = energy consumption in kilograms of coal equivalent per person per year

(a) Plot the data given above in a two-dimensional graph.
(b) Determine the correlation between GNP and energy consumption.
(c) Determine the regression for predicting energy consumption on the basis of per capita GNP output. Draw the regression line on the graph of part (a).
(d) Evaluate the conditional standard deviation $s_{Y|x}$, and sketch the $\pm s_{Y|x}$ band about the regression line of part (c).
(e) Similarly, determine the regression equation for predicting GNP on the basis of energy consumption, and display this graphically with the corresponding $\pm s_{X|y}$ band.

7.6 A tensile load test was performed on an aluminum specimen. The applied tensile force and the corresponding elongation of the specimen at various stages of the test are recorded as follows.

Tensile force (kips) x	Elongation (10^{-3} in.) y
1	9
2	20
3	28
4	41
5	52
6	63

(a) Assume that the force-elongation relation of aluminum over this range of loads is linear. Determine the least-squares estimate for the Young's modulus of this aluminum specimen. The cross-sectional area of the specimen is 0.1 sq in., and the length of the specimen is 10 in. Young's modulus is given by the slope of the stress-strain curve.
(b) In addition to the assumption of a linear relationship between force and elongation, suppose zero elongation should correspond to zero tensile force; that is, the regression line is assumed to be

$$E(Y \mid x) = \beta x$$

What would be the best estimate of Young's modulus in this case?

7.7 The population in a community for the years 1962 to 1972 is tabulated as follows.

Year	Population
1962	24,010
1963	24,540
1964	24,750
1965	25,100
1966	25,340
1967	25,820
1968	26,100
1969	26,200
1970	26,500
1971	26,800
1972	27,450

It is suggested that the population in a given year will depend on the population of the previous year, as predicted from the following model:

$$x_t = a + bx_{t-1} + \varepsilon$$

where x_t and x_{t-1} are the population in the tth and $(t-1)$th year, respectively, and ε is a normal random variable with zero mean and standard deviation σ.

(a) Based on the given population data, determine the estimates for a, b, and σ.
(b) Based on the population model and the estimates from part (a), estimate the population in 1973. What is the probability that the population in 1973 will exceed 28,000?

7.8 The peak-hour traffic volume and the 24-hour daily traffic volume on a toll bridge have been recorded for 14 days. The observed data are tabulated as follows.

X Peak-hour traffic volume (in 10^3)	Y 24-hr traffic volume (in 10^4)
1.4	1.6
2.2	2.3
2.4	2.0
2.7	2.2
2.9	2.6
3.1	2.6
3.6	2.1
4.1	3.0
3.4	3.0
4.3	3.8
5.1	5.1
5.9	4.2
6.4	3.8
4.6	4.2

Assume that the conditional standard deviation $s_{Y|x}$ varies quadratically with x from the origin.

(a) Determine the regression line $E(Y \mid x) = \hat{\alpha} + \hat{\beta}x$.

(b) Estimate the prediction error about the regression line, that is, $s_{Y|x}$.

(c) If the peak-hour traffic volume on a certain day is measured to be 3500 vehicles, what is the probability that more than 30,000 vehicles will be crossing the toll bridge that day?

7.9 The average durations (in days) of frost condition each year at 20 stations in West Virginia were compiled as follows (from Moulton and Schaub, 1969).

Weather station	Elevation (ft)	North latitude (deg)	Average duration of frost (days)
Bayard	2375	39.27	73.0
Brandywine	1586	38.63	29.0
Buckhannon	1459	39.00	28.0
Cairo	680	39.17	25.0
Charleston	604	38.35	11.5
Fairmont	1298	39.47	32.5
Flat Top	3242	37.58	64.0
Gary	1426	37.37	13.0
Kearneysville	550	39.38	23.0
Lewisburg	2250	37.80	37.0
Madison	675	38.05	26.0
Marlington	2135	38.23	73.0
New Martinsville	635	39.65	24.7

Continuation

Weather station	Elevation (ft)	North latitude (deg)	Average duration of frost (days)
Parsons	1,649	39.10	41.0
Pickens	2727	38.66	56.0
Piedmont	1053	39.48	34.0
Rainelle	2424	37.97	37.0
Spencer	789	38.80	16.0
Wheeling	659	40.10	41.0
Williamson	673	37.67	12.0

Perform a multiple linear regression analysis to predict the average duration of frost at a locality in terms of its elevation and latitude.

7.10 The difference between the photogrammetrically triangulated elevation—before adjustment—and the terrestrially determined elevation is an example of measurement error in photogrammetry. This error in elevation E has been observed and theoretically shown to be a nonlinear function of the distance X along the centerline of a triangulated strip as follows:

$$E = a + bx + cx^2$$

Estimate the least-squares values of a, b, and c on the basis of the following measurements. *Ans.* -0.023; 0.235; -0.347.

Distance along centerline of triangulated strip X (km)	Error in elevation E (m)
0	0
0.5	0
1.2	−0.3
1.7	−0.6
2.4	−1.4
2.7	−2.0
3.4	−3.1
3.7	−4.0

7.11 The average distance Y required for stopping a vehicle is a function of the speed of travel of the vehicle. The following set of data were observed for 10 cars at varying speeds.

Car	Speed (mph)	Stopping distance (ft)
1	25	46
2	5	6
3	60	110
4	30	46
5	10	13
6	45	75
7	15	16
8	40	70
9	45	90
10	20	20

(a) Plot the stopping distance vs. speed.

(b) Assume that the mean stopping distance varies linearly with the speed, that is,

$$E(Y \mid x) = \alpha + \beta x$$

Estimate α and β; and $s_{Y|x}$, which may be assumed to be constant.

(c) A nonlinear function is suggested to model the stopping distance-speed relationship as follows:

$$E(Y \mid x) = a + bx + cx^2$$

Estimate a, b, and c; and $s_{Y|x}$, which is assumed to be constant with x.

(d) Plot the two regression curves obtained from parts (b) and (c). Compare the relative accuracy of prediction between these two models.

7.12 The mean rate of oxygenation from the atmospheric reaeration process for a stream depends on the mean velocity of stream flow and average depth of the stream bed. The following are data recorded in 12 experiments (Kothandaraman, 1968).

Mean oxygenation rate X (ppm per day)	Mean velocity V (ft/sec)	Mean depth H (ft)
2.272	3.07	3.27
1.440	3.69	5.09
0.981	2.10	4.42
0.496	2.68	6.14
0.743	2.78	5.66
1.129	2.64	7.17
0.281	2.92	11.41
3.361	2.47	2.12
2.794	3.44	2.93
1.568	4.65	4.54
0.455	2.94	9.50
0.389	2.51	6.29

Suppose that the following relationship is used to estimate the mean oxygenation rate

$$E(X \mid V, H) = \alpha V^{\beta_1} H^{\beta_2}$$

Estimate α, β_1, and β_2 on the basis of the observed data.

7.13 The compressive and flexural strengths of nonbloated burned clay aggregate concrete are measured for 30 specimens after 7 days of curing. The data are (from Martin et al., 1972) as follows.

	7-day compressive strength, X (psi)	7-day flexural strength, Y (psi)
1	1400	257
2	1932	327
3	2200	317
4	2935	300
5	2665	340
6	2800	340
7	3065	343
8	3200	374
9	2200	377
10	2530	386
11	3000	383
12	2735	393
13	2000	407
14	3000	407
15	3235	407
16	2630	434
17	3030	427
18	3065	440
19	2735	450
20	3835	440
21	3065	456
22	3465	460
23	3600	456
24	3260	476
25	3500	480
26	3365	490
27	3335	497
28	3170	526
29	3600	546
30	4460	700

(a) Plot the compressive strength vs. flexural strength on a two-dimensional graph.

(b) Determine the correlation coefficient between the two strengths.

7.14 The settlement of a footing depends on that of the adjacent footing since they are subjected to similar load and soil conditions. Therefore some correlation

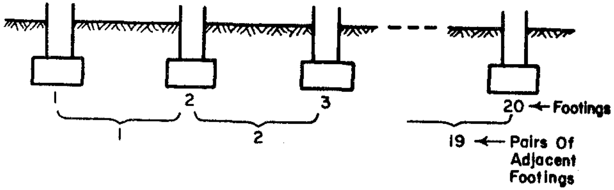


Figure P7.14

exists between the settlement behavior of two adjacent footings. The following is a set of data on the settlement of a series of footings on sand.

Footing	Settlement (in.)	Footing	Settlement (in.)
1	0.59	11	0.93
2	0.60	12	0.78
3	0.54	13	0.78
4	0.70	14	0.77
5	0.75	15	0.79
6	0.80	16	0.79
7	0.79	17	0.78
8	0.95	18	0.77
9	1.00	19	0.63
10	0.92	20	0.73

From a row of 20 footings, 19 pairs of adjacent footings can be obtained as shown in Fig. P7.14. The degree of dependence between the settlements of adjacent footings is described by the correlation coefficient.

(a) Estimate this correlation based on the 19 pairs of data. *Ans. 0.766.*
(b) Estimate the coefficient of variation of the settlement of a footing. *Ans. 0.157.*

8. *The Bayesian Approach*

8.1. INTRODUCTION

In Chapter 5 we presented the methods of point and interval estimation of distribution parameters, based on the *classical statistical* approach. Such an approach assumes that the parameters are constants (but unknown) and that sample statistics are used as estimators of these parameters. Because the estimators are invariably imperfect, errors of estimation are unavoidable; in the classical approach, confidence intervals are used to express the degree of these errors.

As implied earlier, accurate estimates of parameters require large amounts of data. When the observed data are limited, as is often the case in engineering, the statistical estimates have to be supplemented (or may even be superseded) by judgmental information. With the classical statistical approach there is no provision for combining judgmental information with observational data in the estimation of the parameters.

For illustration, consider a case in which a traffic engineer wishes to determine the effectiveness of the road improvement at an intersection. Based on his experience with similar sites and traffic conditions, and on a traffic-accident model, he estimated that the average occurrences of accidents at the improved intersection would be about twice a year. However, during the first week after the improved intersection is opened to traffic, an accident occurs at the intersection. A dichotomy, therefore, may arise: The engineer may hold strongly to his judgmental belief, in which case he would insist that the accident is only a chance occurrence and the average accident rate remains *twice a year*, in spite of the most recent accident. However, if he only considers actual observed data, he would estimate the average accident rate to be *once a week*. Intuitively, it would seem that both types of information are relevant and ought to be used in determining the average accident rate. Within the classical method of statistical estimation, however, there is no formal basis for such analysis. Problems of this type are formally the subject of Bayesian estimation.

The *Bayesian method* approaches the estimation problem from another point of view. In this case, the unknown parameters of a distribution are assumed (or modeled) to be also random variables. In this way, uncertainty associated with the estimation of the parameters can be combined formally